

THE
Modern Navigator's
Compleat TUTOR:

PART II.

Being a Comprehensive

TREATISE
OF

Plain and Spherical TRIGONOMETRY
shewing the different Ways of the
Solution of their several Cases; with the
Demonstration of their Axioms.

Also ASTRONOMY, the Various Questions of
Sun and Fixed Stars, Plainly laid down and Solved
Geometrically, Logarithmically, Instrumentally and
Tabularly. With several Methods to know the
Fixed Stars. And the Projection of the
on any Great Circle.

SPHERE

To which are Annexed,

The Use of the Celestial and Terrestrial Globes,
in Problems of Geography, Navigation, Astronomy,
&c. Great Circle Sailing. Astronomical Paradoxes
answer'd, Use of the Celestial Hemispheres, &c.
The whole made easy to the meanest Capacity, in a
Method not heretofore extant.

By JOSHUA KELLY, Teacher of the Mathematicks.

LONDON,

Printed for W. Mount and T. Page on Tower-Hill; J. Wilcox,
against the New Church in the Strand; J. Clarke, in
Duck-Lane; J. Eade, near King Edward's Stairs, and
B. Macey, on Hermitage-Bridge in Wapping. 1734.

Advertisement.

IN Broad-Street, Wapping, near Wapping New Stairs, are taught these following *Mathematical Sciences*,

V I Z.

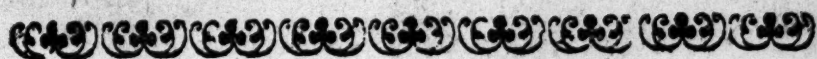
ARITHMETICK,	* DIALLING,
MERCHANTS ACCTS.	* SURVEYING,
GEOMETRY,	* GAUGING,
TRIGONOMETRY,	* GUNNERY,
NAVIGATION,	* FORTIFICATION,
ASTRONOMY,	* ALGEBRA.

The Use of the *GONES*, and all other *Mathematical Instruments*; the *Projection of the Sphere* on any Circle,

By JOSHUA KELLY.

With whom Young Gentlemen and others are well Boarded, and Completely and Expeditionly Quality'd (on Reasonable Terms) for any Business relating to *Accompts* and the *Mathematicks*.





TO THE
MARINERS
OF

Great-Britain *and* Ireland.

GENTLEMEN,



THE Kind Acceptance my former Treatise of *Navigation* hath met with for its usefulness and plainness in the Theory and Practice of this Art, hath encouraged the *Booksellers* and *Proprietors* thereof, at a much greater Charge to Re-print this Second Edition with some Additions and Corrections ; Which hath induc'd me to Add this Second Part or Volume, and to make it as plain and useful as the former Part; not doubting but it will be as well received by You, whose Acknowledgement, and Encouragement will appear by the quick Sale of this. That Prosperity may attend your Sails, and all your Laudable Studies, and Undertakings, is the Hearty Prayer of,

GENTLEMEN,

From my School in Broad-
Street, Wapping,
May 8, 1733.

Your Obliged Servant,

Joshua Kelly.



and I am sure that the
and your own and the
by the way, I am sure
Admitted that the
log but I am sure
it is as plain as the
to add to the
Additional
Charge to the
Change to the

4115

11

THE CONTENTS.

C A A P. I. S E C T. I.

- §. I. **P**lain Trigonometry Rectangular constructed Geometrically, and performed, or solved Mechanically, Literally, Logarithmically, in all the Varieties, of making each Side the Radius of a Circle; also, Instrumentally, by Gunter's Scale, and Arithmetically, by the Natural Sines, (called by some, Given Numbers) and by the Square Root. Page 1
- II. A Triangular Synopsis or View of all the Variety of Propositions, for the Six Cases of Right-Angled-Triangles foregoing. 2

S E C T. II.

- III. Plain Trigonometry Oblique, performed Geometrically, Logarithmically, Instrumentally, and by Natural Sines, Page 23
- IV. A Synopsis or View of Proportions for the Solution of the four Cases of Oblique Plain Triangles, 34

C H A P. II.

- §. V. **S**pherical Geometry; or Stereographick Problems, plainly laid down, in order to Construct Spherical Triangles, or Astronomical Problems, 35

THE CONTENTS.

CHAP. III. SECT. I.

- VI. *Spherick Triangles. I. A Synopsis of the Proportions (or the Literal and Verbal Solutions) of the 16 Cases of Right-Angled Spherick Triangles, according to the Globical Projection of the Sphere, with proper Schemes for the same,*
Page 59

SECT. II.

- VII. *The same performed more easily by the Lord Napair's Catholick or Universal Proposition with Schemes, and Solutions to all the Cases.* 74

CHAP. IV. SECT. I.

- §. VIII. *Right-angled Spherick Triangles, wrought Geometrically, Logarithmically, and Instrumentally,* 81

SECT. II.

- IX. *The Doctrine of Oblique-Spherick Triangles, Logarithmically, and Instrumentally in all their Varieties.* 103

CHAP. V. SECT. I.

- §. X. *Problems in Astronomy, relating to the Sun and fixed Stars, of the various Positions of the Sphere, the five Zones, the different Distinctions of the Inhabitants, and to find by a Map where the Antæci, Peræci and Antipodes, of any given Place are, with Geographical Notes, &c.*
130

SECT. II.

THE CONTENTS

SECT. II.

- XI. *The Doctrine of the Sphere, or Practical Astronomy, or Spherical Trigonometry applied, in great Variety of Astronomical Problems, relating to the Sun and fixed Stars, performed Geometrically, Logarithmetically and Instrumentally,* 138

SECT. III.

- XII. *The Various Questions in Oblique Astronomy solved by proper Axioms, both Geometrical, and Logarithmetical,* 171

CHAP. VI. SECT. I.

- XIII. *To find the Time of the Rising and Setting of the fixed Stars, their Amplitudes, Azimuth, Hour of the Night, &c.* 204

SECT. II.

- XIV. *Various Critical and curious Questions, of the fixed Stars in an Oblique Spherical Triangle solved.* 211

SECT. III.

- XV. *Problems of the Sun and Fixed Stars, Projected upon the Plains of the other Great Circles, viz. the Equinoctial, Horizon, Ecliptick, Prime, Vertical, &c.* 229

SECT.

The C O N T E N T S.

S E C T. III.

- XVI. *The Practical Questions in Astronomy solved by Inspection in the Tables by a general Example,* 247

S E C T. IV.

- XVII. *Of the Poetical Rising and Setting of the Stars, viz. Cosmical, Achronical, Heliacal,* 253

S E C T. IV.

- XVIII. *Of the Images, or Constellations upon the Celestial Globe, Planets, and Circles of the Sphere described in Verse for the help of the Learners Memory,* 256

S E C T. V.

- XIX. *A Table of the Longitude, Latitude and Magnitudes of the fixed Stars Rectify'd to the Year 1733, with their nature, Substance, Number, Parallax, Distance, &c.* 260

- XX. *Problems in Astronomy, wherein by Construction and Calculation, are found, 1. the Right Ascension of the Medium Cæli, what Point of the Ecliptick Culminates, and its Altitude, also what Angle it makes with the Meridian, 2. the Altitude, and Azimuth of the Nonagesime Degree, the Points Ascending and Descending, and the Angle Orient, or Angle of the Ecliptick and Horizon,* 266

- XXI. *The Altitude, and Azimuth of a Planet, Comet, or new Star together with the Time of the Night given to find its Destination Right Ascension, Latitude, and Longitude,* 268

The CONTENTS.

- XXII. *By the Right, Ascension and Declination, or the Latitudes and Longitudes of two Stars, Given, to find their Distance asunder,* 272

S E C T. VI.

- XXIII. *To know the Stars by the Help of a Planisphere and a straight Stick,* 273

C H A P. VII.

- §. XXIV. *To Project the Sphere, on a Plain,* 1. *Upon the Plain of the General Meridian, or Solstitial, Colure,* 2. *On the Plain, of Equinoctial.* 3. *On the Plain of the Horizon.* 4. *On the Plain of the Ecliptick.* 5. *On the Plain of the Prime Vertical.* 6. *Upon an Oblique Circle, which Demonstrates the Difficult part of Dialing,* 276
- XXV. *To Project the Sphere upon one of the Lesser Circles,* 298
- XXVI. *To Project an Eighth Part of the Surface of the Globe on a Plain,* Page 302
- XXVII. *To Project the Sphere Orthographically, on the Plain of the Equator,* 304
- XXVIII. *To Project the Sphere Orthographick, on the Plain of the General Meridian or solstitial Colure, &c.* 306

C H A P. VIII.

- §. XXIX. *The Demonstration of the Axioms used in Oblique Spherick Triangles,* 313

C H A P. IX.

- XXX. *Problems in Geograghy, and Great Circle Sailing; wherein is shewed the great Use and Advantage thereof in Navigation, &c.* 328

The CONTENTS.

CHAP. X.

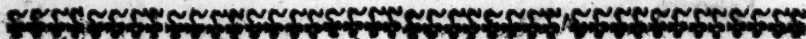
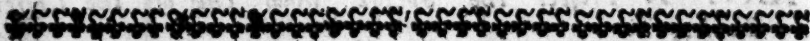
XXXI. *The Use of the Terrestrial, and Celestial Sailing in the various Problems of Geography, Navigation and Astronomy,* 336

CHAP. XI.

XXXII. *Astronomical Paradoxes propos'd and answer'd,*

CHAP. XII.

XXXIII. *The Use of the Celestial Hemispheres.*



E R R A T A

PAGES 7, 12. under Gunter's Scale Instrumental, read, *The Extent from — to — will reach from — to —*; p. 10. Title, for *Triangular*, r. *Rectangular*, p. 26. l. 25. r. *Angle A*; p. 21. l. 7. for *S d* r. *said*; p. 39. l. 10. r. *as in Prob. 8*; p. 42. l. 4. r. *Angle suppose of*; p. 45. l. 10. r. *Prob. 9*; p. 57. l. 4. r. *Equal to the Angles*; p. 61. l. 31. r. *D—C*; p. 65. l. 22. for *F. r. R*; p. *ibid.* a. 5. *Prob.* p. 81. l. 33. r. *Prob. 9. Geom.* p. 115. l. 3. r. *from*; p. *ibid.* l. 5. r. *Prob. 9*; p. 126. l. 2. r. *Case 5*; p. 130. l. 23. for *Q E r. O E*; p. 132. l. 30. r. *March 9*; p. 142. l. 22. r. *Extent*; p. 161. l. 34. r. *Five*; p. 170. l. 14. r. *the Year*; p. 190. l. 1. r. *Axiom 7. Page 323. following*; l. 42. r. *Prob. 13*; p. 217. r. *Case 4. of Oblique Astronomy*; p. 222. f. 6. r. *look further on Page 276*; p. 313. l. 25. for *Cases*, r. *are in proportion to their Opposite Angles*, and *contra*.

B R A T A





*An Explanation of the Symbols or Characters us'd
in this Treatise (for brevity sake.)*

$+$	Signifies	More, or add; as, $A + B$, is A more B.
$-$		Less, as subtract; as, $A - B$, is A less B.
$\times$		In, or multiply; as $A \times B$, is A multiply'd by B.
$\div$		By, or divide; as $A \div B$, is A divided by B.
$∴$		Rule of Three; $A ∴ B ∴ C ∴ D$, i.e. or Direct Proportion; as A is to B, so is C to D.
S.		Sine.
S.C.		Sine Complement.
T.		Tangent.
T.C.		Tangent Complement.
Sec.		Secant.
Sec.C.		Secant Complement.
R.		Radius or Sine of 90 Degrees.
Z.		Sum.
x		Difference.
$=$		Equality; as $A = B$, is equal to B.
$\sqrt{\quad}$		Radicality, or Square Root; as, $\sqrt{144}$ 12 i.e. the Square Root of 144 is 12.
$\angle$		Angle.



THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO
LIBRARY

ALPHABETICALLY BY AUTHOR
AND BY TITLE
AND BY SUBJECT



+
1
X
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100

THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO



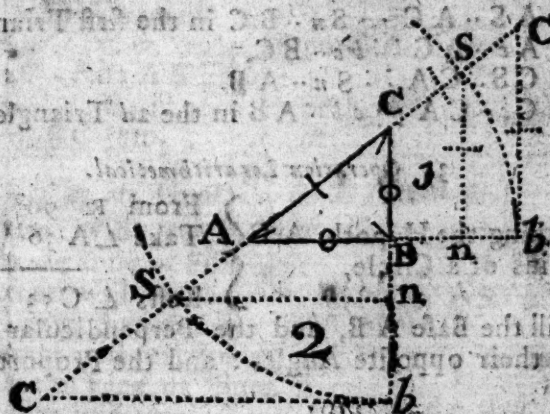
Plain Trigonometry,



N the Right-lin'd Right-angl'd Plain Triangle A B C. (Right-angled at B) there is

Given, the Hypotenuse AC 95m
Parts, and the Angle at A. (or BAC)
 $36^{\circ} 34'$
Requir'd, the Base AB, and Perpendi-
cular BC.

1. Construction Geometrical



1. With 60 Degrees of your Line of Chords, on the Points A C (of the given Line ab), describe the Arch AS; and from the same Line of Chords make the Arch AS.

PART II. A 3

Plain Trigonometry Rectangular.

b S equal to the given Angle *A*, *viz.* $36^{\circ} 34'$. 2. Draw the Hypothenuſal Line *AC* from the Point *A*, thro' *S*, and produce it both ways to *CC*. 3. Make *AC* from any Line of equal Parts equal to 95 Parts (or Miles). 4. From the Point *C* (by Prob. 13. Geom. Part I.) let fall the Perpendicular *Cb*, cutting the Line *A* *b* in *B*; ſo ſhall *AB* be the Length of the Baſe, and *BC* the Length of the Perpendicular required: Which being meaſured by the ſame Line of Equal Parts you took of *AC* from, will give you $36 =$ Parts or Miles for the Baſe *AB*, and $57 =$ Parts or Miles for the Length of the Perpendicular *BC*; which I call the

2. Mensuration Mechanical.

But to find a more exact Answer by Logarithm-Numbers, produce A C both Ways to c , and c A B to b , and C B to b , the Radius of the Circle (or Chord of 60° .) and erect the Perpendicular bc , bc ; and from the Points S S draw the Parallels there to S_n , S_n , in both the first and second Triangles, marked 1, 2. Then 'tis plain by the Definition of a Sine-Tangent and Secant in the foregoing Parr, that nS is the Sine, bc the Tangent, and Ac the Secant of the Angle at C. In the 2^d Triangle likewise, that the Triangles 1, 2, are similar, and proportional, because of the parallel Lines nS and bc . Therefore,

Solution Literal.

1. As $AS \dots AC \dots Sn \dots BC$ in the first Triangle.
2. As $Ac \dots AC \dots bc \dots BC$.
3. As $CS \dots CA \dots Sn \dots AB$.
4. As $Cc \dots CA \dots cb \dots AB$ in the 2d Triangle.

3. Operation Logarithmetical.

1. Making the Hypoth. A C } From R 90° 00'
the Radius of a Circle, } Take $\angle A$ 36 34
} Rests $\angle C$ 53 26

Then will the Base A B, and the Perpendicular B C be Sines of their opposite Angles : and the Proportion is,

Sizes of men: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 84

Plain Trigonometry Rectangular.

As the Radius $\angle B 90^\circ 00'$ 10,00000

To the Hypoth. A C 95 m. Log. 1,97772

So is the Sn. $\angle A 36^\circ 34'$ 9,77506

To the Perpend. B C ——— 11,75278

————— 57 m. 1,75278

As Radius ——— $90^\circ 00'$ 10,00000

Is to the Hypoth. A C 95 m. Log. 1,97772

So is Sn. $\angle C$ ——— $53^\circ 26'$ 9,90480

To the Base A B ——— 11,88252

————— Log. 76 m. 1,88252

4. Second Variety.

Making the Base A B Radius, Perpend. B C is Tang, Hypoth. A C is Secant.

As Secant $\angle A 36^\circ 34'$ Is to the Hypoth. A C 95 m.
So is Radius $90^\circ 00'$ to the Base A B 76 m.

As Secant $\angle A 36^\circ 34'$.. Hypoth. A C 95 m. ::
Tang. $\angle A 36^\circ 34'$.. Perp. B C 57 m.

5. Third Variety.

Making the Perp. B C Radius, Base A B is Tang, Hypoth. A C is Secant.

As Secant $\angle C 53^\circ 26'$.. Hypoth. A C 95 m. :: Tang.
 $\angle C 53^\circ 26'$.. Base A B 76 m.

As Secant $\angle C 53^\circ 26'$.. Hypoth. A C 95 :: Rad. $90^\circ 00'$.. Perp. B C 57 m.

6. By Mr. Gunter's Scale Instrumental.

The Extent will reach from R. Sn. 90° , to Sn. $\angle A 36^\circ 34'$ on the Line of Sines.

The Extent will reach from Hypoth. A C. 95 m. to Perp. B C 57 m. on the Line of Numbers.

The Extent will reach from R. Sn. 90° , to Sn. $\angle C 53^\circ 26'$ on the Line of Sines.

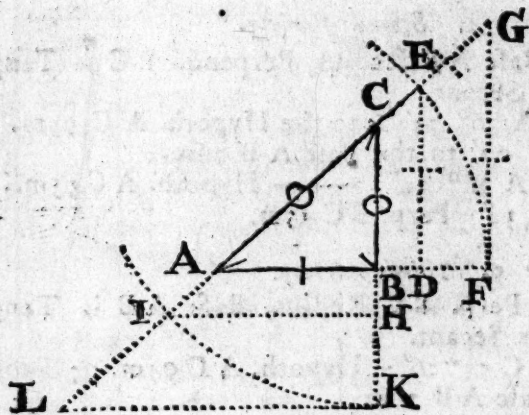
The Extent will reach from Hypoth. A C 95 m. to Base A B 76 m. on the Line of Numbers.

7. *By the Table of Natural Sines Arithmetical.*
In the Table of Natural Sines foregoing, you'll find, the Sine of the Angle A $36,34$ is $,5956$, and of the Angle C $53^{\circ} 26'$ is $,8031$.

poth. A C 95m. : : S L A , 5956 .. Perp, B C 57m. (Or rather 56,572.) 2. ~~As~~ Rad. 1,0000 .. Hypoth. A C 95m.

Reqd. } Hypoth: A C and Perpendicular B C.

Construction Geometrical.



Chord of
60° describe

the Arch FE, which make equal to the Chord of 42° 11'; and draw the Hypothenuſal Line A-G, and 'twill cut the former Perpendicular BC in C; ſo ſhall AC be the Hypoth. and BC the Perpend. required.

4. Produce A B and C B to F and R, the Chords of 60° from the angular Points A and C; and draw the parallel Sines E D, I H, and Tangents F G, K L, as in the last Case was directed.

2. Mensuration Mechanical.

Measure A C, the Hypoth. on the Line of Equal Parts
you plotted A B, and it will give you 102 = Parts or
Miles. Also

Plain Trigonometry Rectangular

Also apply BC, the Perpendicular, on the same equal Parts, and it will give you 151 = Parts or Miles for its Length requir'd.

3. Solution Literal.

1. As IH.. AB :: AE.. AC.
2. As IH.. AB :: ED.. BC.
3. .. AF.. AB :: AG.. AC.
4. .. AF.. AB :: FG.. BC.
5. KL.. AB :: CK.. CB.
6. KL.. AB :: CL.. CA.

4. That is, Logarithmetical.

1. Making the Hypoth. AC the Radius; then shall the Hypoth. and Perpend. BC be the Sines of their opposite Angles in the Proportion.

As the Sn. $\angle C$ $47^{\circ} 49'$ 9.86981

Is to the Base AB 112 m. 2.04921

So is Radius $90^{\circ} 00'$ 10.00000

To the Hypoth. AC 12.04921

Log. 151 m. 2.17940

As the Sn. $\angle C$ $47^{\circ} 49'$ 9.86981

Is to the Base AB 112 m. 2.04921

So is the Sn. $\angle A$ $42^{\circ} 11'$ 9.82704

To the Perp. BC 102 m. 11.87625

Log. 2.00644

2d. Variety. Making the Base AB Radius, BC is Tang. AC is Secant of $\angle A$.

As Radius $90^{\circ} 00'$ to the Base AB 112 m. :: Secant $\angle A$ $42^{\circ} 11'$ Hypoth. AC 151 m.

As Radius $90^{\circ} 00'$ Base AB 112 m. :: Tang. $\angle A$ $42^{\circ} 11'$ Perp. BC 102 m.

3d. Variety. Making the Perp. BC Radius, AB is Tang. AC is Secant to C.

As Tang. $\angle C$ $47^{\circ} 49'$ Base AB :: Secant $\angle C$ $47^{\circ} 49'$ Hypoth. AC 151 m.

As Tang. $\angle C$ $47^{\circ} 49'$ Base AB :: Radius $90^{\circ} 00'$ Perp. BC 102 m.

By

Plain Trigonometry Rectangular.

11

the Arch KI equal to $50^{\circ} 38'$ from the same Line of Chords.

3. Through the Points C and I draw the Hypotheneusal Line GL , to cut the Base Line AF in A ; so shall AC be the Hypoth. and AB the Base required.

4. Draw the similar Triangles ADE , AFG , CHI , and CKL , as was directed in the first Case foregoing.

2. Mensuration Mechanical.

Measure AC and AB on the Scale of equal Parts you plotted BC by, and you'll have 154 for the Length of the Hypoth. AC , and 120 for the Length of AB , the Base required.

3. Solution Literal.

1. $As\ ED :: BC :: AE :: AC.$

2. $As\ ED :: BC :: IH :: AB.$

3. $As\ FG :: BC :: AG :: AC.$

4. $As\ FG :: BC :: AF :: AB.$

5. $As\ CK :: CB :: CL :: CA.$

6. $As\ CK :: CB :: IH :: AB.$

4. Operation Logarithmetical.

1. Making the Hypoth. AC the Radius; then shall AB and AC be in proportion to their opposite Angles! That is,

$As\ Sn. \angle A\ 39^{\circ} 22'$	9.80228
------------------------------------	---------

$Is\ to\ Perp.\ BC\ 98\ m.$	1.00000
-----------------------------	---------

$So\ is\ Rad.\ Sn.\ 90^{\circ} 00'$	10.99122
-------------------------------------	----------

$To\ Hypoth.\ AC$	11.99122
-------------------	----------

$Log.\ 154\ m.$	2.18894
-----------------	---------

$As\ Sn. \angle A\ 39^{\circ} 22'$	9.80228
------------------------------------	---------

$Is\ to\ Perp.\ BC\ 98'$	1.99122
--------------------------	---------

$So\ is\ Sn. \angle C\ 50^{\circ} 38'$	9.88823
--	---------

$To\ Base\ AB$	11.87945
----------------	----------

$Log.\ 120\ m.$	2.07717
-----------------	---------

2d. Variety. Making the Base AB Radius, BC is Tangent, AC is Secant to A .

12

As Tang. $\angle A$ $39^{\circ} 22'$.. Perp. BC $98'$:: Secant $\angle A$ $39^{\circ} 22'$.. Hypoth. AC $154'$.

As Tang. $\angle A$ $39^{\circ} 22'$ Perp. BC $98'$:: Rad. $90^{\circ} 00'$.. Base AB $120'$.

3d Variety. Making the Perp. BC Radius, AB is Tangent, AC is Secant to C .

As Radius $90^{\circ} 00'$.. Perp. BC 98 :: Secant $\angle C$ $50^{\circ} 38'$.. Hypoth. AC $154'$.

As Radius $90^{\circ} 00'$.. Perp. BC $98'$:: Tang. $\angle C$ $50^{\circ} 38'$.. Base AB $120'$.

5. By Gunter's Scale Instrumental.

The Extent will reach from $\text{Sn. } \angle A$ $39^{\circ} 22'$ to Rad. 90° on the Sines.

The Extent will reach from Perp. BC $98'$ to Hypoth. AC $154'$ on the Numbers.

The Extent will reach from $\text{Sn. } \angle A$ $39^{\circ} 22'$ to $\text{Sn. } \angle C$ $50^{\circ} 38'$ on the Sines.

The Extent will reach from Perp. BC $98'$ to Base AB $120'$ on the Numbers.

6. By Natural Sines Arithmetical.

By the foregoing Table of Natural Sines the N. Sine of $39^{\circ} 22'$ is ,6344, of $50^{\circ} 38'$ is ,7731. Then the Proportion is,

As $\text{Sn. } \angle A$.. Perp. BC :: Rad. .. Hypoth. AC .

As ,6344 .. — 98 m. :: 1,0000 .. 154 m.

As $\text{Sn. } \angle A$.. Perp. BC :: $\text{Sn. } \angle C$.. Base AB .

As ,6344 .. — 98 :: ,7731 .. — 120 near.

CASE IV. In the Right-angled Plain Triangle ABC is

Given $\left\{ \begin{array}{l} \text{the Hypoth. } AC \text{ } 154 \text{ Leagues, and Base } AB \\ \text{Requir'd } \left\{ \begin{array}{l} 121,5 \text{ Leagues.} \\ \text{Angles } A \text{ and } C, \text{ and Perpend. } BC. \end{array} \right. \end{array} \right.$

Plain Trigonometry Rectangular.

13

1. Construction Geometrical.

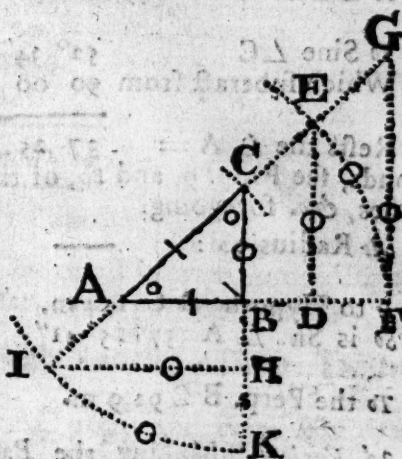
1. Draw A F at pleasure, and make A B equal to 121.5 = Parts,

2. Erect the Perpendicular B C.

3. Take 154 equal Parts in your Compasses, and set one Foot in A, with the other cross the Perpendicular B C in C.

4. Through A and C draw the Hypotenusal Line A C, and continue it to G and I; so shall F G = E D = F E be the equal Angle at A, and B C the Perpend. requir'd.

5. Draw the similar Triangles A D E, A F G, and C H I, as before directed.



2. Mensuration Mechanical.

1. Measure the Chord F E on your Line of Chords; or the Line D E on your Line of Sines; or the Line F G on the Line of Tangents; either of which will give you the Quantity of the Angle at A, viz. $37^{\circ} 25'$.

2. Apply B C to the same equal Parts you take of A B, and 'twill give you the Length of the Perpendicular B C, i. e. 92.9 m.

3. Solution Literal.

1. $\therefore AC \therefore CI \therefore AB \therefore IH = \text{Sn. } \angle C.$
2. $\therefore AE \therefore AC \therefore ED \therefore CB.$
3. $\therefore AF \therefore AB \therefore AC \therefore AG = \text{Secant } \angle A.$
4. $\therefore AF \therefore AB \therefore FG \therefore BC.$
5. Or, $\therefore AG \therefore AC \therefore FG \therefore BC.$

4. That is, Logarithmetical.

Making the Hypoth. A C the Radius, the Angle C shall be in proportion to its opposite Side A B, as Radius 90° is to the Hypoth. A C.

As

14 Plain Trigonometry Rectangular.

As Hypoth. A C 154 m. 3,1846914

Is to Radius 90° 00' 10,0000000

So is Base A B 121,5 m. 3,0845763

To Sine $\angle C$ 52° 34' 19"

Which subtract from 90 00 00

Rests the $\angle A = 37^{\circ} 25' 41''$ To find the Seconds, see Prop. 9, and 10, of the Use of the Tables of Sines, &c. foregoing.

As Radius 90 : 10,0000000

Is to Hypoth. A C 154 m. 3,1846914

So is Sn. $\angle A$ 37° 25' 41" 9,7837355

To the Perp. B C 92,9 m. 2,9684269

2d. Variety. Making the Base A B Radius, B C is Tangent, A C is Secant to $\angle A$.

As the Base A B 121,5 .. Rad. :: Hypoth. A C 154 .. Secant $\angle A$ 37° 25' 41".

As the Rad 90° 00' .. Base A B 121,5 :: Tang. $\angle A$ 37° 25' 41" .. Perp. B C 92,9 m.

5. By Gunter's Scale Instrumental.

The Extent will reach from Hypoth. A C 154 to 121,5 m. the Base A B on the Numbers.

The Extent will reach from Rad. Sn. 90° to Sn. $\angle C$ 52° 34' 19" .. on the Sines.

The Extent will reach from Rad. Sn. 90° to Sn. $\angle A$ 37° 25', &c. on the Sines.

The Extent will reach from Hypoth. A C 154 m. to Perp. B C 92,9 m. on the Numbers.

6. By Natural Sines Arithmetical.

As Hypoth. .. Rad. :: Base to N. Sine,

154 .. 1,0000 :: 121,5 .. ,7889 2 in the Table

AC—Sn. 90—AB—Sn. 52° 34' + } of N. Sines.

As Rad. .. Hypoth. A C :: Sn. $\angle A$ 37° 25' .. Perp. B C

1,0000 .. 154 :: ,6076 .. 92,5704 m. S

7. By Natural Arithmesick:

It will be by 47, 1. Euclid (the Rule) the Square Root of the Difference of the Squares of the Hypoth. and Base will be the Perpendicular. That

Plain Trigonometry Rectangular.

**That is,
By**

154	121,5
155	121,5
616	607,5
770	1215
154	2430
	1215

From Sq.

Take Sq.

Refts

23716
14762,25 14762,25

• 8953.75

✓ 8953,75 (92,6 Перп. BC

The $\angle A$ may be found by the Theorem preceding the first Axiom : Or hereafter, by *Arith. Trigonometry*.

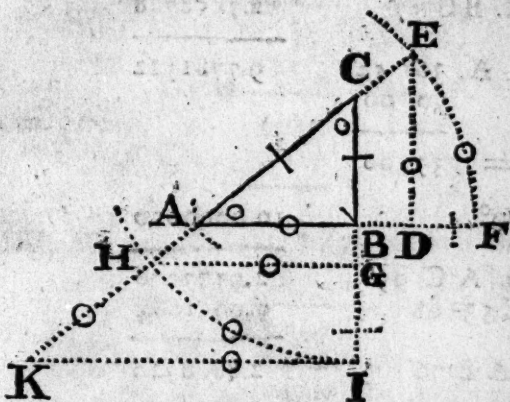
CASE V. In the Right - angled Plain Triangle
ABC is

Given } The Hypoth. AC 95 m. and Perp. BC 57 m.
Required } Angles A and C, and Base AB.

I. Construction Geometrical.

First, Draw A F; and on the Point B erect the Perpendicular BC, which make equal to 57 = Parts.

2. With 95
= Parts in
your Compas-
ses, and one
Foot in C, with
the other cross
A B in A;
then through
the Points A
and C draw



the Hypothenusal Line KE; so shall AB be the Base required. Draw also the similar Triangles ADE, CBH, and CIK; so shall FE be the Chord, DE the Sine of the $\angle$ at A, and IH the Chord, GH the Sine, IK the Tangent, and CK the Secant of the $\angle$ at C, to the Radius CI=AF.

2. Men-

2. Mensuration Mechanical.

2. Measure the Chord FE on your Line of Chords, or the Sine DE on your Line of Sines; either of which gives the Quantity of the $\angle A$, viz. $36^\circ 52'$. 2. Measure AB on your Line of equal Parts, and you have the Length of the Base AB 86'.

3. Solution Literal.

1. $\therefore AC \therefore AE \therefore BC \therefore DE = \text{Sn. } \angle A.$
2. $\therefore CH \therefore CA \therefore GH \therefore BA.$
3. $\therefore BC \therefore CI \therefore CA \therefore CK = \text{Secant } \angle C.$
4. $\therefore CK \therefore CA \therefore KI \therefore AB.$
5. Or, $\therefore CI \therefore CB \therefore IK \therefore BA.$

3. Operation Logarithmetical.

1. Making the Hypoth. AC the Radius, the Angle A shall be in proportion to its opposite Side BC, as Radius $\angle B 90^\circ$ to its opposite Side the Hypotheneuse AC.

As Hypoth. AC 95' 2.9777236

Is to the Radius 90° 10.0000000

So is Perpend. BC 57' 2.7558748

To the Sn. $\angle A 36^\circ 52'$ 9.7781512

Sub. from 90 00

Refts the $\angle C = 33^\circ 08'$

As Radius 90° 10.0000000

Is to Hypoth. AC 95' 2.9777236

So is Sn. $\angle C 53^\circ 08'$ 9.9031084

To the Base AB 76' 2.8808320

2d. Variety. Making the Perpendicular BC Radius, AB is Tangent, CA is Secant to $\angle C$.

As the Perp. BC 57' .. Radius 90° .. Hypoth. AC 95' to .. Secant C $53^\circ 08'$.

As Radius 90° 00' .. Perp. BC .. 57 .. Tang. $\angle C 53^\circ 08'$.. Base AB 76 miles.

Plain Trigonometry Rectangular.

17

4. Instrumental, by Gunter's Scale.

The Extent from Hypoth. A C 95' to Perp. B C 57' on the Numbers,

Will reach from Sn. 90° to Sn. $\angle A 36^\circ 57'$ on the Sines.

The Extent from Radius Sn. 90° to Sn. $\angle C 53^\circ 08'$ on the Sines,

Will reach from Hypoth. A C 95' to Base A B 76' on the Numbers.

5. By Natural Sines Arithmetical.

As Hypoth. .. Radius .. Perp. .. Sn. $\angle A 36^\circ 57'$ in the Table of N. Sines.

— 95 — 1,0000 .. 57 .. ,6000 (near ,5999) $\angle C 53^\circ 08' = ,8000$.

As Radius .. Hypoth. .. Sine $\angle C$.. Base ?
— 1,0000 .. 95 .. ,8000 .. 76 }

6. Arithmetical, by the Square Root.

As in the last Case,

1. Multiply	95	57
By	95	57
	475	399
	855	285
From Square	9025	3249
Take Square	3249	

Rems $\sqrt{\quad}$ 5776 (76 = A B Base.

49

146) 876

876

...

2. For the $\angle A$
Hypoth. A C = 95
Half Base BA 76 = 38.

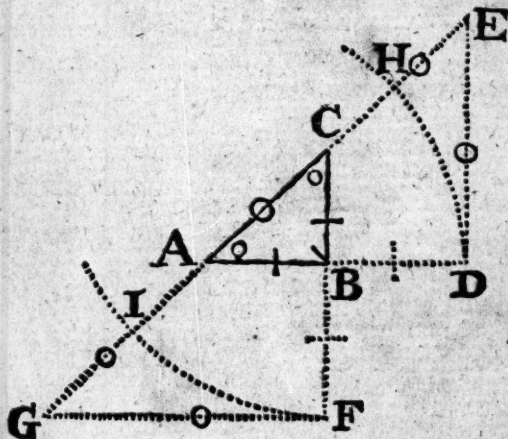
Sum = 133

BC
 8. At 133, — 57 : : 86.
 86
 —
 342
 456
 —
 133) 4902 (36° 52' = $\angle A$
 —
 .912
 —
 114 rem.
 60 x Minutes.
 —
 133) 6840

CASE VI. In the Right-angled Triangled ABC is
 Given } the { Base AB 112' and Perp. BC 102'.
 Req'd. } Angles A and C, and Hypoth. AC.

1. Construction Geometrical.

First, Draw AD at pleasure ; and on the Point A with



112 equal Parts
make AB 2. Er-
ect the Perpen-
dicular BC,
which make e-
qual to 102 from
the same Line of
=Parts. 3. From
the Points A and
C draw the Hy-
pothenusal Line
GE; so shall AC
be the Hypoth.
required. Compleat the similar
Triangles ADE

and CFG ; so shall DH be the Chord and DE the Tangent of $\angle A$, and FI the Chord, FG the Tangent of $\angle C$; alio AE the Secant of $\angle A$, and CG the Secant of $\angle C$ required.

2. Mensuration Mechanical.

The Chord Line DH measured on your Scale of Chords; or, DE measur'd on the Line of Tangents will give the Quantity of the Angle A $42^{\circ} 19'$; and AC being measured on your Scale of equal Parts gives the Length of the Hypothenuſe 151'.

3. Solution Literal.

1. $AB \cdot AD :: BC \cdot DE = \text{Tangent } \angle A.$
2. $AD \cdot AB :: AE \cdot AC = \text{Hypoth.}$
3. $CB \cdot CF :: BA \cdot FG = \text{Tangent } C.$
4. $CF \cdot CB :: CG \cdot CA = \text{Hypoth.}$

4. Operation Logarithmetical.

1. Making the Base AB Radius, BC is in proportion a Tangent, and AC a Secant to the Angle at A.

As Base AB 112'	2,049218
Is to Radius 90°	10,000000
So is Perp. BC 102'	2,008600
To Tang $\angle A 42^{\circ} 19'$	9,959182
As Radius 90°	10,000000
Is to the Base AB 112'	2,049218
So Secant $\angle A 42^{\circ} 19'$	10,131100
To Hypoth. AC 151'	2,180318

2. Making the Perp. BC Radius, AB is Tang. and AC is Secant to Angle C.

As Perp. BC 102'	2,008600
Is to Radius 90°	10,000000
So Base AB 112'	2,049218
To Tang. $\angle C 47^{\circ} 41'$	10,040618

As Rs Radius 90°	10,000000
Is to Perp. B C 102	2,008600
So is Sec. $\angle C 47^\circ 41'$	10,171838
To Hypoth. A C 151	2,180438

5. By Gunter's Scale Instrumental.

The Extent from Base A B 112, to Perp. B C 102 on the Numbers,

Will reach from $\angle$ Tang, 45° , to Tang. $42^\circ 19' \angle A$ on the Tangents.

The Extent from Sn. $\angle A 42^\circ 19'$ to $\angle$ Sn. 90° on the Sines,

Will reach from Perp. B C 102 to Hypoth. A C 151 on the Numbers.

Note. If the Perpend. B C be greater than the Base A B, the Angle at A must be accounted from Tangent 45° , increasing towards the Left hand: but if the Perpendicular be less than the Base (as in this Case) then count it from the beginning of the Tangents, increasing towards the Right hand.

6. By Natural Tangents Arithmetical.

As Base .. Radius :: Perp. .. Tang. $\angle A 42^\circ 19' = 42^\circ 11 \frac{1}{2}'$

— 112 .. 1,0000 :: 102 .. 9107 (nearest in the Table, 9063)

As Sn. $\angle A$.. Perp. :: Radius .. Hypoth. A C.

2d by Natural Sines, $42^\circ 19' = 6732$.. 102 :: 1,0000 .. 151 m.

7. Arithmetical, by the Square Root, and Nr. 86' before demonstrated) per Euclid, 47.1.

RULE. The Square Root of the Sum of the Squares of the Base and Perpendicular, is the Hypotenuse.

Thus $112 \times 112 = 12544$ } Add

And $112 \times 102 = 10404$ }

$\sqrt{22948} (151 = \text{Hypoth. A C.})$

Hypoth. A C $151 + \frac{1}{2} (112) = 56$ is 107.

BC

As $\angle 207$.. 102 :: 86 .. $42^\circ 22'$ — the Angle A.

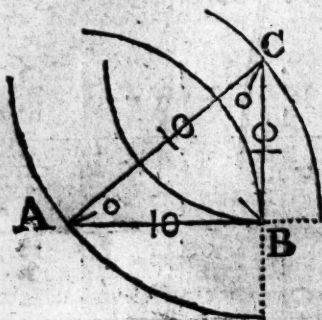
A Triangular Synopsis, or View of all the Variety of Proportions for the six Cases of Right-angled Plain Triangles foregoing.

Varieties and Proportions.			
Case	Given	Reqd	Radius
1.	$\angle A$ $\angle C$	$A B$ and $B C$	$A C$ $A B$ $B C$
2.	$\angle A$ $\angle C$	$A C$ and $B C$	$A C$ $A B$ $B C$
3.	$\angle A$ $\angle C$	$A C$ and $B C$	$A C$ $A B$ $B C$
4.	$A B$ and $A C$	$A C$ and $B C$	$A C$ $A B$ $B C$
5.	$A C$ and $B C$	$A C$ and $B C$	$A C$ $A B$ $B C$
6.	$A B$ and $B C$	$A C$ and $B C$	$A C$ $A B$ $B C$

1. $R \dots A C :: S n \dots L A \dots B C$.	3. secant $L A \dots A C :: R \dots A B$.
2. $R \dots A C :: S n \dots L C \dots A B$.	4. secant $L A \dots A C :: T L A \dots B C$.
5. secant $L C \dots A C \dots R \dots B C$.	6. secant $L A \dots A C :: T L C \dots A B$.
1. $S n \dots L C \dots A B$.	2. $S n \dots L C \dots A B :: S n \dots L A \dots B C$.
3. $R \dots A B ::$ secant $L A \dots A C$.	4. $R \dots A B :: T L A \dots B C$.
5. $T L C \dots A B ::$ secant $L C \dots A C$.	6. $T L C \dots A B :: R \dots B C$.
1. $S n \dots L A \dots B C :: R \dots A C$.	2. $S n \dots L A \dots B C :: S n \dots L C \dots A B$.
3. $T L A \dots B C ::$ secant $L A \dots A C$.	4. $T L A \dots B C :: R \dots A B$.
5. $R \dots B C ::$ secant $L C \dots A C$.	6. $R \dots B C :: T L C \dots A B$.
1. $A C \dots R :: A B \dots S n \dots L C$.	2. $R \dots A C :: S n \dots L A \dots B C$.
3. $B C \dots R :: A C \dots$ secant $L C$.	4. secant $L A \dots A C :: T L A \dots B C$.
5. $\sqrt{A C q - A B q} = B C$.	6. $C A + A B g r \dots B C \dots 86 \dots L A$.
1. $A C \dots R :: B C \dots S L A$.	2. $R \dots A C :: S n \dots L C \dots A B$.
3. $B C \dots R :: A C \dots$ secant $L C$.	2. $R \dots B C :: T L C \dots A B$.
5. $\sqrt{A C q - B C q} = A B$.	6. $A C + A B g r \dots B C :: 86 \dots L A$.
1. $A B \dots R :: B C \dots T L A$.	2. $R \dots A B ::$ secant $L A \dots A C$.
3. $B C \dots R :: A B T L C$.	4. $R \dots B C ::$ secant $L C \dots A C$.
5. $\sqrt{A B q + B C q} = A C$.	6. $A C + A B g r \dots B C :: 86 \dots L A$.

Plain Trigonometry Rectangular.

Application to Navigation.



The 1, 2, 3, 4, 5, and 6th Cases of Plain Sailing; Likewise Traverse - Sailing, Mercators Sailing, Parallel Sailing, and Middle Latitude Sailing, are all resolved by the foregoing Six Cases of Right-angled Triangles.



Plain

Plain Trigonometry Oblique.

C A S E I.

IN the Oblique-angled Plain Triangle A B C

Given $\left. \begin{array}{l} \text{Angle } A = 33^\circ 45', \text{ and Angle } B = 45^\circ \\ \text{and Side opposite } A C = 40 \text{ m.} \end{array} \right\} \text{ the } \left\{ \begin{array}{l} \text{Sides } A B \text{ and } B C. \end{array} \right.$

Preparation. The $\angle A + \angle B = 180^\circ = 101^\circ 15' = \angle C$.

Construction. Draw the Base-Line A B, and make the Angle at A $= 33^\circ 45'$ = given Angle.

2. Make the Side A C equal to 40 = Parts.

3. Make the Angle A C B = $101^\circ 15'$ (or the

Angle D C B $= 78^\circ 45'$, the Comp. of $101^\circ 15'$) and draw the Line or Side C B, cutting the Base A B in B; so shall the $\angle B$ be $= 45^\circ 00'$, and the Side A B and B C the Sides required. To find which, 1. *Mechanical Measure* A B and B C on the same equal Parts you plotted A C by, and you will have their Lengths, viz. A B $55 \frac{1}{2}$, B C $31 \frac{1}{2}$.

2. *Literal*, it is $\left\{ \begin{array}{l} \dots B \dots AC : \dots Sn. C \dots A B ? \\ \dots B \dots AC : \dots Sn. A \dots B A \end{array} \right\}$ requir'd.

3. *Logarithmetical*, by Artificial Sines and Numbers, Proportions to find A B and B C by Axiom 3d, are,
As the Sn. $\angle B$ $45^\circ 00'$

Is to the Side A C 40 Miles

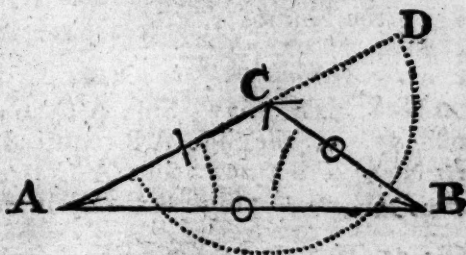
So is the Sn. $\angle C$ $101^\circ 15'$ or $78^\circ 45'$

To the Side A B =

Log. 55.5 m.

PART II.

B 4



9.84948

1.6206

9.99157

11.39303

1.74445

Plain Trigonometry Oblique.

As the Sn. $\angle B$ $45^{\circ} 00'$	9.84948
Is to the Side A C 40 m.	1.60206
So is the Sn. $\angle A$ $33^{\circ} 45'$	9.74473
To the Side B C	11.34679
Log. 31.2 m.	1.49731

6. By Mr. Gunter's Scale Instrumental,

The Extent from Sn. Angle B 45° , to S. Angle C $78^{\circ} 45'$ on the Sines,

Will reach from Side A C 40 m. to Side A B $55 \frac{1}{2}$ on the Numbers.

The Extent from Sn. Angle B 45° , to Sn. Angle A $33^{\circ} 45'$ on the Sines,

Will reach from Side A C 40 m. to Side B C $31 \frac{1}{4}$ on the Numbers.

5. Arithmetical, by the Table of Natural Sines.

As N. S. $\angle B$.7071... Side A C 40 m. : : N. S. $\angle C$.. 9808 .. 55.5 m. the Side A B required,

As N. S. $\angle B$.7071... Side A C 40 m. : : N. S. $\angle A$.. 5556 .. 31.2 m. the Side B C required.

CASE II. In the Oblique-angled Plain Triangle ABC is

Given $\left\{ \begin{array}{l} \text{the Side A B} = 100 \text{ m. Side A C } 80 \text{ m. and An-} \\ \text{gle B opposite} = 45^{\circ} 00'. \end{array} \right.$

Requir'd $\left\{ \begin{array}{l} \text{the Angle C Obtuse, and Side B C.} \end{array} \right.$

Note, The Given Angle Obtuse, the Angle sought is Acute.

But when it's Acute and Opposite to the Greater Side, the requir'd Angle is Acute.

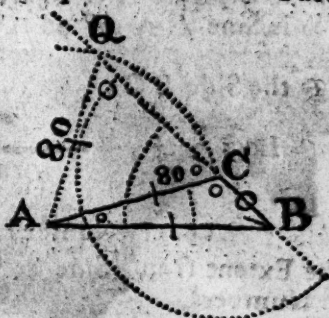
When it's Acute, and Opposite to the Lesser Side, the requir'd Angle is Doubtful.

Whether Acute or Obtuse, it ought to be determin'd before the Operation.

Plain Trigonometry Oblique.

1. Construction Geometrical.

1. Draw A B, which make equal to 100 = Parts,
2. Make the Angle B = $45^{\circ} 00'$, and draw B C produced.
3. With 80 equal Parts in your Compasses, and one Foot in A, crois B C produced in the Points C and Q.
4. Draw A C and A Q, so shall the Angle A Q B = (Angle A C Q) in the superior Triangle be Acute, and the Angle A C B in the Inferior Triangle, Obtuse ; which with the Side B C is required.
-



2. Mensuration Mechanical.

Measure the Angle $\angle ACQ = \angle AQC$ on the Line of Chords; and it will be $62^{\circ} 6'$, whose Complement to 180° is the Measure of the Angle $\angle ACB$ $117^{\circ} 54'$ Otherwise, (because the outward Angle $\angle ACQ$ is equal to the two inward and opposite Angles $\angle BAC$ and $\angle ABC$.) and BC measured on your Scale of equal Parts, will give you $33 \frac{1}{4}$ Miles; *q. e. q.*

7. Solution Literal.

Sd. $AQ \cdot Sn \cdot \angle B : Sd. AB \cdot Sn \cdot \angle AQC = ACQ$ Acute.
and $ACQ - 180 = \angle ACB$ Otrufe.
 $Sn \cdot \angle B : Sd. AC : Sn \cdot \angle BAC : Sd. BC.$

4. Operation Logarithmetical, by Axiom 3d.

As Side A Q 80 m.	1 90309
Is to Sn. $\angle B 45^{\circ} 00'$	9 84948
So is Side A B 100 m.	2.00000
To Sn. $\angle Q 62^{\circ} 06'$ Acute	9 94639
Subt. from 180 00'	
The $\angle C$ 117 54 Obruse $\angle C + B - 180 = 17^{\circ}$	

Plain Trigonometry Oblique.

As Sn. $\angle B 45^{\circ} 00'$	9.84948
Is to the Side A C 80 m.	1.90309
So is Sine $\angle A 17^{\circ} 06'$	9.6840
To the Side B C	11.37149
Log. $33 \frac{1}{4}$	1.52201

5. By Gunter's Scale Instrumental.

The Extent from Side 80 m. to Side 100 m. on the Numbers,

Will reach from $\angle B 45^{\circ}$, to $\angle Q 62^{\circ} 06'$ on the Sines.

The Extent from $\angle B 45^{\circ}$, to $\angle A 17^{\circ} 06'$ on the Sines,

Will reach from Side A C 80, to Side B C $33 \frac{1}{4}$ on the Numbers.

5. By Natural Sinus Arithmetical.

As Sd. 80 m. .. N.S. $45^{\circ} 7071 = 100$ m. .. N.S. 8838 = $\angle Q 62^{\circ} 06'$.

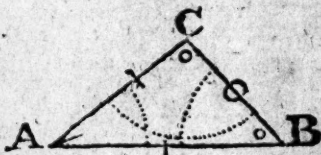
As Sn. N. $45^{\circ} = 7071$.. 80 m. : N.S. $17^{\circ} 06' = 940$.. Sd. B C 33.2 m.

CASE III. In the Oblique Right-lin'd Triangle ABC is

Given $\left\{ \begin{array}{l} \text{the} \\ \text{Angle A} = 33^{\circ} 45', \text{ and containing Sides} \\ \text{AB} = 25 \text{ m. and AC} = 20 \text{ m.} \end{array} \right.$
 Requir'd $\left\{ \begin{array}{l} \text{Angles C and B, and Side B C.} \end{array} \right.$

Construction Geometrical.

1. Make the Angle $= 33^{\circ} 45'$ from your Line of Chords. 2. Make A B = 25 m. and A C = 20 m. from your Line of equal Parts; and lastly, join the Points B and C, and it is done.



1. Mensuration Mechanical.

The Angles C and B. measur'd on the Line of Chords, are $93^{\circ} 13'$ and $53^{\circ} 01'$; and Side B measur'd on the equal Parts (you took of A B and A C) gives the Side B C = 14 Miles.

2. Solu

3. *Solution Literal.*

$AB + AC = \left\{ \begin{array}{l} 45 \text{ Sum} \\ 5 \text{ Diffs.} \end{array} \right\}$ of the Sides.

Sum $\angle s = 180^\circ$ $\angle A = 33^\circ 45' = 146^\circ 15'$, whose half = $73^\circ 7'$, the half Sum Unknown $\angle s$.

Then, $As AB + AC \therefore AB - AC \therefore \text{Tang } \frac{1}{2} Z \text{ Unk. } \angle s \therefore$
 $\text{Tang. } X \therefore \text{Sd. } \angle s;$

Then $\frac{1}{2} Z + \frac{1}{2} X = \left\{ \begin{array}{l} \text{Great Angle C,} \\ \text{Less Angle B.} \end{array} \right.$

3. *Operation Logarithmetical by Axiom 4, and 7.*

As the Sum of the 2 Sides $AB + AC = 45'$ 1.653212

Is to their Diffs. $AB - AC = 5'$ 0.698970

So is $\text{Tang. half Sum Unk } \angle s C + B = 73^\circ 07'$ 10.517833

To the $\text{Tang. half Diffs. } C - B = 20^\circ 06'$ 11.216803

$\frac{1}{2}$ Sum and $\frac{1}{2}$ Diffs. add. give $\angle C = 93^\circ 13'$ 9.563591
 Subducted $= \angle B = 53^\circ 01'$ 9.902444

As $\text{Sn. Angle } B = 53^\circ 01'$ 9.902444

Is to Side $AC = 20'$ 1.301030

So is $\text{Sn. } \angle A = 33^\circ 45'$ 9.744739

To Side $BC =$ 11.045769

$\text{Log. } 14$ 1.143325

5. *By Gunter's Scale Instrumental.*

The Extent from Sum Sides 45 m. to their Differences,

Will reach from Tangent 45° , to Tangent $6^\circ 20'$.
 Then extend from Tang. $6^\circ 20'$, to Tang. $7^\circ 07'$
 (half the Sum of the Angles,) the same Extent will
 reach again from Tang. 45° , to Tang. of $20^\circ 6'$;

2. The Extent from S. Angle $B = 53^\circ 01'$, to S. Angle $A = 33^\circ 45'$ on Sines, will reach from Sd. $AC = 20$, to Sd. $BC = 14$ m. on the Numbers.

5. By Natural Sines Arithmetical.

As Sum Sides 45 m. .. their Diffs, 5 : : N. T. $73^{\circ} 07'$
 $= 3,1963$.. N. T. $0,3774 = \text{Tang. } 20^{\circ} 06'$.

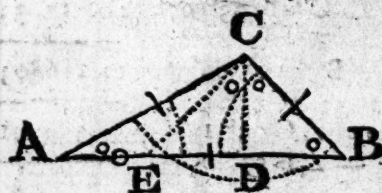
As N. S. Angle B $53^{\circ} 01'$ = .7988 .. Sd. AC 20 m. : :
 N. S. $33^{\circ} 45' = 5556$.. Sd. BC = 13,9.

CASE IV. In the Oblique Plain Triangle ABC is
 Given { The Base AB 64', Sd. AC 47', and Sd. BC 34',
 Requir'd { Angle A, Angle B, and Angle C.

PREPARATION.

Side AC $\pm$ BC = $\frac{Z}{X}$, viz. $47 \pm 34 = \left\{ \begin{array}{l} 81 \text{ Sum} \\ 13 \text{ Diffs.} \end{array} \right\}$ Sides.

1. Construction Geometrical.



1. Make AB 64 equal Parts. 2. From your equal Parts take 47; and one Foot in A, strike an Arch at C. 3. Take off also from the same Line of = Parts 34, one Foot in B, and cross the former Arch in C. 4. Join A C and B C, and the Triangle A B C is made. 5. Demit from the Point C the Perpend. C D, which divides the Oblique Triangle A B C into two Right-angled Triangles ADC, and BDC, both Right-angled at D. 6. Make D E equal to D B; so shall A E be the Diffs. of the Segments of the Base A B.

2. Mensuration Mechanical.

With a Chord of 60° describe Arches from the Angular Points A, B, C, to cut the containing Sides of the said Angle; these Arches respectively measur'd, will give the Quantity of the Angles, viz. Angle A $= 53^{\circ} 20'$, Angle B $51^{\circ} 02'$, Angle C $97^{\circ} 18'$.

3. Solution Literal.

m.

1. As AB .. AC + BC : : AC - BC .. A E & A E $\pm$ AB its
 $= \left\{ \begin{array}{l} AD = 40 \\ DB = 24. \end{array} \right.$

2. Then .. AC .. R : : AD .. Sn. $\angle ACD$, whose Comp. to a Quadrant $= \angle A$ required,

3. Again,

Again, $\therefore BC \therefore \text{Sn. } \angle A : \therefore AC \therefore \text{Sn. } \angle B \text{ and } \angle A + \angle B - 180^\circ = \angle C.$ W. W. R.

4. Operation Logarithmetical, by Axiom 3.

	m.	Co- <i>Ar.</i>
As the Base A B	64	3.19382
Is to the Sum of the Sides AC and BC	81 m.	1.90848
So is the Diff. of the said Sides	13 m.	1.11394
To the Diff. Segm. of Base A E	16	1.21614
As Hypoth. AC	47 m.	1.67209
Is to Radius	90°	10.00000
So is Base AD	40 m.	1.60206
To Sn. $\angle CD$	$= 58^\circ 20'$	9.92997
Subt. from R	90 00	
Gives $\angle A$	$= 31^\circ 40'$	

Find the Angle B by Case the Second.

As Side BC 34 $\therefore \text{Sn. } \angle A 31^\circ 40' : \therefore \text{Side AC } 47 \text{ m.} \therefore \text{Sn. } \angle B 46^\circ 31'.$ Then the Sum of the Angles A $31^\circ 40'$, and B $46^\circ 31'$, subtracted from 180° , gives the $\angle C = 101^\circ 29'.$

5. By Gunter's Scale Instrumental.

The Extent from Base A B 64 m. to Sum Sds. 81 on the Numbers,

Will reach from Diff. Sides 13 m. to Diff. Segments Base 16 m. on the Numbers.

The Extent from Hypoth. AC 47', to Base AD 40' on the Numbers,

Will reach from R Sn. 90° , to Sn. Angle ACD $58^\circ 20'$ on the Sines.

The Extent from Side BC 34', to Side AC 47' on the Numbers,

Will reach from Sn. $\angle A 31^\circ 40'$, to Sn. Angle B $46^\circ 31'$ on the Sines.

4. Arithmetical, and by Natural Sines.

As Base $\therefore$ Sum Sides $: : \text{Diff. Sides} \therefore \text{Diff. Segments Base.}$

$\therefore 64 \therefore 81 \text{ --- } : : 13 \text{ --- } 16,4 = \text{A E, and Half } 64 + 16 = 40 = \text{A D.}$

As

As $AC\ 47' :: N. S. 1000 :: AD\ 40' .. N. S. 8510 = 58^\circ 20'$, the Angle ACD .

As $BC\ 34' .. N. S. Angle\ A\ 31^\circ 40' = 5250 :: AC'\ 47' .. N. S. 7257 = 46^\circ 31'$, Angle B requir'd.

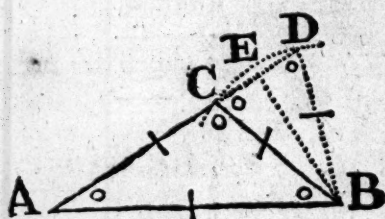
I shall next give the Solution of this Fourth Case, according to the Methods mention'd after *Axiom 5*.

CASE IV. Example II.

Let there be given the Base $AB\ 64$ m. Side $AC\ 47$ m. Side $BC\ 34$ m. as before, to find the Angles ABC .

CONSTRUCTION.

Project the Triangle, as is directed in the last. Then



2. With the Distance BC , and one Foot in B describe the Arch CD , intersecting AC produced in D . 3. Bisect CD in E , and draw the Perpend. EB , which divides the Oblique or Isosceles Triangle BCD

into two Right-angled Triangles CEB and DEB , each Right-angled at E .

Then is AC the true Base, and AD the alternate Base. To find which,

Preparation and Operation Arithmetical and Logarithmetical.

Side $AB\ 64$ }
Side $BC\ 34$ }

Sum Side $= 98$ }

Diff. Sides $= 30$ }

As $.. Base .. Sum\ Sides :: Diff. Sides .. Altern. Base.$
 $AC\ 47 - 98 .. :: 30 .. 62,5 = AD.$

From which subtr. true Base $AC\ 47,0$

Remains $CD\ 15,5$

Whose Half is $CE = ED = 7,75$

2. For the Angles BCE , and ACB .

As the Side CE 7,75

0.889302

Is to Radius $90^\circ 00'$

10,

So is Side BC 34

1.531479

To the Secant $\angle BCE$ $78^\circ 07'$

10.642177

Which taken from $180^\circ 00'$

Remains $\angle ACB = 101^\circ 53'$ requir'd; and the other Angles are easily found by Case 2d. foregoing.

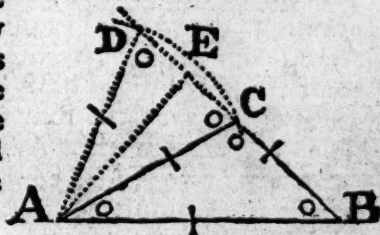
The Outward Angle $BCE = \angle BAC + \angle ABC$, and $BAC + \angle ABC - 180^\circ = \angle ACB$, as hath been said before.

CASE IV. Example III.

Given the Side AB 64, Side AC 47, and Side BC 34, to find the Angles C , &c. as before.

CONSTRUCTION.

Having made the Triangle, with the Distance AC , and one Foot in A , describe the Arch CD , and draw DA , and Perpend. EA , as in the last; so shall BC be the true Base, and BD the alternate Base. To find which, proceed as in the last Example.



$AB + AC = \text{Sum} \left. \begin{matrix} 64 \\ 47 \end{matrix} \right\} \text{Sds.} \left. \begin{matrix} 111 \\ 17 \end{matrix} \right\} \text{And } BC = 34. \text{ Then}$

As Base .. Sum Sides : : Diff. Sides .. Alternate Base.

$BC \ 34 - \dots 111 - \dots : 17 - \dots 55,5 = BD.$

From which take $BC = 34,0$

Remains CD 21,5

Whose Half is $CE = DE$ 10,75

Logarithmetical. As for Axiom 6th, 7th, and 8th, I shall omit their Operations, to avoid Prolixity; and only give the Explanation and Solution to Axiom 9th, it being more practicable than the others.

$A C E 10'$	—	1.000000
$Is\ to\ R\ 90^\circ$	—	10.000000
$So\ is\ A C\ 47'$		1.707570
$To\ Secant\ \angle\ A C E$		10.707570
$78^\circ\ 41'$		
Subt. from $180\ 00$	—	

Rem. $\angle ACB 101\ 19$, &c. as in the last.

CASE IV. Example IV.

Let the Sides $AB\ 64'$, and $AC\ 47'$, and $BC\ 34'$, be given to find the Angle C , &c. as before.

As the Product of the two Sides containing the Angle requir'd, Is to the Square of the Radius; So is the Product of half the Sum of the Three Sides, and Difference of the Opposite Side to the Angle requir'd, To the Square of a Sine, whose Complement to 90° doubled is the Angle required.

To resolve this Axiom, First, set down the two Sides containing the Angle required. Then the Opposite thereto: add these three Sides together, and take half thereof; and from the half Sum subtract the Opposite Side, and subscribe the Remainder.

Then to perform this Axiom by Logarithms, First, set down the two Sides containing the Angle required; and take the Complement Arithmetical of their Logarithms (by Prop. 3. of the Use of the Table of Logarithms foregoing;) to which add the Logarithms of the half Sum, and Remainder: Half the Sum of these four Logarithms is the Sine-Complement of half the Angle required.

OPERATION.

Side	$AC\ 47$	Co-Ar. 8.327902
	$BC\ 34$	Co-Ar. 8.468521
	$AB\ 64$	
	Sum 145	
	Half Sum 72.5	Log. 1.857634
	Remainder 8.5	Log. 0.903361
		Sum 19.557418
	$\frac{1}{2}$ Sum = Co-S. of $\frac{1}{2}\angle C = 53^\circ\ 05'$	9.778709
Which doubled is	2	$53\ 05$
Equal to the $\angle C$	5	$106\ 10$ required,

After

Note, This Difference of $\angle C$ from the former Work is, by reason I have wrought the Logarithms with the Decimals annexed; but there only by the whole Numbers.

After the same manner may the other Angles A and B be found: Or you may find them by Case 2. of *Oblique Triangles* foregoing; which is left to the Learner's Exercise And next, to perform the foregoing Work by *Gunter's Scale*.

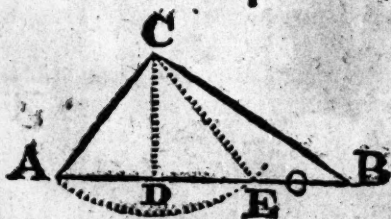
1. The Extent of the Compasses from the greater of the Containing Sides (A C 47) to half the Sum of the three Sides (72,5) shall reach from the Remainder (8,5) to a fourth Number (12,2 m.) on the Numbers.

2. The Extent from the 4th Number (12,2) to the lesser of the two Contained Sides (34) shall reach from the Radius Sine 90°. or Versed Sine (0) to 106° 10' on the Versed Sines, increasing towards the Left hand; which is the Angle C required.

Note, The Line of Versed Sines is annexed to the Line of Sines on the *Gunter's Scale*.

A Synopsis, or View of the Proportions for the Solutions of the Four Cases of Oblique Plain Triangles.

Case	Ax.	Given	Reqd.	Proportions.
1	3	$\angle A, B$ C & AC	$\angle B$ BC	$\text{Sn. } \angle B \dots AC :: \text{Sn. } \angle C \dots AB$ And so is $\frac{AC}{AB} = \frac{\text{Sn. } \angle A}{\text{Sn. } \angle B}$
2	3	$\angle C$ & AB Op $\angle B$	$\angle C$ & A Sd. BC	$AC \dots \text{Sn. } B C :: AB \dots \text{Sn. } A$ $\text{Sn. } \angle B \dots AC :: \text{Sn. } \angle A \dots BC$
3	4	$\angle A, B$ & AC	$\angle C, B$ & AB	$\text{Sn. } A B + AC \dots AB - AC :: \text{T half } \angle Z$ $\angle \angle \text{ Opposite } \dots \text{T half } \angle X \angle \angle \text{ Op. Then}$ Half $Z + \text{half } X = \text{greater } \angle C \text{ find } BC \text{ by Case 1.}$
4	5	$\angle A, B$ & BC	$\angle A, B$ & C	$\text{Sn. } A B \dots AC + BC :: AC \dots BC \dots FB$ Then $AB + EB = AD$ Find the Two Angles B and A The half $\dots$ Thus $\text{Sn. } AC \dots R :: AD \dots \text{Co-Sine } A$ And $BC \dots R :: DB \dots \text{Co-Sine } B$ Angle $B + A - 180 = \text{Angle } C$
4	9	alias		$\text{As } AC \times B \dots C :: Rq \dots \text{half } Z \text{ Sides } \times \text{Remainder.}$ $\dots q \text{ Co-Sine half Angle } C, \& \dots$



Spha

Spherical Geometry : Or Stereographick Problems.

AS a Plain Triangle is made by Three Right Lines, intersecting each other in a Plain; so a Spherical Triangle is made, by the mutual Intersection of three Lines on the Surface of a Sphere or Globe; and consequently, as the Superficies of a Globe is a perfect round Body, so the Sides of any Triangle described thereon must needs be Parts or Arches of Three Great Circles of the said Sphere.

It will be necessary therefore before the Learner enters on Spherical Trigonometry, and the Delineation of the Sphere on a Plain, (which may be performed several ways, viz. Stereographick, Orthographick and Gnomonical, of which we shall treat hereafter in its proper place.) Stereographical, that he should understand how to make and measure a Spherical Triangle; in order thereunto, we shall begin with Stereography or Spherical Geometry, by which the Circles of the Sphere are described and drawn, and projected on a Plain.

DEFINITIONS.

The Projection of the Sphere, of what kind soever, is no more but the Representation of those Circles and Points upon all the Points upon a Plain; which are imagined to be describ'd on the Sphere, or upon the Celestial or Terrestrial Globe. As for instance, in Astronomy.

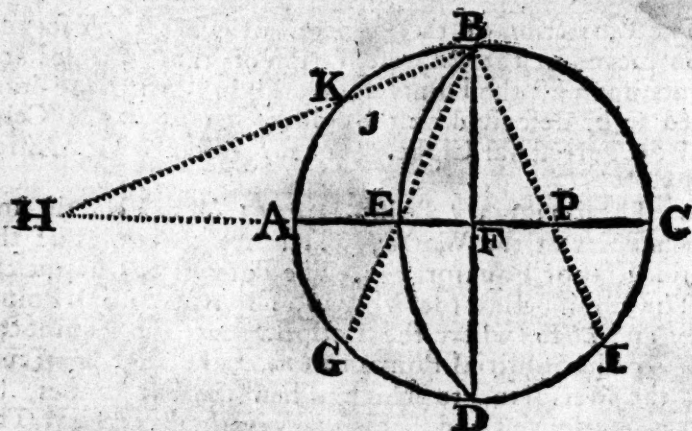
2. The imaginary Points are, 1. The North and South Poles of the World, which are the Poles of the Equinoctial or Equator. 2. The Poles of the Ecliptick. 3. The Equinoctial (or Vernal and Autumnal) Points *Aries* and *Libra*, where the Ecliptick cuts the Equinoctial. 4. The Solstitial Points of *Cancer* and *Capricorn* (cutting the Meridian, to which when the Sun comes, he makes our longest and shortest Day in the Year.) 5. The Zenith and Nadir, &c.

3. The Circles of the Sphere are either greater or lesser Circles. The greater Circles are those that divide the Sphere it self into two equal Parts; as, 1. The Horizon, which divides that part of the Sphere, or Heaven visible to us, from that part that is invisible. 2. The Meridian, which divides the East Half from the West. 3. The Equator, or Equinoctial, which divides the North Half from the South. 4. The Ecliptick, in which the Sun is always moving. 5. The Prime Vertical, &c.

The Lesser Circles divide the Sphere in two unequal Parts; as, the Almicanthars, or Parallels of Altitude, the Parallels of Declination, so called, because parallel to the Equator, &c. Of all which, see a more plenary and particular Description hereafter, in the Use of the Globes and Astronomy.

4. The Stereographical Projection of the Sphere upon the Plain of every Great Circle; as, 1. Upon the Meridian; 2. Upon the Equinoctial; 3. Upon the Horizon; 4. Upon the Ecliptick; 5. Upon the Prime Vertical; the first being mostly used, and indeed most useful in Trigonometrical Operations; in which we make use of a Scale of Chords, Tangents, half Tangents and Secants.

5. Of Great Circles there are three kinds, when projected on a Sphere, viz. 1. That Circle, upon whose Plain the Sphere is projected, is called the *Primitive Circle*, and is always a perfect Circle described about the whole Projection; as the Circle A B C D. 2. A Right

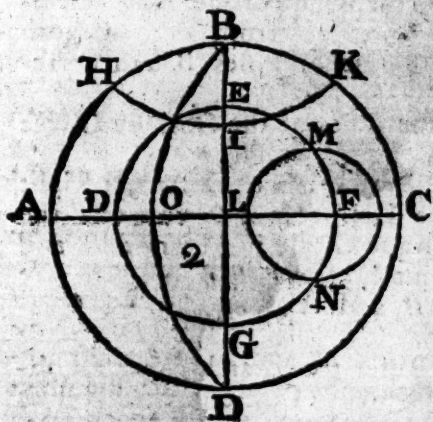


Circle, which appears to the Eye on the Plain a Right Line, or Diameter of the Primitive, cutting the Primitive

primitive at opposite Points, and Right Angles, and passing through the Center thereof, thereby dividing the Superficies of it in two equal Parts, as the Lines or Diameters A F C and B F D are Right Circles.

3. An Oblique Circle cuts the Primitive at Oblique Angles, and in opposite Points; but does not pass thro' the Center of it, and is represented by an Arch, as B E D, Figure 1.

6. Of Parallel or Lesser Circles there are also three sorts; as first, a Parallel to the Primitive, as, the Circle



DEFG. 2. A Parallel to a Right Circle ALC, 3. A Parallel to an Oblique Circle, as is the Arch LMN parallel to the Oblique Circle BOD, and these lesser Circles, neither cut the Primitive at opposite Points, nor (except accidentally) pass thro' its Center. Now these Circles of the Sphere described, considered severally or jointly, will afford us

divers Stereographick Problems, and are the Subject of Spherick Geometry, and are such as follows.

Prob. I. To find the Pole of any Great Circle projected,

In this Problem are three Cases, viz,

Case $\left\{ \begin{array}{l} 1 \\ 2 \\ 3 \end{array} \right\}$ To find the Pole of the $\left\{ \begin{array}{l} \text{Primitive} \\ \text{Right} \\ \text{Oblique} \end{array} \right\}$ Circle $\left\{ \begin{array}{l} \text{ABCD} \\ \text{AFC} \\ \text{BED} \end{array} \right\}$ Fig. 1

Note, That the Pole of any great Circle is the Representation of that Point of the Surface of the Sphere (on the Plain of the Projection) which is 90° distant from that Circle,

2. The Pole of the Primitive is the Center thereof 90° distant from it by a Line of Half Tangents.

3. The Pole of a Right Circle is in the primitive Circle 90° therefrom, by a Line of Chords.

4. The Pole of an Oblique Circle is neither at the Primitive, nor at its Center ; but between the Center of the Primitive and Oblique Circles, in a Diameter which passes thro' the Center of the Primitive.

Fig. 1. Construction. Case I. To find the Pole of the Primitive $A B C D$, find its Center F , by the first Prop. of Book 3 of *Euclid*, which Point F is the Pole.

CASE II. Find the Pole of the Right Circle $A F C$, cross the Diameter $A F C$ with another Diameter $D F B$ at Right Angles ; the two Extremities B and D (in the Primitive) are the Poles of the Right Circle $A F$. Or otherwise, the Chord of 90° . set off from A or C both ways, will determine the Points or Poles B and D required.

CASE III. To find the Poles of the Oblique Circle $B E D$.

1. Let the Diameters $B D$ and $A C$ be drawn at Right Angles to each other ; the Diameter $B D$ connects the Points B and D of the Oblique Circle in the middle and at Right Angles in the Point E ; then $E B$ or $E D$ are each esteemed equal to $A B$, and $A D$ equal to a Quadrant, or 90° .

2. From B or D lay a Ruler to E , and draw an obscure Line $B G$ to cut the primitive Circle in G (which is called Reducing the Elevation of the Oblique Circle $F E$ to the Primitive.) Take 90° from your Line of Chords, and put it off in the Primitive from G to I (or from G to K .)

3. A Ruler laid on B and the Point G , will cut the Right Circle $A C$ in P , the Pole requir'd ; which Point G is the projected Pole that lies above the Plain of the Projection ; but if the projected under Pole had been required, lay a Ruler on B and K , and it will cut the Diameter $C A$ produced in H ; the projected under Pole ; but the Pole G is what we chiefly use.

Otherwise thus : — Since the Right Circle issuing from the half Tangents (accounting the Center of the Primitive the beginning of the Scale of half Tangents) and that every Circle's Pole is 90° from it, as was said before ; therefore measure $F E$ on the half Tangents, which suppose 46° ; then $E F$ 46° subtracted from 90° , rests 44° for $F P$; take 44° from the half Tangents, and set it from the Center F to P ; so shall P be the Pole of the Oblique Circle, as before.

Prob. II.

given Angle 36° from the Chords, as E H; then drawing the Secant-Line D G (the *Primitive* being first cross'd at Right Angles with the Diameters A C and B D) it will cut A C in G, the Center of the Oblique Circle B I D required.

Or, Secondly, 'Tis plain, that E G is the Tangent, and D G the Secant of the given Angle 36° : Therefore the Tangent of 36 , set off from the Center E to G on the Right Circle A C, gives G, the Center of the Oblique Circle.

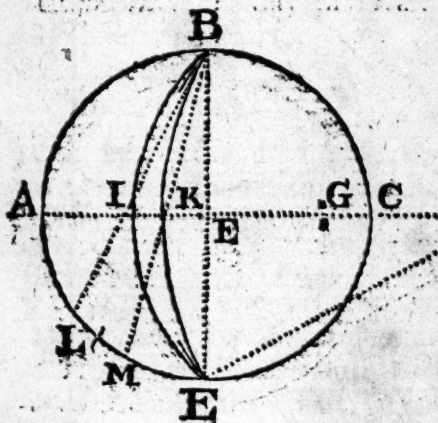
Or, Thirdly, The Secant of the given Angle 36 from your Scale, and one Foot in B or D cross A C in G, the Center, as before.

Lastly, With the Distance or Radius $GD = GB$ describe the Oblique Circle EID , and it shall form an Angle at the Points B or D , equal to the given Angle of $36^{\circ} 00'$.

CASE II. *Variety II.*

To make an Oblique Circle, (as BKE) to make at the Primitive
any given Angle (suppose 23°) with another Oblique Circle
(BLE) given.

Construction. 1. Lay the Ruler from B to I; it shall cut the *Primitive* in L; then set off the Chord of the given Angle, as 23° , from L to M; a Ruler laid from M to B cuts the Diameter of the *Primitive* in K; then thro' the Points B K and E describe the Arch, or Oblique Circle B K E (by *Prob. 2.* of *Plain Geom.*) and 'tis done, and the Angle $IBK = IEK$ is $= 23^{\circ}$, W.W.R.



And thus may you make any Angle, tho' you don't know what Angle the given Oblique Circle BIE makes with the *Primitive*; but if it be known what Angle the given Oblique Circle makes with the *Primitive*, as here suppose 6° . and the Circle requir'd to be drawn is to make at B an Angle of 23° ; then add 36° to 23° , the Sum is 59° ; with the Secant of which 59° find the Center

Center, and describe the required Oblique Circle B K E (which Center falls without the *Primitive*) by the third Direction of the last Case, which will form the Angle B K E, as before $= 23^{\circ}$.

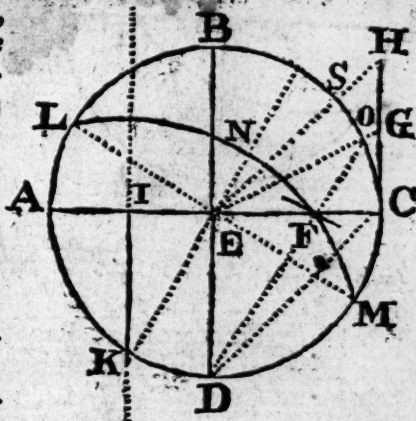
CASE III. Variety I.

To project any given Angle within the *Primitive*, but not at the Center.

First, To draw an Oblique Circle L N M through any given Point F, in a Right Circle A C, to make with the said Right Circle any given Angle, suppose of 50° , is required.

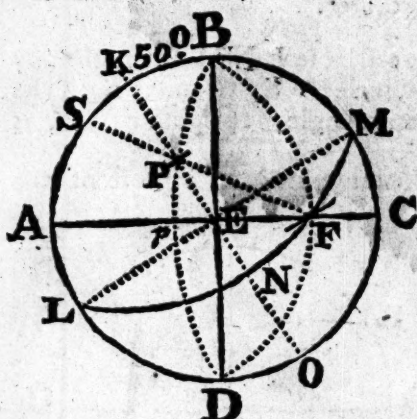
Construction. 1. Lay a Ruler on D, the Pole of the Right Circle A C to F, and it cuts the *Primitive* in O. 2. Draw the Tangent-Line C H from the Point C, and the Secant-Line from the Center of the *Primitive* E through O, to cut the Tangent C H in G. 3. Make C S on the *Primitive* equal to the Chord of the Complement of the given Angle, i.e. 40° , and from the Center of the *Primitive* E through S, draw E H, to meet C H in H; so shall C H be the Tangent-Complement of the given Angle 50° . 4. Make E I equal to the Tangent C G; and on I draw the Perpendicular I K (so is I K a continu'd Line, in which the Centers of all Oblique Circles fall, that pass through the Point F.) 5. Make I K equal to the Tangent-Complement of the given Angle 40° C H; so shall K be the Center of the Oblique Circle required. Lastly, One Foot in K, with the distance K F describe the Oblique Circle M F N L; then shall the Angle E F N $=$ M F C be equal to the given Angle of 50° , as was required.

Or, more briefly thus: Draw the Chord of 90° D C; (then by Theor. 9. of *Plain Geometry*) the Angle O E C at the Center is double the Angle O D C at the Circumference of the *Primitive*, standing upon the same Arch C O; therefore measure the Arch C O on the Chords, (or C F the



the backward way of the half Tangents, suppose it 23° ; then make EI equal to the Tangent-complement of 23° (or else to the half Tangent of 46° , the double thereof,) and IK equal to the Tangent-complement of the given Angle, in this Case 40° , and you have the Center of the Oblique Circle as before.

2. Or otherwise, thus : 1. Through the Poles of the



Right Circle $A C$, B and D , and the given Point F , describe an Oblique Circle as the obscure one $B F D$, and find its Pole p , by *Problem 1.*

Case 3. 2. Draw an obscure oblique Circle $B p D$, thro' the Poles $B p D$. 3. Set off the Chord of the Given Angle 50° from B to S ; lay a Ruler from S to the Given Point F , and

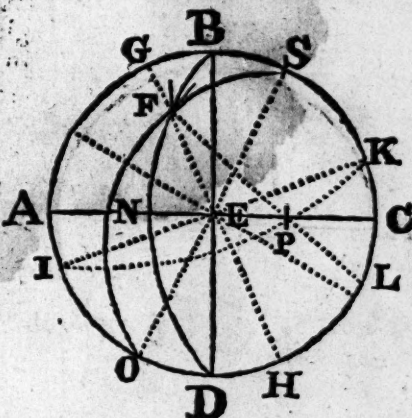
it will cut the Oblique Circle $B p D$ in P ; then through P (which is the Pole of the Oblique Circle requir'd,) and the Center of the *Primitive*, draw the Right Circle $K P N O$; which crosses at Right Angles with the Right Circle $L M$. Lastly, Through the Points M, F, L describe the Oblique Circle M, F, N, L ; so shall the Angle $M F C = N F E$ be equal to the Given Angle of 50° , as before.
Q.E.F.

CASE III.

CASE III. Variety II.

Through any given Point in an Oblique Circle BND, as F, to draw another Oblique Circle, to make with the given Oblique Circle any Given Angle of 27° is required.

Construction. 1. Through the Given Point F, and the Center of the Primitive E, draw the Right Circle GEH; which cross at Right Angles with the Right Circle IEK: Then 2. Measure the Angle GFB by *Prob. 8.* following, which we will suppose to be 48° ; to which add the Angle requir'd to be made, 27° , the Sum 75° is the Angle that the required Oblique Circle is to make with the aforesaid Right Circle, and then proceed in all respects as in the last Variety, by the First or Second Construction, and describe the Oblique Circle OFS to make an Angle of 75° thro' the Point F in the Right Circle GEH, and it's done: For the Angle BFS is 27° , contained between the two Oblique Circles BND and SFO, which was required.



Note. If the nature of the Question had required the Oblique Circle to be projected, to lie nearer the Right Circle than the Given Oblique Circle (as for Instance, had the Oblique Circle SFO, and Point F been given, to make the Oblique Circle BND to make an Angle of 27° , with the former) you must have subtracted the Given Angle 27° from the Angle that the Given Oblique Circle makes with the Right Circle (in this Case equal to 21°) the Remainder is the Angle that the required Oblique Circle is to make with the Right Circle. Then proceed as before.

PROB.

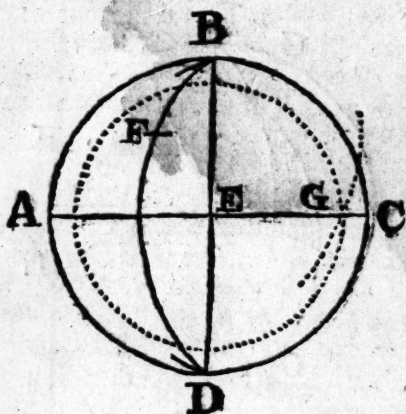


Spherick Geometry : Or,

PROB. III.

To draw an Oblique Circle (BFD) through any given Point F , within the Primitive, to make with the primitive Circle ($ABCD$) any given Angle ; as, suppose of 38° is required.

Construction. 1. With the Tangent of the given Angle 38° , and one Foot in the Center of the primitive Circle. 2. With the Secant of the same Angle, and one Foot in the given Point F , cross the former Circle in G , the Center of the required Oblique Circle ; then with the distance GF describe the Oblique Circle BFD , and it will form at the Points B or D in the Primitive the given Angle. *W.W.R.*

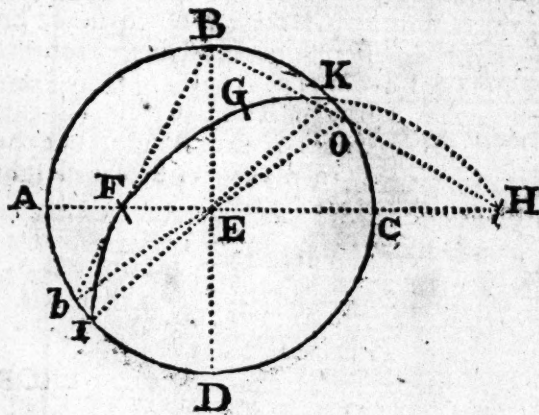


The Reason of which is evident from *Prob. II. Case I.* For the Distance from the Point F (taken any where in the Oblique BFD to the Center G) is equal to the Distance or Secant $GB = GD$ being Radius's of the same Circle or BFD , — .. —.

PROB. IV.

Through any two Points in the Plain of the Projection, to draw a (Great) or Oblique Circle.

Construction. *Case I.* If one of the given Points be in the Center E , and the other in the Primitive, as A , or within the Primitive, as F ; then thro' E , the Center, and the Point A or R given, draw the Right Circle AEC , and the thing is done.



CASB

CASE II. If neither of the Points be in the Center, of the *Primitive*, as F and G, then,

1. Through either Point, as F, and the Center of the *Primitive*, draw the Diameter AFECH continu'd, and cross it at Right Angles with the Diameter BED.

2. From the Point F draw a Line to the Extremity of the second Diameter, as FB.

3. At the End of this Line, viz. at B, erect the Perpendicular BH, to cut the Diameter AC produced in H, the third Point; then (by *Prob. 22. Geom.*) through the Points F, G, H describe a great Circle, as the Oblique Circle I, F, G, K, and 'tis done.

Or otherwise,

Because there will always be 180° of the half Tangents between the first and third Points F and H, having drawn the Diameters BD and AC continued, produce BF to *b*; then lay a Ruler from *b* to the Center of the *Primitive* E, and it will cut the *Primitive* in *o*; a Ruler laid from B to *o*, cuts AC produced in H, the third Point, through which the proposed Circle is to pass,

Or thus:

Measure EF on the half Tangents, which suppose 59° . This subtracted from 180° , the Remainder 121° taken from the Half Tangents, will reach from E to H, the third Point: Then bring the three Points F G H into an Arch of a Circle, as before.

Note, If the two Points of Intersection of the Oblique Circle I and K, being joined with a Diameter IK; if this Diameter or Right Circle pass through the Center of the *Primitive*, 'tis then a true Oblique Circle: otherwise not.

PROB. V.

To draw a Great Circle perpendicular (or at Right Angles) to a Given Great Circle.

This Problem will admit of Four Cases, viz.

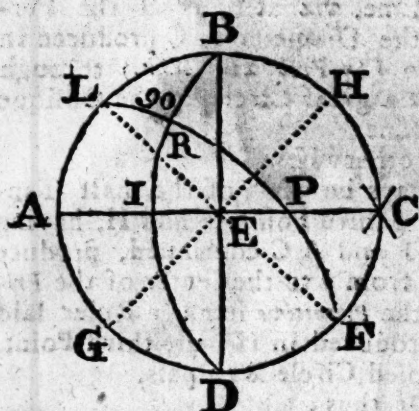
Case	$\left. \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \end{array} \right\}$	$\left. \begin{array}{l} \text{to} \\ \text{draw} \end{array} \right\}$	$\left. \begin{array}{l} \text{a Rt. Circ.} \\ \text{an Ob. Circ.} \end{array} \right\}$	$\left. \begin{array}{l} \text{perp. to} \\ \text{perp. to} \end{array} \right\}$	$\left. \begin{array}{l} \text{the Primitive.} \\ \text{a Right Circle.} \\ \text{a Right Circle.} \\ \text{an Obl. Circle.} \end{array} \right\}$

The

The General Rule is,
 Draw a Great Circle through the Pole of the given Great Circle, and it will be perpendicular to the said Great Circle given.

CONSTRUCTION.

CASE I. To draw a Circle (as AC) perpendicular to the Primitive



lar to the Primitive ABCD, through the Center of the Primitive E draw a Diameter or Right Circle AC, and 'tis done: For the Center of the primitive Circle is its Pole.

CASE II. To draw a Right Circle (as BD) perpendicular to a Right Circle (as AC)

given; cross the Right Circle at AC at Right Angles with the Right Circle BD, and it is done; and B and D are the Poles of the given Right Circle in the primitive Circle.

CASE III. To draw an Oblique Circle (as BID) perpendicular to a Right Circle (as AC) given; find the Poles of the given Right Circle, as BD; with any Distance, and one Foot in B and D, strike two Arches crossing one another in C; then with the Distance CD=CB, describe the Oblique Circle BID, and it's done.

If the Oblique Circle required to be drawn, were to limited, as to make a Given Angle with the Primitive in the Points or Poles B and D, then take the Secant of that Angle in your Compasses; and placing one Foot in B and D, make two Arches crossing each other in C, C shall be the Center. Or, take the Tangent of the said Angle, and set it off in the Right Circle, or Diameter AC from the Center E to D, and it is done.

But if the Point I, in the Right Circle AC were given, through which this Oblique Circle should pass, without any relation to the Angle it should make with the primitive Circle (though that be virtually given)

a Circle struck through the three Points B, I, and D, will answer the Demand.

CASE IV. To draw an Oblique Circle (as L P F) perpendicular, to, or at Right Angles, with a given Oblique Circle, as B I D.

General R U L E.

Find the Pole of the Given Oblique Circle by Prob. I. through that Pole describe an Oblique Circle, that shall make at the Primitive any given Angle, by Prob. III.

C O N S T R U C T I O N.

Having found the Pole of the Oblique Circle given, as P, lay a Ruler over the Center E of the Primitive, any wise cutting the Primitive in the Points L and F, a Circle struck through the three Points L, P and F, shall be the Great Circle (L P F) required, cutting the given Oblique Circle at Right Angles in the Point R.

Note, 1. If the Point R (in the given Oblique Circle) be given, then draw a great Circle through the two Points P and R, by the 4th Problem. Or, 2. If it be required that the Oblique Circle passing through the other's Pole P, shall also make a given Angle with the Primitive, it may be done by Prob. III.

P R O B. VI.

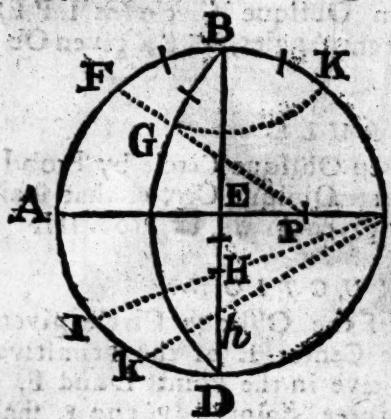
To lay any Quantity of Degrees on a Great Circle.

In this Problem are three Cases, viz.

Case	1	} To lay any Number of De- grees, sup- pose 40°	} on	} the Primitive a Right an Oblique is Required.	} Circle
	2				
	3				

GENERAL RULE.

Lay the Number of Degrees on the primitive Circle,



making two Marks; and find the Pole of the Circle you would lay the Degrees on (by Probl. I.) Then laying a Ruler on the Pole of the Circle, to the two Marks before made on the Primitive (or draw two obscure Lines from the Pole to the Marks) and 'twill cut off the corresponding Degrees, upon either

Primitive, right or oblique Circle.

Construction.

CASE I. Take from your Line of Chords the Number of Degrees (40°) and transfer it to the Primitive (as from the Point B to K or F) and it is done.

CASE II. To lay 40° on the Right Circle BED, and let A and C be the Poles thereof:

Set off the Chord of 40° from A to I on the Primitive. Lay a Ruler from the Pole C to I, and it will cut off EH on the Right Circle BED, equal to 40° .

Or otherwise, seeing that all Right Circles are measured from the Center of the Primitive by a Line of Half Tangents, take the Tangents of its Half, viz. 20° , and lay it off from the Center E to H.

Or from the Scale of Half Tangents take 40° , and it will reach from E to H.

But if it were required to lay off any Number of Degrees from a Point, as H, in a Right Circle, which is not in the Center of the Primitive, suppose of 20° ; then from A or C, the Pole of the Right Circle, and the given Point H, lay a Ruler, and observe where it cuts the primitive Circle, as in I; from this Point I, thus found (as the Case requires that the Degrees should be set towards, or from the Center of the Primitive) set off the Number of Degrees (as here 20°) from I to K; lay a Ruler on the Point K and the Pole C, and where it cuts the Right Circle, as in the Point h, is the required Point; so shall Hh on the Right Circle BED be equal to 20° q. e. f.

CASE

Stereographick Problems.

49

CASE II. To Lay 40° , (or any other Number) on an Oblique Circle as B G D.

1. Draw the Diameters B D and A C at right Angles to each other, and find the Pole of the Oblique Circle as P, by the first Problem.

2. Lay the given number of Degrees (as here 40°) on the Primitive from B to F, by Case 1, of this Prob.

3. Lay a Ruler from P the Pole, to the Point F, in the Primitive, and it shall cut the Oblique Circle in G, so that B G on the Oblique Circle is equal to 40° .

Or thus Thus;

1. Lay off the given Degrees 40, both ways from B to F and K on the Primitive by your Line of Chords.

2. Take the Tangent of the Given Degrees 40° ; and one Foot in F and K describe two Arches above B to intersect each other in X; then one Foot in X with the Radius or Distance X F = X K describe from K to F an Arch (or Parallel) K F, which will cut the Oblique Circle in G, as before, W.W.R.

P R O B. VII.

To measure any Part (or Portion) of a Great Circle projected.

In this Problem are three Cases; and are but the Reverse of the last: Therefore we shall make use of the former Figure.

CASE I. To measure any part of the primitive Circle, as B F, take the Arch (BF) required to be measured; in your Compasses, and apply it to your Scale of Chords, and it shews you the Degrees, as in this case 40° .

CASE II. To measure any part of a Right Circle, (as EH,) Take the Distance EH in your Compasses, and apply it to your Scale of Half Tangents, and you have its Measure, in this case 40° .

But if you would measure H b, neither Point being in the Center, do thus: Measure E b on the Half Tangents, suppose it 60° . Then also measure EH, and let it be 40° on the Half Tangents, from B b 60° take EH 40° , and you have 20° for the part or portion of the Right Circle H b.

CASE III. To measure any portion of an Oblique Circle, as B G.

1. Find the Pole of the Given Oblique Circle, which is P.

2. Lay a Ruler on the Pole P, to the two Extremities of the portion to be measured, as B G.

3. A Ruler laid, will cut the Primitive in two points B and F, whose distance measured on the Line of Chords gives the Quantity of Degrees desired; in this case B G is 40° . W.W.R.

P R O B. VIII.

To measure any Spherick Angle.

General R U L E S.

1. A Spherick Angle in the projection is measured, by measuring the Arch of a Great Circle, passing between the two Sides or Arches containing the Angle, at a Quadrant or 90° distance from the Angular Point (of which consequently the Angular point must be the Pole) if such an Arch be described in the projection. If not, then,

2. Measure the distance between the two Poles of the Circles containing the Angle; and that is the Measure of the Angle required.

We shall instance in all the Five Varieties; as,

1. A Right Circle with the primitive, which makes always Right Angles, as, I L and A C, or, with the primitive A B C D, the Angle B I N, or F A u, is 90° .

2. A Right Circle with a Right Circle; because the Poles of all Right Circles are in the primitive, their distance measured on the Chords is the Quantity of the contained Angle; let the Quantity of the Angle

A E F be required; the Pole of A E C is B or D, and the Pole of F B K is I or L; add B I = D L = A F = C K, measured on the Chords, gives the Quantity of the Angle A E F, in this case 40° .

Otherwise thus: By the First General Rule, A F is an Arch of the Primitive at 90° distance from the Angular Point or Pole of the primitive; therefore A F is the Quantity of the required Angle measured on the Chords.



3. An Angle made by the Intersection of a Right Circle with an Oblique Circle; 'tis requir'd to know what Angle the Right Circle I E L makes with the Oblique BND at the point N.

Find the Pole of the Right Circle, which is F, and of the Oblique Circle, as u , by Prob. I. A Ruler laid from the Angular point N, to the Pole of the Oblique Circle u , will cut the primitive Circle in C, and F, being the Pole of the Right Circle; the Arch C F measured on the Chords, is (56°) the Quantity of the Angle BNI required.

After the same manner you may measure the Angles DTK or BHF.

4. An Angle made by the Intersection of an Oblique Circle with the Primitive, as the Angle NBI.

The Pole of the Primitive is its Center, as E; and the Pole of the Oblique Circle BND is as u . A Ruler laid from the Angular point B to u , cuts the Primitive in O; and from B to E, cuts it in D; the Arch DO measured on the Chords (or the distance of the Marks E u measured on the Half Tangents) is 45° , the Quantity of the Angle NBI required.

After the same manner you may measure the Arch of Angle HBE, or BEG, which being Obtuse, is greater than 90° , the distance of the 2 Poles E u , being measured on the Half Tangents (suppose 60°) and taken from 180° , the Complement, or Residue (120°) is the Angle BEG.

5. And lastly, to measure an Angle made by the Intersection of an Oblique Circle with an Oblique Circle, as the Angle BGF is requir'd.

1. Find the Poles of the Oblique Circles BGD and FGK (by Prob. I.) which are N and q (the Poles of the Circles containing the Angles required.)

2. Apply a couple of Rulers, or draw a couple of obscure (or prickt) Lines from the Angular point G thro' the two Poles N and q , cutting the Primitive in the points r and m ; then the Arch rm applied to the Line of Chords, shews the Number of Degrees of the Angle BGF (as here, 47°) *q. e. i.*

PROB. IX.

To describe a Parallel, or small Circle.

 Parallel Circles on the Projection, are the Representation of those parallel Circles on the Globe, which divides the Surface thereof into two unequal parts, and

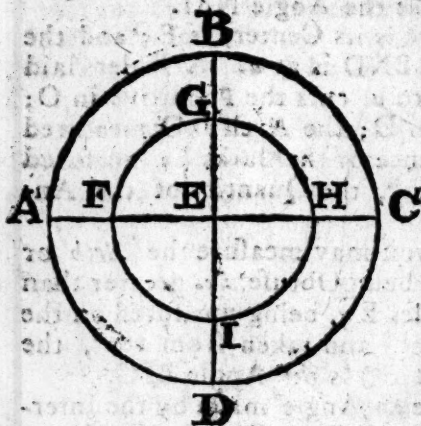
are always parallel to some great Circle, as Parallels of Altitude of the Sun or Stars are parallel to the Horizon; Parallels of Declination, the Tropicks and polar Circles are parallel to the Equinoctial, &c.

This Problem admits of three Cases, viz.

To describe a { the primitive } Circle { 1 } Case.
Parallel to { a Right } { 2 }
 { an Oblique } { 3 }

CASE I. To draw a Circle parallel to the primitive Circle, as FGHI 40° from the Primitive ABCD, or 50° from its Pole or Center E.

Construction. With the Tangent of half the Parallel's Distance from the Pole E (25°) or the Half Tangent of its Distance (50°) and one Foot in the Pole or Center E, describe the Circle FGHI, and it's done.

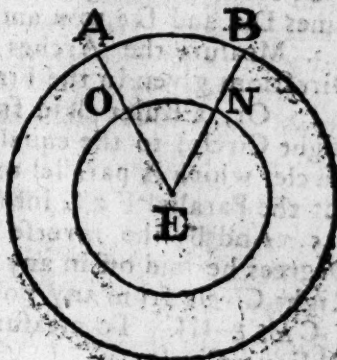


CASE II. To describe a Circle (as FKG) parallel to a Right Circle (AEC) at 40° distant from its pole is required,

Cor.

CASE I. To measure an Arch of a Parallel to the primitive Circle (as O N.)

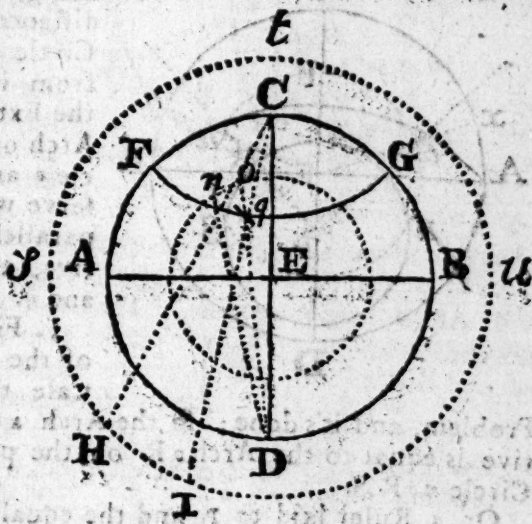
Construction. From the Pole or Center of the primitive Circle E through the two Extremities of the given Arch O and N, draw the Right Lines EO and EN, produced till they cut the Primitive in A and B; then the Arch AB measured on the Chords is the Quantity of the given Arch O N required.



By the Inverse Method may any Number of Degrees be laid down upon any small Circle, parallel to the Primitive from any point given.

CASE II. To measure any Arch of a small Circle, parallel to a Right Circle, as N o; or divide into equal Degrees.

Construction. 1. Describe a Circle parallel to the primitive Circle, as far distant from the Pole of the primitive E, in which the Eye is, as the Parallel F G is from C, the Pole of the Right Circle AEB 50° , and consequently distant from the opposite or (under) Pole, in which the Eye is supposed at 130° (or which is the same) the Tangent of 65° , and one Foot in the Center of the Primitive, describe a Parallel within the primitive Circle also.



2. From C, through the two Extremities n and q , of the Arch nq to be measured; draw the obscure Lines CH , CI to cut the outward Parallel st in H and I .

Or from D through n and q , draw the obscure Right Lines Dn and Dq to n and q .

3. Measure the Arches HI or nq , according to the Directions given in the First Case nq .

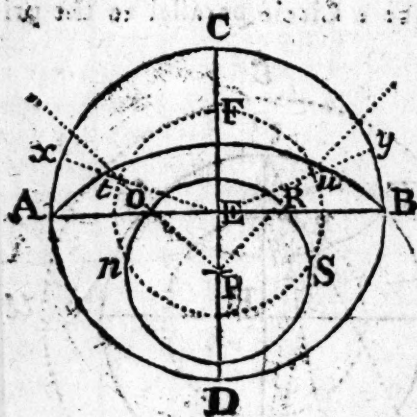
4. Or, a Ruler laid from C (the Pole of the given Right Circle) to the equal Divisions of that Half of the Circle, which is parallel to the Semi-circle ADB , shall cut the Parallel st into its correspondent Degrees.

5. And by the inverse Method may any Number of Degrees be laid off in any Circle parallel to a projected Right Circle from any point given.

CASE III. To measure any Arch or Parallel to an Oblique Circle.

As, suppose it were required to measure the Arch OR of the Parallel st , or to divide the Parallel st or RS , parallel to the Oblique Circle AFB , and distant from its Pole P 50° .

Construction. 1. Describe a Circle parallel to the Primitive, with the Half Tangents of 30° , the distance the parallel Circle st or RS given is from its Pole B , and the Extremities of the Arch of the small Circle st and R ; and observe where it cuts the parallel to the primitive, as in the points x and u .



3. Find the Quantity of the Arch st , by Case the First of this

Problem, and it's done; so the Arch st on the Primitive is equal to the Arch OR on the parallel, or small Circle st or RS .

Or, a Ruler laid to P , and the equal Divisions of the Circle parallel to the primitive before described, shall divide the parallel st or RS into correspondent Degrees.

Here, also by the Inverse Method, may any Number of Degrees be laid off in any small Circle Parallel to an Oblique Circle from a Given Point.

PROB.

PROB. II.

Two Great Circles given by position, forming with each other an Angle given; 'tis required to draw another Great Circle, that shall form two Angles with the former, equal to Angles given; provided the Sum of the three Angles be more than 180 Degrees? Or, which is the same, to construct (make) a Right-angled, or Oblique-angled Spherical Triangle, having the Three Angles given?

EXAMPLE.

Fig. 1.

The Angle $\left\{ \begin{array}{l} F 93^{\circ} 00' \\ B 64^{\circ} 30' \\ G 36^{\circ} 30' \end{array} \right\}$ given the Angle $\left\{ \begin{array}{l} B 128^{\circ} \\ G 56^{\circ} \\ F 34^{\circ} \end{array} \right\}$

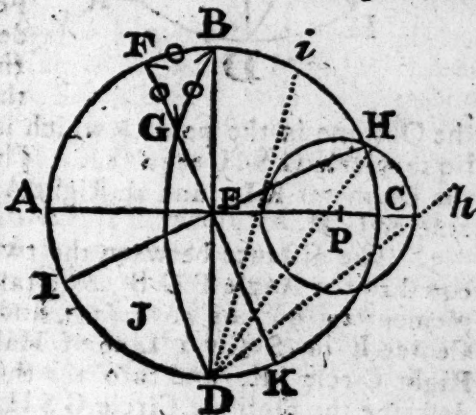
Construction. Make the Angle F B G at the primitive Circle equal to the given Angle B $64^{\circ} 30'$ (by Prob. 2. Case 2.)

2. Find the Pole of the Oblique Circle B, G, D (that makes the foresaid Angle with the Primitive) which is P, and describe a Parallel to an Oblique Circle, distant from it $36^{\circ} 30' =$ the given Angle B G F

(by Prob. 9. Case 3.) to cut the Primitive in H the Pole, of the required Right Circle.

3. Draw the Diameter H I, and at Right Angles to it the Diameter or Right Circle F K, cutting the Primitive in F, and the Oblique circle in G; so shall B F G be the required Triangle, in which

The Angles at $\left\{ \begin{array}{l} F \\ G \\ B \end{array} \right\}$ are equal to $\left\{ \begin{array}{l} 90^{\circ} 00' \\ 36^{\circ} 30' \\ 64^{\circ} 30' \end{array} \right\}$ the given Angles, which was required.



CONSTRUCTION II.

1. With the Secant-comp. of the Angle B, i. e. 128°

$-180^\circ = 52^\circ$) describe the Angle G B F, and find the Pole of the Oblique Circle EFD , which is at p , and describe a Parallel from the Pole p , at $34^\circ \angle B$, distant from it, as in the last.

2. With the Half Tangent of the other Angle (G 56°) given, and one Foot in the Center E describe a parallel to the primitive, to cut the former parallel to

the Oblique in the point P, which is the Pole of the Oblique Circle G S H required. Through P and E draw the Diameter K L, and at Right Angles to this, the Diameter G H.

3. The distance between the two circles E and P, is equal to the Angle F G B 56° ; take therefore the Complement to 90° of $56^\circ = 34^\circ$, and set it off from the Center E to S (by a Line of Half Tangents) on the Right Circle I K, and thro' the three points G, S, and H describe the Oblique Circle G S H, and it's done. (Then in the Oblique Triangle G B F the Angle F B G is $= 128^\circ$, the Angle B G F $= 56^\circ$, and the Angle G F B $= 34^\circ$, which was required.)

Or with the Secant of the given Angle B G F $= 56^\circ$, and one Foot in H and G, describe the Arches to intersect each other in X. Or the Tangent of 56° set off from E to X, on the Diameter I K (produced, if needful) will find the Center of the required Oblique Circle G S H, as before.

And thus much shall suffice for *Problems*, necessary for the making and measuring any Spherical Triangle; and I would advise the Learner to be perfectly acquainted with them; and then the Geometrical Construction, or Mensuration of the following Cases of Spherick Triangles, as also the Projection of the Sphere upon the plain of any Great Circle will be the better understood, and made easy to him.

CHAR.

Spherick Trigonometry: Or, the Doctrine of Spherical Triangles.

CHAP. VI. SECT. I.

Definitions, Affections, and Properties of Spherick Triangles.

1. **A** Plain Triangle is made by the mutual Intersection of three Right (or Streight) Lines upon a plain Superficies; so is a Spherical Triangle formed by the mutual Intersection of the Arches of Three Great Circles on the Surface of the Sphere.

2. Again, as a plain Angle is measured by the Arch of a Circle, described from the Angular Point, and contained between the Lines which form the Angle; so a Spherical Triangle is measured by the Arch of a great Circle, described from the Angular Point on the Surface of the Sphere, and contained between the Arcular Lines which form the Angle.

3. Hence, if the Sides that contain the Spherick Angle are continu'd to Quadrants, *i. e.* 90° (as the Sides GB to I, and GS to S, containing the Angle BGF in the last Projection,) then the Arch of the Great Circle IS (of which the Angular Point G is the Pole) contained between them, is the Measure of the Angle.

4. The opposite Angles (as IGS and IHS in the last Projection) at the Intersections of Two Great Circles (as GIH and GSH) are equal.

5. The Greater Angle of a Spherick Triangle (as well as a Plain Triangle) is opposite to the greater Side; and contrary, the greater Side is opposite to the greatest Angle.

6. Spherick Triangles that are equal-sided, are also equal-angled.

7. An Isosceles (or equicrural Triangle) hath its two Angles at the Base mutually equal; and, on the contrary, if a Triangle has two Angles equal, it has two Sides equal.

8. If

8. If there be two Triangles, and in each one Angle, and the two Sides including, respectively equal; or if one Side and the two Angles adjacent be severally Equal, then the two Triangles are equal: For if laid upon one another, they will agree.

9. The Sum of the Three Angles of every Spherical Triangle is more than two Right Angles, or 180° ; and less than six Right Angles, or 540° .

10. Any Two Angles added together, their Sum is more than the Difference between the third (or remaining) Angle, and 180° .

11. A Great Circle passing through the Poles of another Great Circle, cuts it at Right Angles; and contrary, if it cuts it at Right Angles, it passes through its Poles.

12. When a Circle falls on another Circle, the Sum of the Two Angles made thereby, is equal to Two Right Angles, or 180° .

13. When a Circle crosses a Circle, the Vertical Angles (at the Interfection) made thereby, are mutually equal.

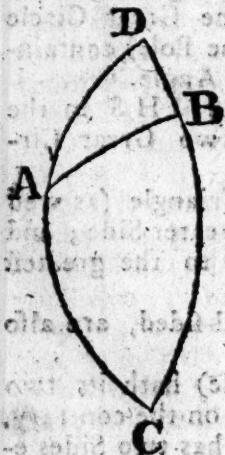
Or the Opposite Angles, made by the Interfection of two Great Circles, are equal.

14. In any Spherick Triangle the outward Angle is less than the Sum of the Two Inward and Opposite.

15. All Great Circles cut each other into equal Parts: Therefore,

16. Every Side of a Spherical Triangle is less than a Semicircle, or 180° .

Let A B D be a Spherick Triangle, and the Sides D A and D B produced to Semicircles intersecting each other in C and D.



Demonstration.

DB is less than DBC, and DA less than DAC.

17. The Sum of the Three Sides of any Spherical Triangle is less than a Circle, or 360° . For BA is less than BC + AC; and consequently BA + BD + DA is less than DBC + DAC; the Sum of which two last (being Semi-circles) make a Circle.

18. In

18. In any Spherical Triangle, if the Sum of the Sides of an Angle be greater than equal to, or lesser than a Semi-circle, the internal Angle at the Base shall be greater than equal to, or lesser (accordingly) than the outward and opposite Angle at the Base.

Demonstration:

1. If $DB + BA = DBC$, then $BA = BC$, and consequently the Angle $BAC = \angle C$, which is $= \angle D$ by the 14th, *q. e. d.*

2. If $DB + BA$ be greater than DBC , then BA is greater than BC , and consequently the Angle BAC less than the Angle C , which is $= \angle D$, by the 14th; and consequently the Angle at D is greater than the Angle BAC , *q. e. d.*

3. If $DB + BA$ be less than DBC , then BA is less than BC ; and consequently the Angle BAC is greater than the Angle C , which is $= \angle D$. Wherefore the Angle D is less than the Angle BAC , *q. e. d.*

Coroll. Hence it is manifest, that if any two Sides of a Spherical Triangle taken together are greater than equal to, or lesser than a Semi-circle, the Sum of the Angles opposite to them shall be accordingly greater than equal to, or lesser than a Semi-circle, or 180° .

Demonstration.

1. When $DB + BA = DBC$, then the Angles at D , and DAB are $= DAB + BAC$, equal to Right Angles, by the 13th.

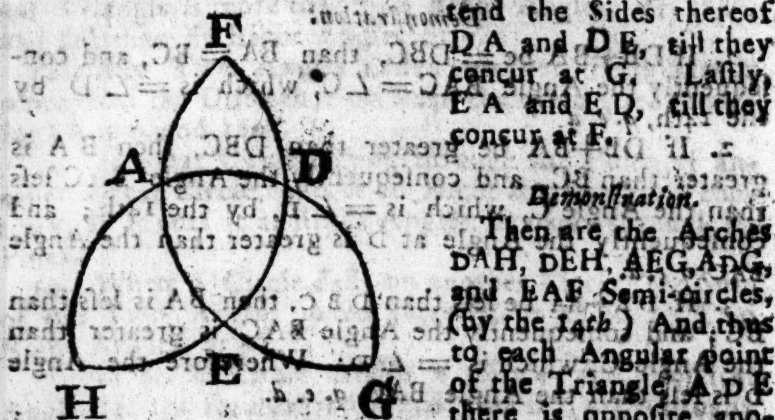
2. When $DB + BA$ is greater than DBC , then the Angles D and DAB is greater than the Sum of DAB and BAC ; for D is greater than BAC .

3. When $DB + BA$ is less than DBC , the Angle at $D - C$ is less than BAC ; and consequently the Angles D and DAB taken together, are less than DAB and BAC taken together; which are equal to two Right Angles, by the 13th.

Coroll. Hence it is manifest, that in an Equilateral Triangle, if one of the equal Legs (Sides) be equal to, greater or lesser than a Quadrant, then the opposite Angles at the Base are each accordingly equal to, greater or lesser than a Right Angle.

19. Every Spherical Triangle hath, opposite to each Angular point another Triangle, having the same Base with the former, and the Angle opposite thereto equal; the other Parts of it are the Complements of the several parts of the former to a Semi-circle, or 180° .

Construction. Let ADE be a Spherical Triangle; extend the Sides thereof DA and DE, till they concur at G. Lastly, EA and ED, till they concur at F.



Then are the Arches DAH, DEH, AEG, and EAF Semi-circles, (by the 1st.) And thus to each Angular point of the Triangle ADE there is opposite another Triangle, having the same Base with the former, &c. that is, to the Angular point E there is opposite the Triangle ADE, whose Angle at F = Angle ADE, and the Base AD is common to both Triangles; and the Sides EA and ED are the Complements of the Sides DA and DE; and the Angles FAD and FEA are the Complements of the Angles EAD and EDA, to a Semi-circle, or 180° .

N.B. If therefore the Triangle to be resolved be Obtuse-angled, or having two of its Sides either of them greater than a Quadrant; tho' you might find out the thing required in that, yet it will be more convenient to resolve one of the least of the three Triangles opposite to his Angular points, as if a Question were proposed in the Triangle ADE, it may more conveniently be wrought in the Triangle AFD, &c.

20. The Poles of the three Sides of any Spherical Triangle, when connected together by Arches of Great Circles, will with those Arches constitute another Triangle, whose Sides shall be equal to the Angles of a Given Triangle; and its Angles, equal to the Sides of the said Given Triangle, if only instead of the greatest Side, or greatest Angle, you take the Complement thereof to a Semi-circle.

Thus

Thus in the last Projection, or Figure in Stereographic Problems, and by *Prob. 8.* of Stereographic Problems, the Distance of the Poles of the Circles forming an Angle, is the Measure of the contain'd Angles.

So in the Triangle GBE forefaid, the Measure of the Angle FBG is the Side Ep; the Measure of the Angle BGF is EP, (and by *Prob. 4.* of Stereographic Problems, drawing a Great Circle through the two Poles Pp) the Measure of the Angle GFB is Pp (the distance of the Poles of the Sides containing each Angle.)

Secondly, by the same way of arguing, likewise, in the Triangle EPe, the Angle eEP is measured by the Side BG, the distance of the Poles of the containing Sides EC and EK, which is B and G; and so in like manner, the Angles EPe and EpP are equal to the Sides GF and BF, as you may easily prove by the 5th Stereographic Problem.

Corol. 1. Hence it is manifest, that the three Angles (of a Spherical Triangle) being given, the three Sides may be easily found; and, on the contrary, the Sides being given, the Angles may be easily had.

2. Hence, and from the 10th, the three Angles of any Spherical Triangle, are greater than Two Right Angles.

For EP and Pp, the Measure of the Angles G and F taken together, are greater than Ep, the Supplement or Complement of the Angle B, to 180° . And since Ep, and its Complement to a Semi-circle make but Two Right Angles (for $AF = Ep$, and the Complement of the Angle B, and $FC = \angle B$) - Now $AF + FC = AC$, Semicircle, or 180° ; therefore the Measure of the Angles EP and Pp, and the Complement (of the Angle B) Ep taken together, shall make more; that is, the Angles BG and F, taken together, make more than Two Right Angles, or 180° .

2d. The internal (or inward) and external (or outward) Angles of any Spherical Triangle are equal to Six Right Angles (or 540°). For $AF + FC = \text{Two Right Angles}$; and $LS + SK = \text{Two Right Angles}$, which are the Measure of the internal and external Angles at B and G; and by the 13th, the Arch BED falling on the Circle GFH, the Sum of the Angles made by their Intersection, is equal to two Right Angles, which make together Six Right Angles, 540° . Whence it's manifest, that their Sum can never exceed Six Right Angles.

Whence

3. If the Hypothenuse is less than 90° , the Legs (and consequently the Angles) are similar; that is, both are either greater or less than a Quadrant: And on the contrary, if the Hypothenuse be less than a Quadrant, the Legs (and consequently their opposite Angles) shall be, one greater, the other less than 90° .

4. If the Hypothenuse be greater than 90° , each of the Legs shall be similar to its adjacent Angle; *i. e.* if the Leg be greater than 90° , the Angle shall be likewise greater; and the contrary. Wherefore *vice versa*, if the Leg of a Right-angled Triangle be of the same kind with its adjacent Angle, the Hypothenuse shall be greater than a Quadrant.

5. If the Hypothenuse be less than 90° , each Leg shall be of different kind to its adjacent; and on the contrary, if the Leg be of different kind to its adjacent Angle, the Hypothenuse shall be less than a Quadrant.

24. Of Spherical Triangles are three sorts, *viz.* Right-angled, Oblique-angled, and Quadrantal.

A Right-angled Triangle hath one Angle 90° ; as the Triangle BFG, Right-angled at F, in Fig. 1. to Prob. 11. *Stereog. Prob.*

A Quadrantal Triangle hath one Side 90° ; as the Triangle EGB in the aforesaid Figure, whose Quadrantal Side is EB.

An Oblique Triangle hath no Side nor Angle 90° ; as the Triangle GBF in Fig. 2. to Prob. 11. of *Stereograph. Probl.*

Of an Oblique Triangle, some of the Angles and Sides are Acute, (*i. e.* less than 90° ;) and some Obtuse, (more than 90° .)

As, the Angles BFG and FGB are Acute, and the Angle FBG is Obtuse.

Triangles made by the Intersection of two lesser Circles, or of a Small Circle with a Great Circle, we take no notice of in the *Doctrine of Spherical Triangles*.

24. In Spherical Triangles are 28 Cases, *viz.* 16 in Right-angled, and 12 in Oblique-angled; all of them may be solved two different Ways, which we come now to shew: First, I begin with the Solution of Right-angled Triangles, by the Globical Projection (called by some,) which depends upon the following *Lemmas*, *Conclusions*, or *Axioms*, *viz.*

As the Sine of the Hypothenuſe AD

L to the Sine of the Hypothenuſe AI :

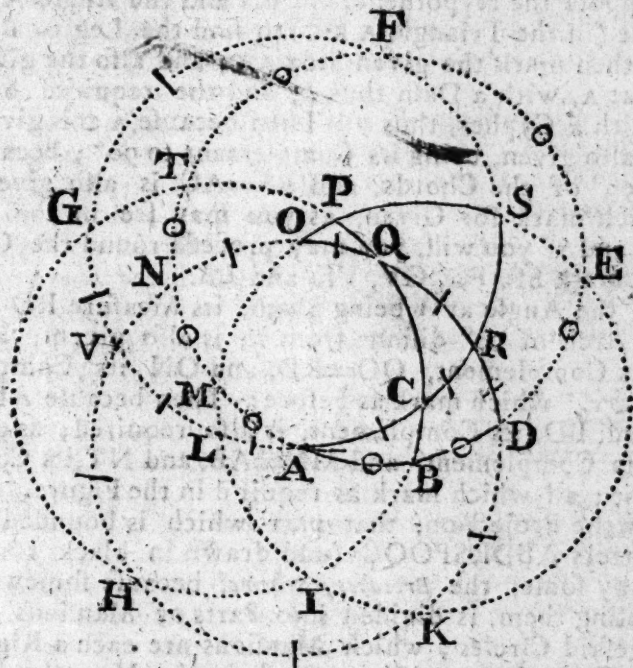
So is the Sine of the Perpend $DC=DB$

To the Sine of the Perp. $IH = LA$ *q. e. d. & contrā.*

3. The Sines of the Angles subtended by the Perpendiculars, are directly proportional to the Sines of the Perpendiculars subtending.

4. And if two Perpendiculars subtend equal *Angles*, the Sines of the Perpendiculars are directly proportional to the Sines of the Hypothenuses.

Next, To describe the General Scheme, or Projection, to find the Proportions for solving the 16 Cases of Right-angled Spherical Triangles, by the (so called) Globical Projection of the Sphere.



Construction. 1. With the Chord of 90° , and one Foot of the Compasses any where, as in A, describe the Circle OHKD.

2. Then with the same Extent, and one Foot any where in the *Periphery* of that Circle, as in O, draw the Circle ARFG; let the Chord of the *Angle* at A (as suppose 25°) from O to Q (although in this you need not be exact; for you may find the proportion as well, if you make the *Angle* A at pleasure;) then 3. With the Chord

of 90° , and one Foot in Q, sweep the Circle AEFN; and by reason that the Triangle ABC given, we would have Right-angled at B.

4. Take the Chord of 90° , and setting one Foot at some convenient distance from A (suppose at B) with the other extend to the Point M, in the same Circle AEFN, upon which the Right Angle is to be; and upon M as a Center, describe the Circle BCPGH; then is the Triangle ABC finish'd.

5. Lastly, With the same Extent of 90° , and one Foot in the Point C describe the Circle OIKE, and then is the Projection fitted for our intended Use.

Then for Calculation, observe these Four Rules following:

1. Observe what parts of your Triangle are given: As, suppose the Hypothenufe AC, and the Angle A at the Base (in the Triangle ABC) to find the Leg or Base AB; then mark the given Side AC, and also the given Angle at A, with a Dash thus /; and the required Side AB, with a Cypher, thus o: Then because AC is given, CR is also given, being its Complement to 90° ; because AR is 90° of the Chords, and RS=AC is also given: all which mark for Given, as you may see in the Figure; and if you will, you may proceed round the Circle, to mark SF, FG, GV, VL, and LA.

And the Angle at A being given, its Measure RD being an Arch of 90° distant from it, is also given; and RQ its Complement, QO=RD, and ON its Complement, &c. which mark as before: Then because AB is required, BD, its Complement, is also required; as also AM, its Complement, and MN=AB, and NT, its Complement; all which mark as requir'd in the Figure.

2. In the Projection, that part which is bounded by the Letters ABDRSPOQC (and drawn in black Lines) called by some, the *Breeches*, *Scheme*, because somewhat resembling them, is divided into Parts or Mansions, by the foresaid Circles; which Mansions are each a Right-angled Triangle, consisting of a Base, an Hypothenufe, and Two Perpendiculars; as do the Mansions ACR, DB, or BCQRD, &c. [Therefore when you are skill'd in this kind of Projection, you may, instead of the Whole Circles, only draw that part represented by the black Arches aforesaid, and omit the prick'd ones: Nor need you be careful to draw all the Arches or Parts therein exactly proportionable; but may draw them with a Pen at pleasure, holding to the same Form, or so

So much of it as you conceive you have Occasion for. Wherefore observe to place the given Triangle at the Bottom-Corner, as here you see is ABC . To find out the Proportion, take notice, in which Mansion you have Three things given, and One required; which is here in the Mansion $QRDBC$, Right-angled at R and D , the Side QD being the Base, and QB the Hypothenuse, and CR and DB are Perpendiculars; and here you have QR given, and also QD the Radius, which is always given; and you have CR also given, to find BD , in which the Proportion is (as in *Axiom 1. Prop. 3.* foregoing) as in Plain Triangles, whose Sides are similar and proportional, *viz.*

As the Short Base $QR = \text{Comp. Angle at } A$
Is to the Long Base QD , or Radius Sine 90° ;
So is the Short Perpend. $RC = \text{Comp. of Hypoth. } AC$
To the Long Perpend. $BD = \text{Comp. of Base } AB$:

3 But to know whether the Proportion lies in Sines, or Tangents, or both, you may observe as in Plain Trigonometry, when the Base is made Radius, the Perpendicular is a Tangent; but when the Hypothenuse is made Radius, the Perpendicular is a Sine.

So here, when the Proportion lies in the Base and Perpendiculars, the Long Base is Radius, and the Short Base a Sine, and the two Perpendiculars are Tangents; But when the Proportion lies in the Hypothenuse and Perpendiculars, the Long Hypothenuse is Radius, and the Short Hypothenuse is a Sine, and the Perpendiculars are Sines. See the Demonstration of *Axiom 1.* foregoing

4. And Lastly, To know whether your Proportion lies in Sines and Tangents, or in their Complements, Observe whether the Parts of the Mansion where the Proportions lie, be the Parts of the Triangle given or required, or the Complements of those Parts; as in this Case, in the Mansion QDB , the Proportion is,

As $QR .. QD :: RC .. DB$, by *Rule II.*

But by *Rule III.* QR is a Sine, QD a Radius, and RC and DB are Tangents.

And by *Rule IV*. QR is the Complement of the Angle at A (because it is the Complement of DR , the Measure of it;) and CR is the Complement of AC , and BD is the Complement of AB .

Therefore the Proportion is,

As (QR) the Sine-Comp. of the Angle A }
 Is to (QD) Radius, } given,
 So is (RC) the Tang. Comp. of AC }
 To (BD) the Tang. Comp. of AB required.

For Solution of the following Sixteen Cases, suppose as before, ACB a Right-angled Triangle, and its Sides produc'd to Quadrants AR, AD, DQ . Suppose also CS, CP, RO , and SO , Quadrants.

Then is $RS = AC$ and the Complement of $AB = BD = \angle BQD$, and $PS = \angle PCS = \angle ACB$, and $OQ = RD = \angle A$, and the Angles at B, D, R, S, P , right.

A Sy-

*A Synopsis of the Proportions (or Literal and Verbal Solutions)
of the 16 Cases of Right-angled Spherick Triangles, according
to the Globical Projection of the Sphere.*

Nr.	Given.	Reqd.	Rough Scheme.	Mansion.	Proportion or Solution.
1	Hypoth AC & $\angle A$.	Perp BC oppo- site.		A D R	.. Sn. AR : AC : Sn. RD.. Sn CB. i.e. As R Sn 90 is to Sn. Hypo. AC ; So is the Sn. giv- en $\angle BAC$ To the Sn. of the oppo ^t Sd BC, by Ax. 1. of P. or S. Trig.
2	Hypoth AC and one side BC.	$\angle A$ oppo- site.		A D R	.. Sn. AC .. Sn. AR : Sn CB.. Sn R.D = $\angle$ A. i.e. As Sn. Hypo. AC, Is to Radius ; So is is Sn. Perp. BC, To Sn. $\angle A$ opposite, by Ax. 1.
3	Sd BC & $\angle A$ A oppo- site.	Hypoth AC		A D R	.. Sn DK .. Sn. BC : Sn. RA .. Sn. CA. i.e. As Sn. given $\angle A$, Is to the Sn. of the oppo- site Side BC ; So is the Radius To the Sn. of the Hypoth AC. Ax. 1.
4	Sd AB and adja- cent $\angle A$.	Perp or Side BC		A D R	.. Sn AD .. Sn. AB : Tang. DR .. Tang. BC. i.e. As Rad. (Ax. 3) Is to Sn. of given Sd AB ; So is Tang. of given $\angle A$ To Tang. of re- quir'd Side BC.
5	Sd BC & $\angle A$ A oppo- site.	Base or Side AB		A D R	.. T. RD .. T. CB : Sn. DA .. Sn. BA. i.e. As Tang. of given $\angle$ A, Is to tang. of giv- en Side BC ; So is R. To the Sine of the Sd AB. by Ax. 3. inverted.

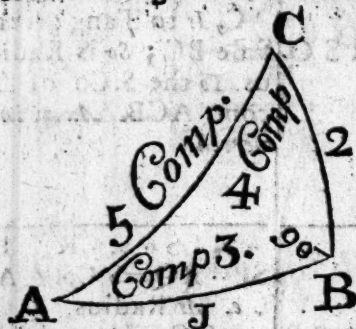
Nr	Given	Required	Rough Scheme	Manifession	Proportions or Solutions
6	The Sds. AB and BC	The $\angle A$ or C opposite		A DR	$\therefore \text{Sn. AB} \therefore \text{Sn. AD} \therefore$ $\text{TBC} \therefore \text{TDR} = \angle A.$ <i>i. e. As Sine on the sides AB is to Radius, So is Tang. of the other side BC, To the Tang. of its opposite side = A. Ax. 3.</i>
7	Hypo. AC & one of the Legs BC.	Base AB		B D Q	$\therefore \text{SQC} \therefore \text{SQB} \therefore \text{S. C}$ $\text{R} \therefore \text{S. BD} = \angle Q = \text{S.}$ $\text{C. AB. i. e. As the S. Co. CB, is to Radius; So is S Co. AC, To the Si. Co. of AB. Ax. 1.$
8	Sds. AB and BC.	Hypoth. AC		B D Q	$\therefore \text{S. QB} \therefore \text{S. QC} \therefore \text{S. B}$ $\text{D} \therefore \text{S. CR} = \text{S. Co. AC.}$ <i>i. e. As Radius, is to the S. Co. of one of the Sds BC; So is S. Co. of other Sd. AB, To S Co. of Hypoth. AC. Ax. 1.</i>
9	One Side AB and adj. $\angle A$	Hypoth. AC.		B D Q	$\therefore \text{S. QD} \therefore \text{SQR} \therefore \text{T. B}$ $\text{CR} = \text{S. C. AC. i. e. As Radius, is to S. Co. given } \angle A; \text{ So is T. Co. of given Sd. AB, To T. Co. of Hypoth. AC. Ax. 3.}$
10	Hypoth. AC and $\angle A$.	Adj. Side AB		B D Q	$\therefore \text{SQR} \therefore \text{QD} \therefore \text{T. R}$ $\text{C} \therefore \text{T. DB} = \text{S. C. AB.}$ <i>i. e. As the S. Co. given $\angle A$, is to Radius; So is T. Co. of Hypo. AC, To T. Co. of the Sd. AB. Ax. 3.</i>
11	One Side BC and adj. $\angle A$ CB.	The other $\angle A$		C S P	$\therefore \text{S. CP} \therefore \text{CQ} \therefore \text{S. PS}$ $\therefore \text{SQR} = \text{S. C. RD} = \angle A. i. e. As Radius, is to Sn. } \angle \text{QCR} = \text{given } \angle \text{ACB; So is S. Co. of given Sd. BC To S. Co. of the opposite } \angle \text{BAC. Ax. 1.}$

Nr. Giv.	Req.	Rough Scheme	Mainfain	Proportions or Solutions
One Side BC and $\angle A$ opp. Amb.	The other $\angle C$ adjacent.		CSP	$\therefore S.CQ \dots S.CP \dots S.$ $QP \dots S.PS = \angle ACB.$ <i>i.e. As S.Co. given Sd.</i> <i>BC, Is to Rad so is</i> <i>S.Co. given $\angle BAC$,</i> <i>To Sine adj. $\angle ACB$.</i> <i>Ax. 1.</i>
One side BC and Hypoth AC	The contained $\angle C$.		RSO	$\therefore T.RS \dots T.QP \dots S.$ $SO \dots PO = S.C. \angle AC$ <i>i.e. As Tan. Hypo.</i> <i>AC, Is to Tang given</i> <i>Side BC; So is Radi-</i> <i>us, To the S.Co of $\angle$</i> <i>cont: ACB. Ax. 3. in-</i> <i>verted.</i>
Hypoth AC and one of the $\angle$ s C.	The other $\angle A$.		CSP	$\therefore S.CS \dots S.CR \dots T.$ $SP \dots RQ = S.Co. \angle A.$ <i>i.e. As Radius, Is to</i> <i>the S.Co. Hypoth. A</i> <i>C; So is T. given $\angle$</i> <i>ACB, To the T.Co. of</i> <i>Req. $\angle$ at A. Ax. 3.</i>
The $\angle$ s A & C.	The Hypoth AC		CSP	$\therefore T.PS \dots T.QR \dots S.$ $C.S. RC = S.C$ <i>AC. i.e. As T. one of</i> <i>given $\angle$s C, Is to T.</i> <i>Co. of the other $\angle A$;</i> <i>So is Radius, To the</i> <i>S. Co. of Hypoth. A</i> <i>C. Ax. 3. inverted.</i>
The $\angle$ s A & C.	One of the sides BC.		CSP	$\therefore S.SP \dots S.RQ \dots P$ $C.QC = S.Co. BC.$ <i>i.e. As $\angle$ adj. to Sd.</i> <i>Req. ACB, Is to S.Co.</i> <i>oppof. $\angle A$; So is</i> <i>Radius, To S.co Req.</i> <i>Sd. BC. by Ax. 1. 2</i> <i>E.I.</i>

In

In the preceding Solutions of Right-angled Spherick Triangles, you may perceive, that the Sides of the Triangle must be produced, that so there may be divers Triangles made by their Intersections, consisting of the Parts of the first, or of the Complement those Parts diversly. And then it must be considered, to which of all those Triangles one of the foresaid Axioms may apply and immediately be apply'd, for finding the Thing requir'd, or the Complement thereof.

But the Noble Lord *Napair* (Inventer of Logarithms from a diligent Observation of the Proportions afore-



said found, they might be solved by one general Proposition as a Confectory from Axiom 2. and 3. foregoing; which is thus: The Radius multiply'd by the middle Part, is equal to the Product of the Tangents of the Extrems joyn'd, and equal to the Product of the S. Comp. Extrems disjoyn'd. For the understand-

ing of which, 'tis necessary to premise, that as a Triangle hath Five Circular Parts, besides the Right-angle; (which being known, we take no notice therefore of the Right-angle,) which are, *viz.* the Sides or Legs AB and BC, together with the Hypotenuse AC, and two Oblique Angles. BAC and ACB, esteem'd the Five Circular Parts of every Right-angled Spherick Triangle: The same Lord likewise taking notice of those Parts of the Right-angled Triangles, which in Conversion of the Sides do change their Proportion from Sines or Tangents to their Complements, *viz.* the Hypotenuse AC; and the two Oblique Angles at A and C, are noted by Complements; and of these Five Circular Parts (in every Question of Right-angled Spherick Triangles) there are always Two given, and One requir'd, and of these Three, One is always the middle Part, and the other Two are either Extrems joyn'd, or Extrems disjoyn'd, or opposite. Now, if the three Parts in the Question lie together, (not separated by any Side or Angle) as in the former Figure, Suppose the Angles A and C were given to find the Hypotenuse AC, then AC is made middle Part, and the other two Parts, *viz.* the Angles A and C are Extrems joyn'd; and the Proportion

tion by the General Proposition is,

Axiom 1. *As Radius, Is to the Tangent of one of the Extrems adjacent, or join'd ; So is the Tangent of the other Extream To the Sine of the Middle Part.*

But because in this Case the Question falls among those Parts that are noted by their Complements, you must instead of Tangent, use Tangent-Complement ; and instead of Sine, use Sine-Complement. Or, in case the Parts given and requir'd, are separated by one of them (for there will be two together) that part which is separated, is the Middle part, and the other two Extrems disjoyn'd, or opposit ; and the Proportion is,

2. *As S. Comp. one Extream, Is to Radius ; So is S. Middle part, To the S. Comp. of the other Extream.*

Or, if the Middle part is requir'd,

3. *As Radius Is to S. Comp. of one Extream ; So is S. Comp. the other Extream, To the Sine of the Middle part.*

Hence you may observe, to find an Extream either join'd or disjoin'd, you must begin the Proportion with the other Extream ; if the Middle part is requir'd in either, to begin the Proportion with Radius.

Note, 1. If either of the Extrems disjoyn'd be in the places noted by Complements (which are always the Hypothenuse and the two Oblique Angles) instead of Sine-Comp. you must use Sine : For the Comp. of a Sine-Comp. is the Sine it self.

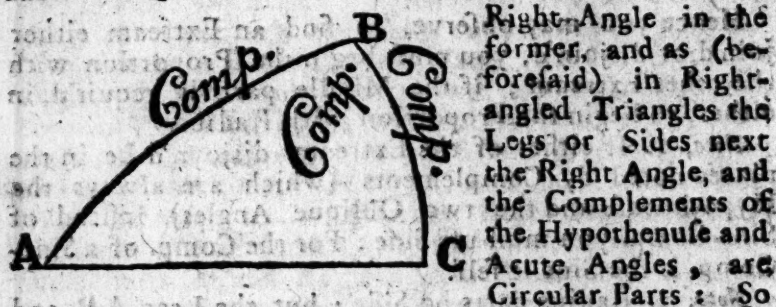
Note, 2. Radius parts no Side ; but the Legs A B and B C are supposed to joyn together.

And for the easier understanding the foregoing Rules and Directions, observe the Table following, wherein are plac'd the five Circular parts of a Triangle, under their respective Titles, whether they be taken for the Middle part, or for Extrems, either join'd or disjoin'd ; and unto those Parts are prefix'd Sine, Sine-Complement, as they should be.

A Table of all the Varieties of *Extreams* joyn'd
or disjoyn'd.

Nr.	Middle Part	Extr. Joyn'd.	Extr. Disjoyn'd
1	Sn. A B.	Tang. B C T. Co. B A C	Sn. A C B Sn. A C
2	Sn. Co. B A C	T. Co. A C Tang. A B.	Sn. Co. B C Sn. A C B
3	Sn. Co. A C	Tang. Co. B A C Tang. Co. A C B	Sn. Co. B C Sn. Co. A B
4	Sn. Co. A C B	Tang. Co. A C Tang. B C	Sn. B A C Sn. Co. A B
5	Sn. B C	Tang. A B Tang. Co. A C B	Sn. B A C Sn. A C

Likewise Quadrantal Triangles are also wrought by the same General Proposition (of the Lord Napair) if you allow the Quadrantal Side in these to be the same as the



in Quadrantal Triangles the Quadrantal Side is omitted; and the two Angles adjacent to the Quadrantal Side, and the Complements of the other two Sides, and of the Angle opposite to it, make the Five Circular Parts, and come under the Rules foregoing.

As, suppose the Triangle A B C to be a Quadrantal Triangle; and Let the Side A C be 90° ; then is the Angle A and the Angle B, with the Complement of the Side A B, and the Complement of Angle B, and the Complement of the Side C B, the five Circular Parts; and so of any other.

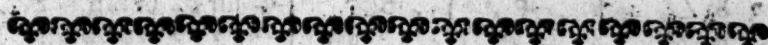
Take

Take the Sum and Substance of the Catholick Proposition in the Verse below ; which committed to Memory, will be of great use to the Learner.

Extreams, when joyn'd, are Tangents, which, I say,
Disjoyned, are Sine-Complements alway.
Middle Parts are Sines continually also.
But if you seek the Middle Part, put Radius first, you'll
know.

If either is Extream, observe then hereby,
To take the other Extream, if you will work truly.

To render this yet more plain, I shall for the Learner's Exercise, lay down a Table of Proportions for Solution of the Sixteen Cases of Right-angled Spherick Triangles.



A TABLE

A Table of the Proportions for the Solution of the 16 Cases of Right-angled Spherick Triangles, according to the Lord Napier's Proposition.

Nr	Given	Required	Rough Scheme.	Med. Pt.	Ext. joyn	Ext. disj.	Proportions.
1	Hypoth. A C and $\angle$ adj. A.	Perp. B C opp.				AC and A	.. R. S. Hyp. AC : : S $\angle$ A S Perp. BC, as in <i>Pl. Trian.</i> Or, as R. S. one Ext. A C : S oth. Ext. $\angle$ A : S. Mid. Pt. BC. Ax. 3.
2	Hypoth. A C and one side B C	$\angle$ A opp.			.. T. one Ext. : Rad. : S. M. Pt. T. oth. Ext.	AC and A	.. Sn. Ext. AC : : R. : Sn. Mid. Pt. BC to Sn. oth. Ext. $\angle$ A. Or, as in <i>Pl. Tri.</i> As Sn. Hyp. A C : R. : Perp. BC : Sn. $\angle$ A reqd.
3	One side B C & $\angle$ A opposite	Hypoth. AC.				AC and A	.. S. Ext. $\angle$ A : : R. : S. Mid. Pt. Sn. Extr. AC. Or, as in <i>P. Trian.</i> : S. $\angle$ A : Perp. BC : : R. : S. Hypoth. A C.
4	One side A B and $\angle$ A adj.	Perp. or side B C.			BC and A		.. T. Co. Extr. $\angle$ A : Radius : : S. Mid. Pt. A B : T. other Extr. reqd.
	One side B C and $\angle$ A opp.	Base or side A B			BC and A		.. Radius : T. Co. Extr. $\angle$ A : : T. other Extr. B C : S. Mid. Part A B by Ax. 1.

Nr	Given.	Req.	Rough Scheme.	Mid Ext	Ext	Proportions.	
				Pr.	joyn	disj	
6	The Sds. AB and BC	The $\angle A$ or C opp.		AB and $\angle A$	BC and $\angle A$		.. T. Extr. BC .. Radius; :: Sn. Mid Part AC, .. T. Co. other Extr. $\angle A$.
7	Hypoth AC, one Leg BC	Base AB		AC	BC and AB		.. S. Co. one Extr. B C, .. Radius; :: S. Co. Mid. Part AC, .. S. Co. other Extr. AB.
8	Sds. AB and BC	Hypoth AC		AC	BC and AB		.. Radius, .. S. Co. one Extr. (BC) AB; .. S. Co. (AB) BC other Extr. .. S. Co. Mid. Part AC.
9	One Side AB and adj. $\angle A$	Hypoth AC		$\angle A$ and AC	AB & AC		.. T. one Extr. AB .. Radius; :: S. Co. Mid. Part $\angle A$, .. T. Co. other Extr. AC.
10	Hypoth AC & $\angle A$	The adj. side AB		$\angle A$ and AC	AB & AC		.. T. Co. one Extr. AC, .. Radius; :: S. Co. Mid Part $\angle A$.. T. other Extr. AB.
11	One Side BC and adj. $\angle C$	Other $\angle A$		$\angle A$	BC and $\angle C$		.. Radius, .. S. Co. one Extr. BC; :: S. Co. other Extr. $\angle C$.. S. Co. Mid. Part $\angle A$.
12	One side BC and $\angle A$ opp.	Other $\angle C$ adj.		$\angle A$	BC and $\angle C$		.. S. Co. one Extr. B C, .. Radius; :: S. Co. Mid. Part $\angle A$, .. S. other Extr. $\angle C$.

Nr	Giv.	Req.	Rough Scheme.	Mid Pt.	Ext joyn	Ext disj	Proportions.
13	One side BC and Hypoth AC.	The con- ta. C			AC and BC		.. Radius, .. T. one Extr. BC ; :: T. C. other Extr. AC ; .. Mid. Part C.
14	Hypoth AC and one C	The other A			ACA	C & A	.. T. C. one Extr. C, .. Radius ; :: S. C. Mid. Part AC, .. T. C. other Extr. A.
15	The A and C	Hypoth AC			ACA	C & A	.. Radius, .. T. C. one Extr. A ; :: T. C. other Extr. C, .. S. C. Mid. Part AC.
16	A and C	One of the sides as B C.			A	C BC	.. S. one Extr. C C, .. Radius ; :: S. C. Mid. Part A, .. S. C. other Extr. BC.

Right-

Right-angled Spherick Triangles, wrought Geometrically, Logarithmically, and Instrumentally.

In the Right-angled Spherick Triangle ABC .

Case I. **T**HE Hypotenuse and Angle at the Base given, to find the opposite Side, or Perpend.

Example.

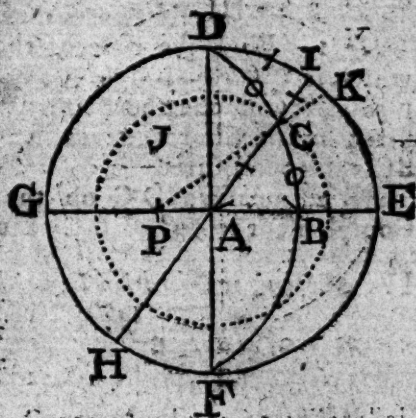
Given $\text{Hypo. } AC \ 71^\circ 74'$ Requir'd the Perp. BC , less the $\text{Adj. } \angle A \ 54^\circ 46'$ than a Quadrant.

Construction Stereographical.

I. To make the Triangle ABC at the Center of the primitive Circle, Fig. 1.

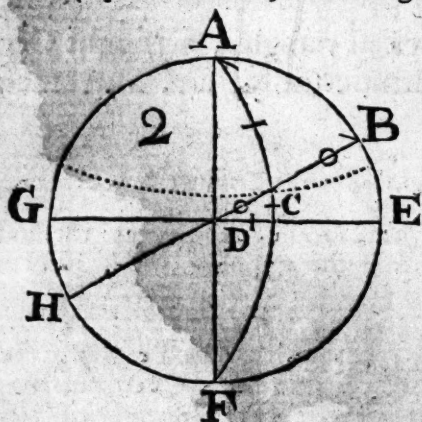
1. Describe the primitive Circle $GDEF$ with a Chord of 60° , or a Sine of 96° , or Half Tangent of 90° , which are all equal to each other. (Do this first in every Problem following.)

2. Make the Angle $BAC = EAI = 54^\circ 46'$ by Prob. II. Case I. of Stereog. i. e. make $EI =$ Chord of $54^\circ 46'$, and draw the Right Circle IH . (3.) By Prob. 9. of Stereog. Case I. with the Half Tangent of $71^\circ 54'$ and one Foot in the Center A , describe a Parallel to the Primitive, to cut the Right Circle CA . 4. Through the Points D (the Pole of AE) C and F (the Pole of AE) draw an Oblique Circle $DCBF$ (by Prob. 11. of Geom-try) and 'tis done: For ABC is the Triangle requir'd.



II. To make the Triangle, that the given Angle A shall be at the Primitive.

1. (By Prob. II. Case 2. Stereog.) make an Angle BAC



at the Primitive, equal to the given Angle, i. e. $54^{\circ} 46'$, with the Secant of the same. 2. By Prob. IX. Case 2. of Stereog. describe a Parallel to the Right Circle GE at $18^{\circ} 06'$ from it, or $71^{\circ} 54'$ distant from its Pole A, to cut the Oblique Circle in C. 3. Through the Center of the Primitive D, and the Point C, draw the Right Circle

BCDH, and it's done; and ABC is the required Triangle, Fig. 2. and Fig. 1. both are Right-angled at B.

III. Mensuration Stereographical.

Fig. 1. Find the Pole of the Oblique Circle DBF, which is P (Prob. I. Case 3. Ster.) And lay a Ruler from P to C, 'twill cut the Primitive in K; then KE measured on the Chords, gives the Quantity of the Perpend. BC $50^{\circ} 56'$ required.

Fig. 2. measure DC on the Half Tangents, it's the Complement to 90° of BC required.

IV. Solution, or Operation Logarithmetical, per Synopsis and Table, Nr. 1.

As $\angle B$ 90°	$\angle B$	10.000000
Is to the S. Hypoth. AC $71^{\circ} 54'$		9.977959
So is the S. $\angle A$ $54^{\circ} 46'$		9.912121
To S. opp. Side Perp. BC $50^{\circ} 56'$		9.890080

V. Instrumental, by Gunter's Scale.

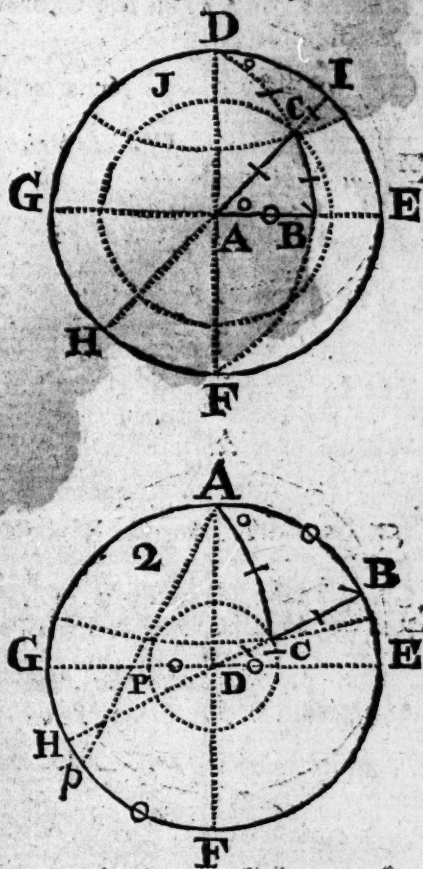
The Extent from $\angle B$ 90° to Sn. $\angle A$ $54^{\circ} 46'$ on the Line of Sines,

Will reach from Hypoth. AC $71^{\circ} 54'$ to Perp. BC $50^{\circ} 56'$ on the Line of Sines.

CASE 2. In the Right-angled Spherick Triangle ABC
Given $\angle$ Hypoth. AC $71^{\circ} 54'$ and Sd or Perp. BC $50^{\circ} 56'$
Reqd. $\angle$ A opposite less than 90 Degrees.

Construction. To avoid needless Repetitions, let P c Q
signifie (in this and the following Problems) to draw the primitive Circle and quarter it, viz. cross it at Right-angles with two Diameters. To project the Triangle at the Center of the Primitive. 1. P c Q; then with the Half Tangent of $71^{\circ} 54'$ (the given Hypoth. AC) by *Prob. IX. Case 1. Stereog.* Describe a Parallel to the Primitive. 2. (By *Case 2. of Prob. IX. Ster.*) describe a Parallel to the Right Circle GE at $50^{\circ} 56'$ from it, or $39^{\circ} 04'$ distant from its Pole C, which cuts the former Parallel in the Point c. 3. Thro' the Points D, c, and F, draw a great Circle, so shall ABC be the Triangle sought.

II. To project the Triangle at the Primitive, 1. P c Q. Then describe a Parallel to the Right Circle GE, at $71^{\circ} 54'$ distant from its Pole A (*Prob. IX. Case 2. Ster.*) 2. Draw also a Parallel to the Primitive at $50^{\circ} 56'$ distant from it, or $39^{\circ} 04'$ (its Comp.) from its Pole D, or Center D, which cuts the former Parallel in c. 3. Through the Points A, c, and F, draw an Oblique Circle, and it's done: For ABC is the Triangle, and A the Angle required.



Right-angled Spherick Triangles

III. Mensuration.

Fig. 1. The Arch of the Primitive *E I* measured on the Chords. Or, *Fig. 2.* *D P* (the Distance of the Poles of the contained Sides *A B*, *A C*) measured on Half Tangents, or *F p* on the Chords, will give $54^{\circ} 46'$ for the Angle at *A*.

Operation, as in Nr. 2 of Synopsis and Table,

IV. Logarithmical.

As <i>S.</i> Hypoth. <i>A C</i> $71^{\circ} 54'$	9.077959
Is to Radius <i>S</i> 90°	10.
So is <i>S.</i> Perp. <i>B C</i> $50^{\circ} 56'$	9.890093
To <i>S.</i> oppos. $\angle$ <i>BAC</i> $54^{\circ} 46'$	9.912134

V. Instrumental, by Gunter's Scale.

The Extent from *Sn.* Hypoth. *A C* $71^{\circ} 54'$, to *S.* Perp. *B C* $50^{\circ} 56'$ on the Sines,
Will reach from *R.* $S.$ 90° , to *S.* Angle *A* $54^{\circ} 46'$ on the Sines.

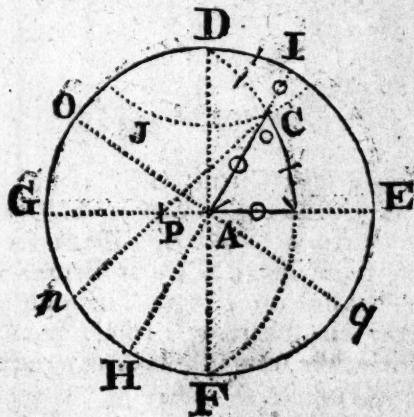
CASE III. In the Right-angled Spherick Triangle *ABC*,

Given $\left\{ \begin{array}{l} \text{One Sd. or Perp. } B C \ 50^{\circ} 56' \text{ and } \angle A \text{ op. } 54^{\circ} 46' \\ \text{Requ. } \left\{ \begin{array}{l} \text{Hypoth. } A C \text{ less than a Quadrant.} \end{array} \right. \end{array} \right.$

I. To make the Angle *A* at the Center.

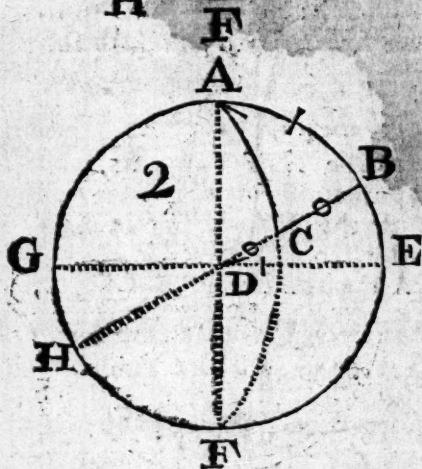
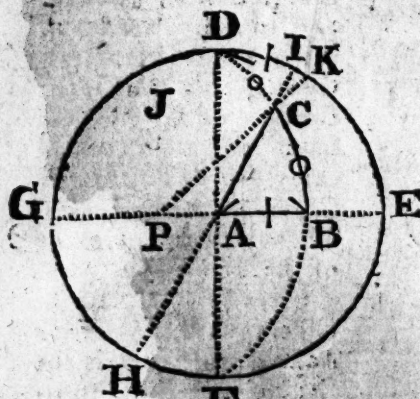
Construction. *P C Q.* Then in *Fig. 1.* at the Center *A* make the Angle *BAC* equal to $54^{\circ} 46'$, as in Case I. 2 Draw a Parallel to the Right Circle *G E*, at $39^{\circ} 04'$ distant from its Pole *D*, to cut the Right Circle *A I* in *C*. 3. Then draw an Oblique Circle through the Points *D*, *C*, and *F*, as before, and you have the Triangle *ABC* concluded.

II To project the Triangle, so that the Angle *A* shall be at the Primitive, *Fig. 2.* 1. As in Case the



I. To project the Triangle at the Center A. Fig. 1.

Construction. PCQ. 1. Make the Angle $A = 54^\circ 46'$ (as in the last Case 3.) 2. Make AB equal to the Half Tangent of $60^\circ 02'$, from your Scale. 3. And thro' the Points D, B, and F, draw an Oblique Circle, as before, and it's done, Fig. 2.



II. To make the Triangle ABC at the Primitive. 1. With the Secant of the given Angle $A = 54^\circ 46'$ describe the Oblique Circle AC D (as in the last Case 3, Fig. 2.) 2. Make AB on the Primitive equal to $60^\circ 02'$, from your Line of Chords. And Lastly, from B, through the Center D draw the Right Circle BH, and you have formed the Triangle ABC. q.e.d.

III. Mensuration. Fig. 1.

1. Find P, the Pole of the Oblique Circle DBF (as in Case 1.) and lay a Ruler from P, over the Point C, to cut the Primitive in K. 2. Then EK measured on your Chords is $BC = 50^\circ 48'$. q.e.f.

Fig. 2. Measure DC on the Half Tangents, it gives the Comp. to 90° , of BC $50^\circ 48'$ (as before in Case 1, Fig. 2.) Or measure BC the backward way of the Half Tangents, from 90° , counting 80° to be 10° , 70° to be 20° , 60° to be 30° , &c. it gives the Degrees requir'd.

IV. Oper-

IV. Operation. 1. By the Synopsis, or Globe-Projection, Nr. 4.

As Radius	—	90° 00'	10.000000
Is to the S. given Side AB	60° 02'		9.937676
So is T $\angle$ A	—	54° 46'	10.151014

To T. reqd. Sd. or Perp. BC 50° 48' 10.088690

2. By the L.N. Universal Proposition, per Table, Nr. 4.

As T.C. $\angle$ A (Extr.)	54° 46'	9.848985
Is to Radius	90° 00'	10.
So is S. Side AB (Mid. Pt.)	60° 02'	9.937676

To T. Side BC (other Extr.) 50° 48' 10.088691

Here you see, both Ways give the same.

V. Instrumental, by Gunter's Scale.

The Extent from R. S. 90°, to Sn. AB 60° 02' on the Sines,

Will reach from T. $\angle$ 54° 46', to T. Side BC 50° 48' on the Tangents.

In the Right-angled Spherick Triangle ABC.

CASE 5. and 12. Given one Side (the Perp. BC 50° 48') and the opposite Angle (A 54° 46') to find the Base, or other Side AB, and Angle C.

I. Construction. This is projected, both at the Center, and Primitive, by the Directions in Case 3. here being the same things given as there.

II. Mensuration, in Case 3. Fig 1. AB being measured on the Half Tangents, gives 60°.

And for the Angle ACB, find the Pole of the Right Circle o or q , (by crossing it at Right Angles with the Right Circle o , q , Prob. 1. Case 2. Stereog.) 2. Find the Pole of the Oblique Circle DBF, which is p . Lay a Ruler from the Angle-point C over P, 'twill cut the Primitive in n ; then no measured on the Chords, gives the Quantity of the Angle C = 65° 53', by Prob. 8. Case 3. Stereog. Or, to measure AB, and Angle C in Case 3. Fig. 2. find, as before, the Pole of the Right Circle BH, which is m or r , and the Pole of the Oblique Circle ACF, which is P; a Ruler laid over the Angle-point C and P, cuts the Primitive in k ; then km measured on the Chords, is equal to the Angle C = 65° 53', as before; and AB measured on the same Chords, is equal to the Side AB = 60°.

III. Operation. 1. By the Glob. Project. or Synop. for AB, in Nr. 5. and 12.

As the Tang. $\angle A$ $54^{\circ} 46'$	10.151014
Is to Tang. of Sd. BC $50^{\circ} 48'$	10.088533
So is Radius S. $90^{\circ} 00'$	10.

To the Sn. of the Sd. AB 60°	9.937519
---------------------------------------	----------

2. For the Angle at C.

As S C Side BC $50^{\circ} 41'$	9.800737
Is to Radius $90^{\circ} 00'$	10.
So is S.C. Angle A $54^{\circ} 46'$	9.761106

To the S. Angle C $65^{\circ} 53'$	9.960369
------------------------------------	----------

3. By the Univ. Propof. for AB, Table Nr. 5. and 12.

As Radius $90^{\circ} 00'$	10.
Is to T.C. $\angle A$ $54^{\circ} 46'$	9.848985
So is T. BC $50^{\circ} 48'$	10.088533

To the S. AB 60°	9.937518
---------------------------	----------

4. For the Angle C, the Proportion is exactly the same as in the Globical Projection, or Synopsis.

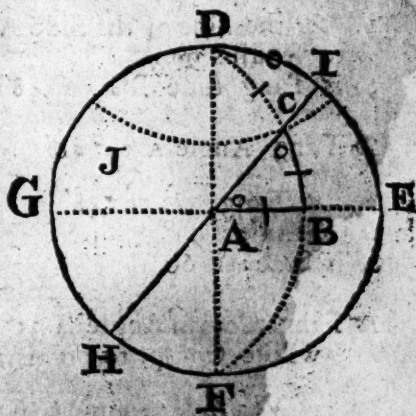
IV. Instrumental, by Gunter's Scale.

The Extent from Tang. $\angle A$ $54^{\circ} 46'$, to Tang. Side BC $50^{\circ} 48'$ on the Tangents,
Will reach from R 90° , to Sine Side AB 60° on the Sines.

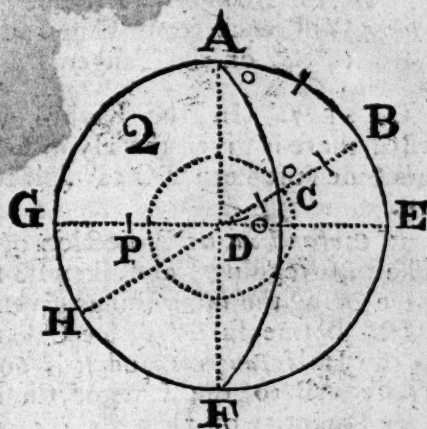
The Extent from S.C. BC $39^{\circ} 12'$, to R Sn. 90° on the Sines,
Will reach from S.C. $\angle A$ $35^{\circ} 14'$, to the Sn. $\angle C$ $65^{\circ} 53'$ on the Sines.

CASE VI. In the Right-angled Spherick Triangle ABC given, the Sides (or Base and Perp.) AB 60° Bc $50^{\circ} 48'$, to find the Angle A (or c.)

I. *Construction.* Fig. 1. at the Center P c Q, make AB on the Right Circle GE equal to the Half Tangent of 60° , and through the Points DB and F describe the Oblique Circle D c D F. 2. (by Prob. 9. Case 2. *Stereog*) describe a Parallel, distant from the Pole of the Right Circle GE, that is, $D 39^\circ 12'$ (the Comp. of the Perp. or Side Bc) to cut the Oblique Circle DBF in c. Lastly, thro' the Center A and c, draw the Right Circle IH, and the Triangle A B c is finish'd.



II. To make the Triangle A B c at the Primitive, 1. Take 60° from the Chords, and lay it on the Primitive from A to B, and from B thro' the Center D, draw the Right Circle BH in (in Fig. 2.) 2. Take the Half Tangent of the Comp. of BC, i.e. $39^{\circ} 12'$, and sweep a Parallel to the Primitive, to cut the Right Circle BH in C. Lastly, Through the Points A, c and F, draw the Oblique Circle A c F, and the Triangle A B c is made. The same Projection for Case 8.



III. *Mensuration of the Angle A required, Fig. 1. (as in Case 2.)*

E I measured on the Chords, is the Quantity of the Angle $A = 54^{\circ} 46'$; Fig. 2. the Distance of the Poles of the contained Sides AB and AC, which is DB measured on the Half Tangents, gives the Degrees of the Angle $A = 54^{\circ} 46'$.

IV. Ope-

IV. Operation. 1. By Synopsis, Nr. 6. 2. By Table Nr. 6. or Univ. Prop.

1. As the Sine of ths Side AB 60° 9.937531
 Is to Radius 90° 10.
 So is Tang. Side BC $50^{\circ} 48'$ 10.088533

 To Tang. Angle A $54^{\circ} 46'$ 10.151003

2. As Tang. BC $50^{\circ} 48'$ 10.088533
 Is to Radius 90° 10.
 So is Sine AB 60° 9.937531

 To Tang. Comp. Angle A $35^{\circ} 14'$ 9.848998
 whose Comp. to 90° is $= 54^{\circ} 46'$.

V. Instrumental, by Gunter's Scale.

The Extent from Sine A B, 60° . to Radius 90° on the Sines, Will reach from Tang. BC $50^{\circ} 48'$, to Tang. $\angle A$ $54^{\circ} 46'$ on the Tangents.

CASE 7. In the Right-angled Spherick Triangle ABC, there is given the Hypothenuse (AC $71^{\circ} 54'$) and one Side (the Perp. BC $50^{\circ} 48'$) to find the Base (or Side) AB.

I. Construction, or Projection of this, is the same as in Case 2. foregoing; and likewise for Case 13. following; there being the same things given in each.

II. To measure AB, see Case 2. Fig. 1. Apply A B to the Half Tangents, and it is $60^{\circ} 34'$. Or Case 2. Fig. 2. Apply AB to the Line of Chords, and you have the same Quantity of Degrees, i. e. $60^{\circ} 34'$.

III. Operation. 1. by G. P. 2. by Univ. Prop. Synopsis, and Table, Nr. 7. The Proportion is the same in both, i. e.

As S. C BC $39^{\circ} 12'$ 9.800737
 Is to Radius $90^{\circ} 00'$ 10.
 So is S. C. AC $18^{\circ} 06'$ 9.492108

 To S. C. AB $22^{\circ} 26'$ 9.691571
 Comp. to 90° is $= 60^{\circ} 34'$.

IV. Instrumental, by Gunter's Scale.

The Extent from S. C. BC $39^{\circ} 12'$, to R. S. 90 on the Sines,

Will reach from S. C. A C $18^{\circ} 06'$. to S. C. AB $22^{\circ} 26'$ on the Sines.

CASE

CASE VIII. In the Right-angl'd Spherick Triangle ABC, are given the Sides (AB $60^{\circ} 34'$, and BC $50^{\circ} 48'$) to find the Hypoth. AC.

1. This is projected in Case 6. And 2. Measured. viz. AC = $71^{\circ} 54'$, as in Case 3. to which I refer you, and pass on to the

III. Operation, by Synopsis and Table, Nr. 8. which are the same, i. e.

As Radius	$90^{\circ} 00'$	10.000000
Is to S. C. AB	$60^{\circ} 34'$	9.691571
So is S. C. BC	$50^{\circ} 48'$	9.800737
To S. C. AC	$18^{\circ} 06'$	9.492308
Comp. to 90°	$= 71^{\circ} 54'$	

IV. By Gunter's Scale.

The Extent from Radius 90° , to S. C. AB $29^{\circ} 26'$ on the Sines,
Will reach from S. C. BC $39^{\circ} 11'$, to S. C. AC $18^{\circ} 06'$ Sines.

CASE IX. In the Right-angled Spherick Triangle ABC, is given one Side (AB $60^{\circ} 34'$) and adjacent Angle (A $54^{\circ} 46'$) to find the Hypoth. AC.

1. This is also projected by the Directions in Case 4. and 2. the Hypoth. measured, as in Case 3, which is $71^{\circ} 58'$, and the

III. Operation by G. P. 2. by Univ. Prop. as in Synopsis and Table, Nr. 9. i. e.

1. As Radius	$90^{\circ} 00'$	10.
Is to S. C. $\angle A$	$54^{\circ} 46'$	9.761106
So is T. G. Side AB	$60^{\circ} 34'$	9.751462
To T. C. Hypoth. AC	$71^{\circ} 58'$	9.512568
2. As Tang. AB	$60^{\circ} 34'$	10.248538
Is to Radius	$90^{\circ} 00'$	10.
So is S. C. $\angle A$	$54^{\circ} 46'$	9.761106
To T. C. Hypoth. AC	$71^{\circ} 58'$	9.512568

IV. Instrumental, by Gunter's Scale.

The Extent from Radius S. 90° , to S. C. $\angle A$ $35^{\circ} 14'$ on the Sines,

Will

Will reach from T.C. $29^{\circ} 26'$, to T.C. AC $18^{\circ} 02'$ on the Tangents.

CASE X. In the Right-angled Spherick Triangle ABC, given the Hypoth. (AC $71^{\circ} 54'$) and adjacent Angle (A $54^{\circ} 46'$) to find the adjacent Side AB.

I. Construction. This is projected as in Case 1. and II. the Side AB measured as in Case 5. viz. AB at the Center, by the Half Tangents, and at the Primitive on the Chords, and will be found $60^{\circ} 28'$.

III. Operation, 1. by G.P. Nr. 10. 2. by Table of Univ. Prop. Nr. 10.

1. As S.C. $\angle A$	$54^{\circ} 46'$	9.761106
Is to Radius	90	10.
So is T.C. Hypo. AC	$71^{\circ} 54'$	9.514349
To T.C. Side AB	$29^{\circ} 32'$	9.753243
whose Comp. is	$60^{\circ} 28'$	
2. As T.C. AC	$71^{\circ} 54'$	9.514349
Is to Radius	90	10.
So is S.C. $\angle A$	$54^{\circ} 46'$	9.761106
To Tang. AB	$60^{\circ} 28'$	10.246757

IV. Instrumental, by Gunter's Scale.

The Extent from S.C. $\angle A$, $35^{\circ} 14'$ to Radius S.90, on the Sines,

Will reach from T.C. Hypoth. AC $18^{\circ} 06'$, to T.C. Side AB $29^{\circ} 32'$ on the Tangents.

CASE XI. In the Right-angled Triangle ABC, Given one Side (BC $50^{\circ} 48'$) and the adjacent Angle (C $65^{\circ} 53'$) to find the other Angle A.

Note; If there had been given, the Side AB, and adjacent Angle A, to find the Angle C; then the Triangle is made, as in *Case 4.* and the Angle C is is measured as before taught in *Case 5.* and the Operation is the same as in this *Case 11.* above.

CASE XII. is solved as aforegoing, in *Case 5.* therefore,

CASE XIII. In the Right-angled Spherick Triangle ABC, there is given the Hypoth. (AC $71^{\circ} 54'$) and Side (BC $50^{\circ} 56'$) to find the contained Angle c:

I. This Triangle is made, as in *Case 2.* And II. the Angle c measured as in *Case 5.* foregoing.

III. Operation, 1. By Synopsis. 2. By the Table Nr. 13. In both,

1. As Tang. Hypoth. AC $71^{\circ} 54'$	10.485651
Is to Radius	90 00
So is Tang. Side BC	50 56
	10.090598

To the S.C. conta. $\angle c$	23 44	9.604947
Comp. to 90, is ..	66 16	

2. As Radius	90	10.
Is to Tang. BC	50 56	10.090598
So is T.C. AC	18 06	9.514349

To S.C. $\angle c$	23 44	9.604947
the same as before,		

IV. By Gunter's Scale.

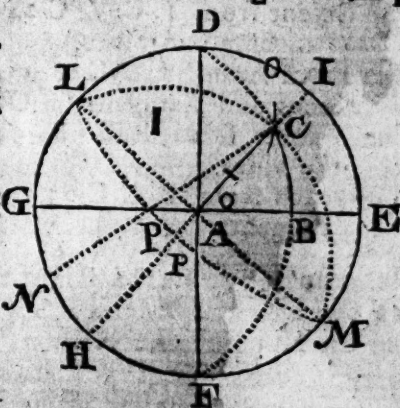
The Extent from T. AC $71^{\circ} 54'$, to T. BC $50^{\circ} 56'$ on the Tangents,
Will reach from R.S. 90, to S.C. $\angle c$ $23^{\circ} 44'$ on the Sines.

CASE XIV. In the Right-angled Spherick Triangle ABC, given Hypoth. (AC $71^{\circ} 54'$) and one Angle (C $65^{\circ} 53'$) to find the other Angle A.

Wrought Geometrically, Logarithmically, &c.

I. Construction. 1. To make the Triangle at the primitive Circle's Center.

Draw the Right Circle H I, and set off the Half Tangent of $71^{\circ} 54'$ (the given Hypoth. AC) from the Center A, to C. 2. And on the Point C, (by *Prob. 2. Case 3. of Stereog. Prob.*) make an Angle equal to the given Angle $c\ 65^{\circ} 53'$, by drawing the Oblique Circle DBF, as is there directed, and 'tis done, as in *Case 11. Project. 1.*

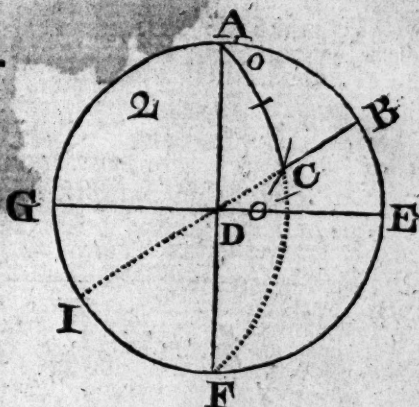


II. To make the Triangle at the Primitive.

It's done, as in *Case 11.*

Projection 1.

III. To measure the Angle A, in Fig. 1. and in Fig. 2. 'Tis done by the Directions in *Case 2.* and is $55^{\circ} 15'$.



IV. Operation.

1. *Per Synopf. and Table, Nr. 14.*

As Radius	90°	10.
Is to S.C. Hypoth. AC	$71^{\circ} 54'$	9.492308
So is T. $\angle C$	$65^{\circ} 53'$	10.349041

To T. C. $\angle A$	$55^{\circ} 15'$	9.841349
---------------------	------------------	----------

Per Glob. Projection.

2. As T. C. $\angle C$	$65^{\circ} 53'$	9.650959
Is to Radius	90°	10.
So is S.C. $\angle AC$	$71^{\circ} 54'$	9.492208

To T.C. $\angle A$	$55^{\circ} 15'$	9.841349
--------------------	------------------	----------

Per Univ. Prop.

V. In.

Right-angled Spherick Triangles

V. Instrumental, by Günter's Scale.

The Extent from T. C. $\angle C$ 24 07, to Radius Sn. 90° on Tangents and Sines,

Will reach from S. C. AC 18 06, to T. C. $\angle C$ 34 45 on Sines and Tangents. Or thus:

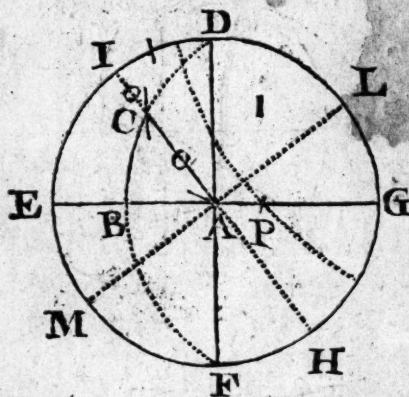
The Extent from Rad. S. 90° , to S. C. AC 18 06 on the Sines,

Will reach from T $\angle C$ 65 53, to T. $\angle A$ 55 15 on the Tangents.

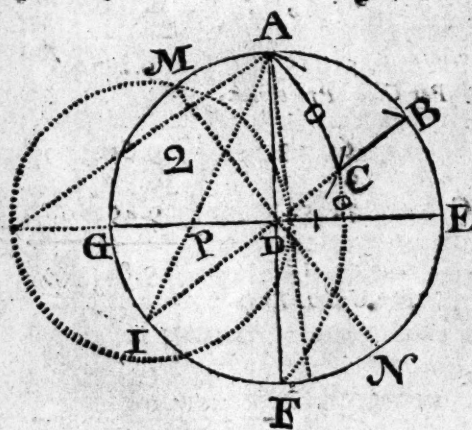
CASE XV. In the Right-angled Spherick Triangle ABC , given the Angles (A $55^\circ 15'$ and C $65^\circ 53'$) to find the Hypothenufe AC.

I. Construction, or Projection.

To make the Triangle at the Center of the Primitive, Fig. 1.



Quantity of the Given Angle) to



1. Make the Angle A at the Center equal to (the Given Angle) $55^\circ 15'$, as in Case the first or third. 2. Cross the Right circle IH at Right Angles with the Right Circle LM. 3. By Case 1. or Prob. 9. Case 2. Stereog. describe a Parallel to the Right Circle IH, at $65^\circ 53'$ distant from the Pole thereof L (*viz.* the Quantity of the Given Angle) to cut the Right Circle EG in P, the Pole of the requir'd Oblique Circle DBF. 4. Measure the Distance of the Poles A and P on the Half Tangents, & subtract it from 90° , and set off the Half Tangent of the Remainder from the Center A, to B, (on the Right Circle AE.) Lastly, through the Points DBF draw the Ob-

Oblique Circle DCF; so shall ABC be the Triangle sought.

II. To make the Triangle at the Primitive, Fig. 2.

1. Make the Angle A at the Primitive equal to the given Angle of $55^{\circ} 15'$ with the Secant thereof, as in Case 1: Fig. 2:

1. Find the Pole of the Oblique Circle ACF, which is P.

3. By Prob. 9. Case 3. of Stereog. describe a Parallel to the Oblique Circle ACF, at $65^{\circ} 53'$ (the other Angle) distant from the Pole P; to cut the Primitive in M.

4. From M, through the Center D draw the Right Circle MN, and cross it at Right Angles; with the Right Circle IB, to cut the former Oblique Circle in C; so shall ABC be the Triangle requir'd.

III. To measure the Hypoth. AC in either. Observe the Directions in Case 3. foregoing.

IV. Operation. 1. By G. P. 2. By U. P. it will be,

1. As T. one $\angle C$	$65^{\circ} 53'$	10.349041
Is to T. C. other $\angle$	$34^{\circ} 45'$	9.841187
So is Radius	90 00	10.
To S.C. Hypoth. AC		9.490146
Comp.	71 54	
2. As Radius	90 00	10.000000
Is to T. C. one $\angle C$	$24^{\circ} 07'$	9.650959
So is T. C. other $\angle A$	$34^{\circ} 45'$	9.841187
To the S.C. Hypo. AC		9.492146

V. Instrumental, by Gunter's Scale.

The Extent from T. $\angle C$ $65^{\circ} 53'$, to Radius S. 90 on the Tangents and Sines,

Will reach from T. C. $\angle A$ $34^{\circ} 45'$, to S. C. Hypoth. AC $18^{\circ} 06'$ on the Sines and Tangents.

CASE XVI. In the Right-angled Spherick Triangle ABC, Given the Oblique Angles A $55^{\circ} 15'$, and C $65^{\circ} 53'$, to find one of the Sides, as BC.

I. Projection. Here are the same Data, or Given things, as in the last Case 15; and therefore take the same Directions for the making it.

II. To measure the Side BC, see Case the First, or Fourth.

PART M.

G

III. Ope-

III. Operation, by G. P. and U. P. the same, i. e.

As S. $\angle C$	65° 53'	9.60335
Is to Radius	90 00	10.
So is S. C. $\angle A$	34 45	9.755872
To S. C. Side BC	38 88	9.795537
Comp. to 90° is	51 22	

IV. By Gunter's Scale.

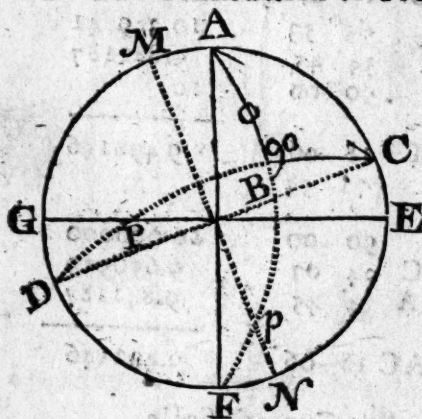
The Extent from S. $\angle C$ 65 53, to Radius S. 90 on the Sines,
Will reach from S. C. $\angle A$ 34 45, to S. C. BC 38 38 on the Sines.

CASE XVI.

Example 2.

Let the same things be given, as in Case next above, to find the Side AB.

I. The Construction or Projection may be the same



as in Case 15: Otherwise, for Variety-sake, you may make the Triangle thus:

1. Make the Angle A at the Primitive equal to 55 15 by drawing the Oblique Circle ABE with the Secant thereof. 2. Find the Pole of the Oblique Circle ABE, which is P. 3. By Prob. 3. of Stereog. Prob. draw an

Oblique Circle thro' the Pole or point P, to make at the Primitive an Angle equal to the other given Angle (in this Case, 65 53,) and this last Oblique Circle shall intersect the former at Right Angles, in the Point B, and ABC is the Triangle sought. II. And AB is measured as in Case 3. Fig. 2. for AC. In this Case the Operation is the same as in the last Case (16) i. e. by Logarithms:

IV. As S. $\angle A$ Is to Rad. So is S. C. $\angle C$, To S. C. Side AB.

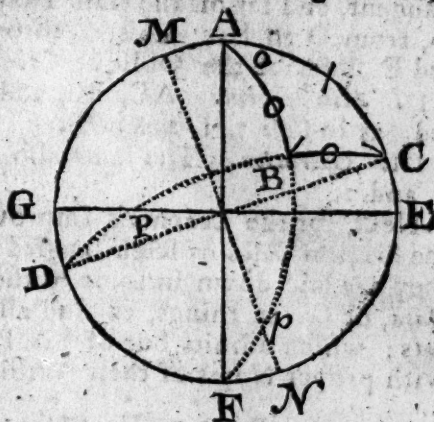
V. By Gunter's Scale.

The Extent from S. $\angle A$, to R. on the Sines,
Will reach from S. C. $\angle C$, to S. C. AB on the Sines.

Ex.

EXAMPLE 3.

In like manner, in Case 14, where there is given the Hypotenuse (AC $71^{\circ} 54'$) and the Angle (C $65^{\circ} 53'$) to find the Angle A, &c. it may be projected thus; 1. P CQ. Then lay the Hypoth. AC $71^{\circ} 54'$ on the Primitive, from A to C. 2. Draw the Right Circle CD, and at Right Angles to it, the Right Circle MN. 3. Make the Angle C at the Primitive equal to $65^{\circ} 53'$, as in Case the First, Third, or Fourth, find the Pole P of the Oblique Circle last drawn, CBPD. Lastly, Thro' the Points or Poles A P F draw the Oblique Circle ABPF, as before taught, and 'tis done.



2. The Angle ABC is measured, as in Case 2. the Side AB, as in Case 3. Fig. 2. and the Side BC the same way.

3. The Operation, to find AB, is the same as in Case 10. and to find BC, as in Case 1. and BAC, as in Case 14. foregoing.

EXAMPLE IV.

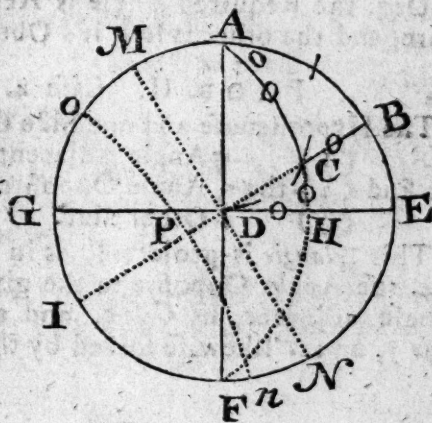
Given the Base (AB $60^{\circ} 10'$) and Angle (C $65^{\circ} 53'$) to make, measure, and solve the Spherical Triangle.

1. Projection, PCQ. Make AB on the Primitive equal to $60^{\circ} 10'$, the Given Side, from the Line of Chords.

2. Draw from the Point B, through the Center of the Primitive, the Right Circle BI.

3. And cross it at Right Angles with the Right Circle MN.

4. By Prob. 9. Case 2. of Stereog. describe a Parallel to the Right Circle BN, at $65^{\circ} 53'$ (the Given Angle's) Distance from the Pole I, and 'twill cut the Right Circle GE in P, the Pole of the requir'd Oblique Circle.



PART II.

G 2

Mea-

III. Operation, by G. P. and U. P. the same, i. e.

As S. $\angle C$	65° 53'	9.60335
Is to Radius	90 00	10.
So is S. C. $\angle A$	34 45	9.755873

To S. C. Side BC	38 88	9.795537
Comp. to 90° is	51 22	

IV. By Gunter's Scale.

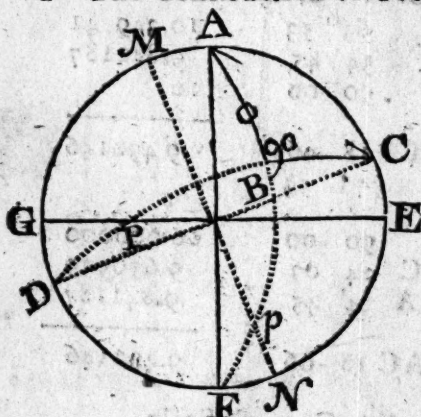
The Extent from S. $\angle C$ 65 53, to Radius S. 90 on the Sines,
Will reach from S. C. $\angle A$ 34 45, to S. C. BC 38 38 on the Sines.

CASE XVI.

Example 2.

Let the same things be given, as in Case next above, to find the Side AB.

I. The Construction or Projection may be the same as in Case 15: Otherwise, for Variety-sake, you may make the Triangle thus:



1. Make the Angle A at the Primitive equal to 55 15 by drawing the Oblique Circle ABE with the Secant thereof. 2. Find the Pole of the Oblique Circle ABE, which is P. 3. By Prob. 3. of Stereog. Prob. draw an

Oblique Circle thro' the Pole or point P, to make at the Primitive an Angle equal to the other given Angle (in this Case, 65 53') and this last Oblique Circle shall intersect the former at Right Angles, in the Point B, and ABC is the Triangle sought. II. And AB is measured as in Case 3. Fig. 2. for AC. In this Case the Operation is the same as in the last Case (16) i. e. by Logarithms:

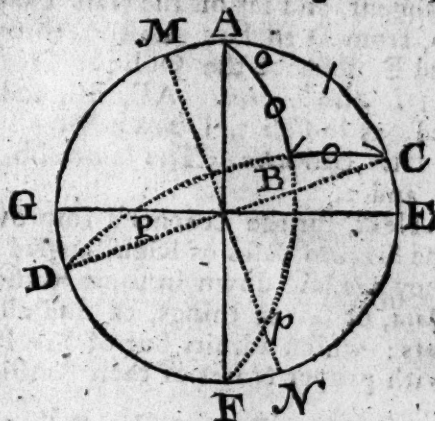
IV. As S. $\angle A$ Is to Rad. So is S. C. $\angle C$, To S. C. Side AB.

V. By Gunter's Scale.

The Extent from S. $\angle A$, to R. on the Sines,
Will reach from S. C. $\angle C$, to S. C. AB on the Sines.

EXAMPLE 3.

In like manner, in Case 14, where there is given the Hypotenuse (AC 71 54') and the Angle (C 65 53') to find the Angle A, &c. it may be projected thus; 1. P CQ. Then lay the Hypoth. AC 71 54' on the Primitive, from A to C. 2. Draw the Right Circle CD, and at Right Angles to it, the Right Circle MN. 3. Make the Angle C at the Primitive equal to 65 53', as in Case the First, Third, or Fourth, find the Pole P of the Oblique Circle last drawn, CBPD. Lastly, Thro' the Points or Poles A & P draw the Oblique Circle ABPF, as before taught, and 'tis done.



2. The Angle ABC is measured, as in Case 2. the Side AB, as in Case 3. Fig. 2. and the Side BC the same way.

3. The Operation, to find AB, is the same as in Case 10. and to find BC, as in Case 1. and BAC, as in Case 14. foregoing.

EXAMPLE IV.

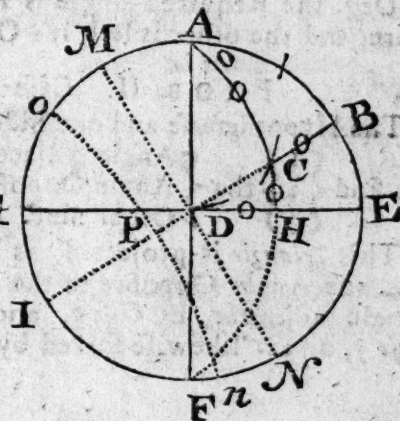
Given the Base (AB 60 10') and Angle (C 65 53') to make, measure, and solve the Spherical Triangle.

1. Projection, PCQ. Make AB on the Primitive equal to 60° 10', the Given Side, from the Line of Chords.

2. Draw from the Point B, through the Center of the Primitive, the Right Circle BI.

3. And cross it at Right Angles with the Right Circle MN.

4. By Prob. 9. Case 2. of Stereog. describe a Parallel to the Right Circle BN, at 65 53' (the Given Angle's) Distance from the Pole I, and 'twill cut the Right Circle GE in P, the Pole of the requir'd Oblique Circle.



PART II.

G 2

Mes-

5. Measure the Distance of the Poles DP on the Half Tangent, and set off the Half Tangent of its Compl. to 90, from D to H. Lastly, through the points A, H, and F describe the Oblique Circle AHF, and it's done.

II. *Mensuration.* AC, BC, and Angle A, are measured, as in Case 3, 1, 2, *Figures.*

III. *Operation.* The same also, as in the said Cases 3, 1, and 2.

Next, for the Learner's Improvement, we shall insert the Sixteen Cases of Right-angled Spherick Triangles, as they are laid down in some Authors; that is, from the *Data*, or Given things, to find all the wanting Particulars; which admits but of Six Problems, and are thus, with proper Notes of their Ambiguities. *Vide Atk. Epit.*

PROB. I. Case 1, 2, and 3.

The Hypothenuse, and one Angle given,

To find $\begin{cases} 1 \\ 2 \\ 3 \end{cases}$ the $\begin{cases} \text{Sd. opposite} \\ \text{Sd. adjacent} \\ \text{Other Angle} \end{cases}$ to the Given Angle.

This Triangle is made, as in Case 1. and measured, as in Case 1, 5, and 12, and its Solution, by the said Cases 1, 5, and 12. foregoing.

Note, the $\begin{cases} \text{Acute} \\ \text{Giv. Angle} \end{cases}$ $\begin{cases} \text{Sd. Oppos.} \\ \text{Obruse} \end{cases}$ $\begin{cases} \text{less} \\ \text{found, is} \end{cases}$ $\begin{cases} \text{more} \end{cases}$ than 90°.

Note, 2. When each thing is less, or more than 90°, the Requir'd adjacent Side is less than 90°; but if one is less, and the other more, it's more than 90 Degrees.

Note, 3. When each Given thing is less or more than 90 Deg. the Required Angle is Acute; but when one is more, and the other is less, it's Obruse.

PROB. II. Case 4, 5, and 6.

The Hypothenuse and one Side Given,

To find $\begin{cases} 1 \\ 2 \\ 3 \end{cases}$ the $\begin{cases} \text{Angle Adjacent} \\ \text{Angle Opposite} \\ \text{Other Side.} \end{cases}$ to the Given Side.

This Triangle is projected, as in Case 2. and measured, viz. the Angle Opposite to the given Side, by Case 2. the Angle adjacent, by Case 5, and also the other Side, by Case 5, or 12. likewise solved by the foresaid Cases.

Note,

Note 1. $\begin{cases} \text{less} \\ \text{more} \end{cases}$ than 90° , the reqd. $\begin{cases} \text{acute} \\ \text{obtuse} \end{cases}$.
Giv. $\begin{cases} \text{Side} \\ \text{Oppos.} \end{cases}$ $\angle$ is

2. Hypoth. $\begin{cases} \text{each} \\ \text{one} \end{cases}$ more $\begin{cases} \text{or the} \\ \text{other} \end{cases}$ less than 90° , the
and Giv. $\begin{cases} \text{Side} \\ \text{Angle} \end{cases}$ reqd. $\begin{cases} \text{acute} \\ \text{obtuse} \end{cases}$.
 $\angle$ is

PROB. III. Case 7, 8, 9,

A Side, and its Opposite Angle given,

To find $\begin{cases} 1 \\ 2 \\ 3 \end{cases}$ the $\begin{cases} \text{Side} \\ \text{Angle} \\ \text{Hypoth.} \end{cases}$ $\begin{cases} \text{Adjacent to} \\ \text{Given Angle} \end{cases}$ $\begin{cases} \text{Angle} \\ \text{Side} \end{cases}$.

This Triangle is made, as in Case 3. and measured, viz. the Side adjacent by Case 5. and Angle adjacent to the Given Side, by Case 5, or 12. and the Hypotenuse by Case 3. and its Solution by the same Cases.

Note, 1. If the unknown Angle be Acute or Obtuse, the required Side accordingly is less or more than 90° Degrees.

Or, the Hypotenuse and Given Side, each less, or each more than 90° Degrees, the Required Side is less than a Quadrant; but if one is more, and the other less, it's more than 90° Degrees.

2. If the Hypotenuse and Given Angle be each less, or each more than 90° Degrees, the Angle required is Acute; but when one is more, and the other less, it's Obtuse.

3. If both Sides, or both Angles be of one kind, the Hypotenuse is less than 90° Degrees; but if they be of different Kinds, it's more.

PROB. IV. Case 10, 11, 12,

A Side, and its adjacent Angle given,

To find $\begin{cases} 1 \\ 2 \\ 3 \end{cases}$ the $\begin{cases} \text{Side} \\ \text{Angle} \\ \text{Hypoth.} \end{cases}$ $\begin{cases} \text{Oppos. to the Given} \\ \text{Side} \end{cases}$ $\begin{cases} \text{Angle} \\ \text{Side} \end{cases}$.

This Triangle is projected, as in Case 4. and the Side opposite measured by the same Case, and the opposite Angle, as in Case 5. or 12. the Hypotenuse, as in Case 3. its Solution by the said Cases.

Note, 1. The Given Angle acute, the Required Side is less than 90° Degrees; but when obtuse, then more than 90° Degrees.

2. The Given Side less, or more than 90 Degrees, the Angle Required is Acute or Obtuse accordingly.

3. When each given thing is less or more than 90 Degrees, the Hypothenufe, is less than 90 Degrees; but if one be less, and the other more, it's more than a Quadrant, or 90 Degrees.

P R O B. V. Cafes 15, and 16.

Both the Sides Given, to find, 1. Either of the Angles
2. the Hypothenufe.

This Triangle is Projected as in Case 6. and Angles Measured, as in Case 2, 5, or 12. The Hypothenufe as in Case 3. and may be Solved by the same Cafes.

Note, 1. If the Side opposite to the Required Angle be less than a Quadrant, the Angle Sought is Acute; but when greater, then it's Obtuse.

2. If both the Sides be of one Kind, the Hypothenufe is less than a Quadrant; but when of Different Kinds (that is, one Side more, the other less than a Quadrant.) then it's more than 90 Degrees.

P R O B. 6. Case 15, and 16.

Both the Oblique Angles given, to find, 1. Either of the Sides. 2. The Hypothenufe.

This Triangle is made as in Case 15, or 16. and the Sides measured as in Cafes 1. and 10. the Hypothenufe as in Case 3. or Case 2.

The Operation to find the Sides in the same as in Case 16. and to find the Hypothenufe, as Case 15.

Note, 1. If the Angle Opposite to the Required Side be Acute, the Side sought is less than a Quadrant; but if Obtuse, then it's greater.

2. The Angles of one Kind, the Hypothenufe is less than a Quadrant, but when of Different Kinds, they are more than 90 Degrees.

Thus have we finish'd Right-angled Spherick Triangles, in which are Contain'd 30 Astronomical Problems, as shall be shewn hereafter. We Pass on to

SPHERICAL TRIGONOMETRY OBLIQUE,

S E C T. II.

The Doctrine of Oblique-Spherick Triangles. Projected and Measured Stereographically, and their Solutions, Logarithmically, and Instrumentally,

Method I. **A**NY Oblique-Spherical Triangle may be reduced to two Right-angled-Spherical Triangles, by letting fall a Perpendicular (from some one of its Angles upon the Opposite Side produced, if need be) which Perpendicular either divides the Oblique Proposed into two Right-angled ones, or makes two Right, by Adding a Right-Angle-Triangle to it.

2. The Perpendicular is a Great Circle passing thro' the Pole of that Circle it falls upon; which Side we may call the *Base*, and the Angle adjacent the *Angle at the Base*, and that adjoining to the Perpendicular, we call the *Angle at the Perpendicular*.

3. Observe in all Cases, the Perpendicular must fall from the end of a given side (adjacent,) to a Given Angle; and then it falls Opposite to the said Angle, so as to make the Given Side the Hypotenuse, and the Given Angle, the Angle at the Base: For then we have enough given, to find the Side or Angle Required at two Operations.

4. The Oblique-Spherick Triangle being thus divided into two Right ones, we call that which contains the Hypotenuse and Angle at the Base, the *First Triangle*; the other, the *Second Triangle*.

In the First Triangle there are given the Hypotenuse and Angle at the Base; to find either the Base, or Angle at the Perpendicular.

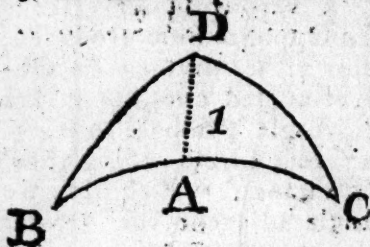
In the Second Triangle, there is either a Side or Angle given; or else the Base, or Angle at the Perpendicular found in the First, gives the Base or Angle at the Base in the Second Triangle, by Addition or Subtraction; according as the Perpendicular falls within, or without the Oblique Triangle. A Side or Angle being thus known, in the Second Triangle, the Side or Angle-required, is found by Comparing the Similar Parts of the first and second Triangle.

For two Right-angled Triangles, having a Common Perpendicular, are Proportional, (as hath been shewn in Plain Triangles) and by taking in the Common Perpendicular by the Side or Angle Given, and the Side or Angle Required, the Proportion to find the Side or Angle, may easily be deduced from the first Axiom or the Lord *Napier's* Universal Proposition.

That these Prenotions may suffice, however, to render this more Perspicuous, to the Learner, we shall make a more easy Explanation.

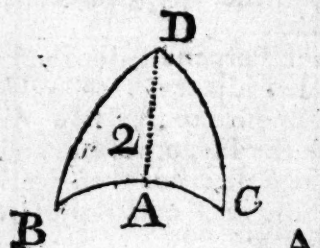
CONCLUSIONS.

1. If the Perpendicular falls within the Triangle,

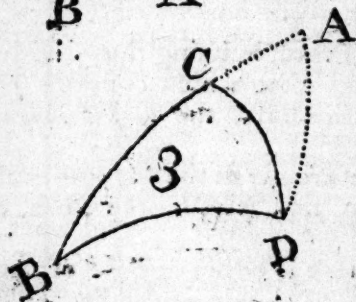


(as in the First and Second Figures) it divides the Triangle BCD, into two Right-angled ones BAD, and CAD.

2. If the Perpendicular falls without the Triangle (as in the Third Triangle,) it produces two Right-angled Triangles BAD, and CAD, by Adding one Right-angled Triangle CAD, to the Triangle CDC, Given.



3. If the Angles at the Base, B and C of the Oblique Triangle BDG are Similar, that is, both Acute (as in Fig. 1.) or both Obtuse (as in Fig. 2.) then the Perpend. let fall from the Angle Opposite to the Base, falls within the Triangle. But,



4. If the Angles at the Base are Dissimilar, that is, one Obtuse, and the other Acute, (as in Fig. 3.) then the Perpendicular let fall from the Angle Opposite to the Base, will fall without the Triangle, and consequently the Base (BC) must be produced to meet it. And this is evident from the 23d. Definition, and Fig. annex in Page .

There

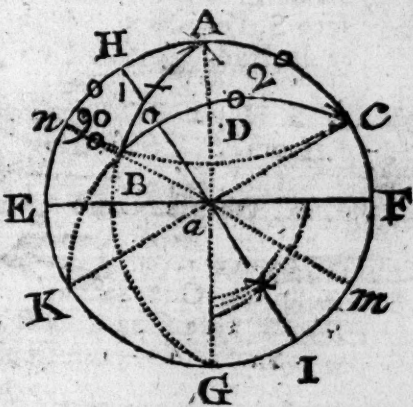
There are 12 Cases in an Oblique Triangle, 10 of which we shall Solve by this Method, and afterwards give Solution to the 12 Cases by Axioms for that Purpose.

You may easily solve the First and Second Case following, by Axiom the First, by the Proportion of Opposite Sides and Angles.

CASE. 1. In the Oblique-angled Spherick-Triangle ABC, there are Given Two Angles ($BAC\ 148^{\circ}.59'$. $ACB\ 29^{\circ}.55'$) and Side Opposite to one of them ($AB\ 60^{\circ}.10'$. To find the other Opposite Side, greater than 90 Degrees.

I. CONSTRUCTION.

PCQ, Make the Angle BAC equal to $148^{\circ}59'$ by Drawing the Oblique Circle ABG with the Secant of its Complement, viz. $31^{\circ}01'$ (because the given Angle A is Obtuse (by Prob. 2. Case 2. Stereog. Prob) 2. and Prob. 6. Case 3. of Stereog. Prob. Lay off $60^{\circ}10'$ (the given Side) on the Oblique Circle ABG from A to B . 3. On the



Point B (by Prob. 3. Stereog.) Draw an Oblique Circle KBC , that shall make at the Primitive an Angle of $29^{\circ}55'$ (the other Given Angle ACB) and the Triangle ABC is Made. 4. Let fall the Perpendicular from the Point B to the Primitive, by Drawing the Right Circle mBn to cut the Primitive in n ; so shall BnA be the first Triangle and BnC the Second.

II. Mensuration. The Side or Hypoth. AC is Measured by Prob. 7. Case 3. of Stereog. which is $116^{\circ}36'$, because the Side CB is greater than a Quadrant.

II. Operation. By Case 1. of Right-angled Spherical Triangles, Given Hypoth. $AB\ 60^{\circ}10'$, and the Angle at the Base $ABn = 31^{\circ}01'$ (the Comp. of the given Angle $A\ 148^{\circ}59'$ to 180) to find the Perpend. Bn in the first Triangle BnA , i. e. As Rad. $SAB :: S\angle BAn :: Bn$.

That

That is,

As Radius	90° 00'	10.
Is to Hypoth. AB	60 10	9.938258
So is S. $\angle$ A	31 01	9.712049
To Perp. Bn Oppof.	26 33	9.650307

The Perpend. Bn being thus obtain'd, we have Given in the Second Triangle Bnc the Angle at the Base BCA 29 55' and the Perp. Bn 26 33' to find the Hypoth. or Opposite Side CB by Case 3. of *Right-angled Spherical-Triangles*, i. e. As S. $\angle$ BCA .. Bn :: Rad. :: CB.

As S. $\angle$ B C A	29° 55'	9.697874
Is to S. Perp. B n	26 33	9.650307
So is Rad.	90 00	10.

To S. Hypo. B C	63 24	9.951433
Subt. from	180 00	

Rem. BC Obtuse 116 36 required.

Or, by Ax 1.

As Sn. $\angle$ C	29° 55'	9.697874
Is to S. Side Oppof. A B	60° 10'	9.938258
So is Sn. $\angle$ A	148° 59'	9.712049

To S. Oppof. Side B C	19.650307
-----------------------	-----------

Acute 63° 24'	9.951433
---------------	----------

III. Instrumental, by Gunter's Scale.

The Extent From S. $\angle$ C 29° 55', to R S. 90° on the Sines.

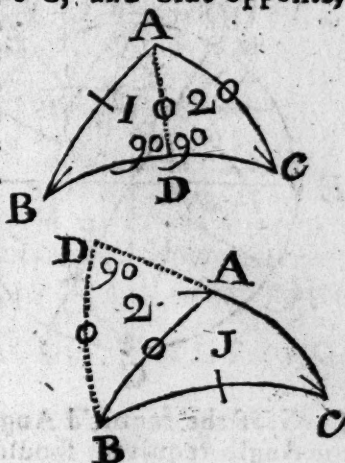
Will Reach from S. Perp. Bn 26° 33', to S. Hypo. A C 63° 24' on the Sines.

Or, the Extent from S. $\angle$ C 29 55, to S. $\angle$ A 31 01 on the Sines.

Will Reach from S. Side AB 60 10, to S. Side B C 63 24 on the Sines.

CASE I. Example 2. Fig. 1.

Given the Angle B, and Angle C, and Side opposite, BA, to find the opposite Side CA; the Perpend. AD falls within; BDA is the first Triangle, in which is given the Angle at the Base B, and Hypoth. BA, to find the Perp. D A; which found in the second Triangle DCA, will be given the Angle C at the Base, and Perpendicular DA, to find the Side or Hypothenufe AC, as above.



CASE I. Example 3. Fig. 2.

Also in the Triangle BCA given, the $\angle C$, $\angle A$, and Side BC opposite, to find the Hypoth. or Side AB, in the first Triangle BDC right-angled at D (the Perpend. BD falls without the Triangle) Data, $\angle C$ at the Base, and Hypoth. BC. Quere the Perp. BD; which obtained in the second Triangle BAD, you'll have given the Perp. BD, and $\angle BAD$, the Complement of the $\angle A$ to 180 Degrees, to find the Hypothenufe or Side BA; which I leave to the Reader's Exercise.

CASE II. In the Oblique Spherick Triangle ABC there are given two Sides (AB 34 59', and AC 60 10') and the opposite Angle (ACB 29 55') to find the opposite Angle ABC, Acute.

Note. this Case is ambiguous: For the requir'd Angle may be either Acute or Obtuse, as you may see in the Projection.

I Construction. PCQ. Make AC on the primitive

Circle equal to $60^{\circ} 10'$; and on the Point C in the Primitive, make an Angle of $25^{\circ} 55'$ (Prob. 2. Case 2. Stereog.)

2. Draw (by Prob 9. Case 2. Stereog.) a Parallel to the Right Circle EF at $34^{\circ} 59'$ distant from its Pole A, 'twill cut the former Oblique Circle in the Points B and n.

3. Through the Points

($A\pi G$, if the requir'd Angle must be Obtuse, or) ABG , if the Angle required should be Acute, as in this Case, draw the Oblique Circle ($A\pi G$, or) ABG ; so you have made the required Triangle ABC .

And because the Angles at B and C are similar, *i. e.* Acute, draw the perpendicular Great Circle ADG thro' the Point A, and Pole P and G, to fall within the Triangle upon the Base BC (by *Conclusion 3.*) so shall AD be the perpend. and CDA the first Triangle and BDA the Second.

II. The *Angle* CBA is measured by Prob. 8. *Variety* 5;
Stereog. in this case is $48^{\circ} 59'$.

III. Operation, as in Case 1. or Ax. 1. As R. SAC ; ;
SG--SDA, Perp. in the first Triangle DCA.

1. As Radius	90°	10.
Is to Hypo. or Side AC	60 10'	9.938258
So is the S. $\angle$ C	29 55	9.697874
To the Perp. DA	25 38	9.636132
Again, As S. Hypo. BA	34 59	9.758411
Is to Radius	90 00	10.
So is S. Perp. DA	25 38	9.636097
To S.op. $\angle$ ABD or ABC	48 59	9.877686 Tri. 2.

2. Then in the Right-angled Triangle BAD is given the Hypoth. AB 34 59', and Perp. DA 25 38', to find the opposite $\angle$ DBA required,, by Case 2. Right-angled Spe-
vick Triangles.

Or

Or thus, by *Axiom 1.*

As S. Side AB	34° 59'	9.758411
Is to S. $\angle$ BCA	29 55	9.697874
So is S. Side AC	60 10	9.938258
To S. Op: $\angle$ ABC	48 59	9.636132
As before		9.877721

IV. *Instrumental, by Gunter's Scale.*

The Extent from Rad. S. 90° , to S. $\angle$ C 29 55' on the Sines,

Will reach from S. Hypoth. AC 60 10', to S. Perp. DA 25 38' on the Sines.

2. The Extent from S. Hypoth. AB 34 59', to Rad. S. 90° on the Sines,

Will reach from S. Perp. AD 25 38', to S. Oppof. $\angle$ B 48 59 on the Sines.

Or,

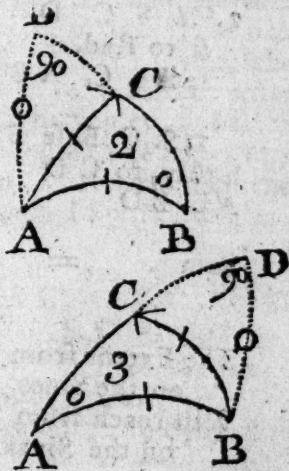
3. The Extent from S. Side AB 34 59', to S. $\angle$ C 29 55' on the Sines,

Will reach from S. Side 60 16', to S. $\angle$ B 48 59' on the Sines.

CASE II. *Example 2. Fig. 2. and 3.*

Given AB, and AC, and $\angle$ ACB, or its Complement to 180° , the $\angle$ ACD, to find the Perp. AD in the first Triangle ACD; which found, the second Triangle ABD, will be given the Perp. AD, and Hypoth. AB, to find the opposite *Angle B.* Fig. 2.

Or, in Fig. 3. Given AB, BC, and *Angle* BCD, to find the Perp. BB, which obtained, you have given, as before, the Hypothenufe AB, and Perpendicular BD, to find the opposite *Angle A.*



CASE III. In the Oblique Spherick Triangle ABC are given Two Sides (AB $34^\circ 59'$, and AC $60^\circ 10'$) and the Opposite $\angle$ ACB $29^\circ 55'$) to find the adjacent Side to the Given *Angle* BC.

I. Con-

I. *Construction*, the same as in the Last, *Case 2.* here being the same things given in this.

II. The Required Side BC is Measured, by *Prob. 7. Case 3. of Stereog.*

III. The Perpendicular AD being let fall from the End of the Side AB, and Opposite to the Angle at C; AB CA become Hypotenuses, in both the first Triangle CDA and the Second Triangle BDA and then to find the Perpend. AD, the Operation is the same as in the Last, *i. e. Case 2. viz. As R 90° .. S. Hypo. AC 60° 10' :: S. L C 29° 55' S. Perp. DA 25° 38'* which being found, we have in each Right-angled Triangle CDA and BDA, the Hypoth. cA and BA, and Perpendicular DA (Common to both Triangles) to find DC and DB, the Bases whose Sum is equal to the Side BC Required.

Therefore. IV. Operation by *Case 7. Right-angled Sph. Triangles.*

1. As S. C. DA .. Rad. :: S. C. AC .. S. C. DC
2. As S. C. DA .. Rad. :: S. C. BA .. S. C. BD *i. e.*

1. As S. C. Perp. DA 25° 38'	9.955005
Is to Radius 90° 00'	10.
So is S. C. Hypo. AC 60° 10'	9.696774

To S. C. Base CD	56° 31'	9.741769
------------------	---------	----------

2. As S. C. DA 25° 38'	9.955005
Is to Rad. 90° 00'	10.
So is S. C. AB 34° 59'	9.913453

To S. C. Base BD	24° 40'	9.958448
Lastly, to CD	56° 31'	
Add BD	24° 40'	

Sum = BC 81 11 requir'd.

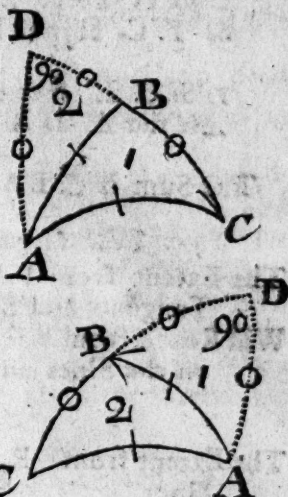
By Gunter's Scale.

The Extent from S. C. DA 64° 22', to Rad. S. 90° 00' on the Sines,
Will reach from S. C. AC 29° 50', to S. C. CD 33° 29' on the Sines.

The Extent from S. C. DA 64° 22', to Rad. S. 90° 00' on the Sines,
Will Reach from S. C. AB 55° 01', to S. C. BD 65° 20' on the Sines.

Example II.

Example II. In the Triangle, $A C D$ Right-angled at D , given $A C$, and adjacent Angle C . to find the Perpend. $A D$ and Base $C D$; which Obtain'd in the Triangle $A B C$, Given the Hypo. $A B$, and Perp. $A D$, to find the Base $B D$. Then, $B D \pm C D = B C$ required *i, e.*



General Rule.

If the Perpend. fall within, then $B D$ added to $D C$, gives $B C$.

If the Perpend. fall without, then $B D$ subtr. from $D C$, gives $B C$. *q. e. i.*

CASE 4. In the Ob. Sph. Triangle $A B C$ there are given Two Sides ($A C 60^{\circ} 10'$ and $A B 34^{\circ} 59'$) And an Angle, ($A C B 29^{\circ} 55'$ Opposite to one of them, to find the Contained Angle $B A C$.

I. *Construction.* This is Projected as before in *Case 2.*

II. And the Required Angle $B A C$ Measured as in *Prob. 8. Variety 4. of Stereographick Problems.*

The Perpendicular $A D$ being let fall as before Directed. Then III. the Operation, to find the said Perpendicular is the same as in *Case 2.* and *3.* which is $25^{\circ} 38'$. Hence in the Triangle $A D C$ given the Hypoth. $A C$ and adjacent Angle C . to find the other Angle $D A C$ and by *Case 14.* Right-angled Spherick Triangles; and in the second Triangle $B D A$ will be given the Hypoth. $B A$, and Perp. $D A$, to find the Angle $B A D$, by *Case 13.* of Right-angled Sph. Triangles; that is,

1. *As* $T.C. \angle C :: R :: S.C. A C :: T.C. \angle D$
 $A C$. 2. *As* $R T. A D :: T.C. B A :: S.C. \angle B A D$.

1. <i>As</i> $T.C. \angle C$	$29^{\circ} 55'$	10.240028
<i>Is to</i> Rad.	$90^{\circ} 00'$	$10.$
<i>So is</i> $S.C. A C$ Hypo.	$60^{\circ} 10'$	9.696774
<i>To</i> $T.C. \angle D A C$	$74^{\circ} 02'$	16.456753

2. As Rad. S	90° 00'	10.
Is to T. Perp. AD	25 38	9.681092
So T. C. Hypo. BA	34 59	10.155042
To S. C. $\angle$ BAD	46 43	9.836134
Add the $\angle$ DAC	74 02	
The Sum is $\angle$ BAC	120 45	Obtuse. Required.

IV. Instrumental, by Gunter's Scale.

The Extent from T. C. $\angle$ C 60° 05' to R. S. 90° on the Tangents and Sines,
Will Reach from S. C. AC 29° 50' to T. C. $\angle$ DAC 15° 58' on the Sines and Tangents.

Or,

The Extent from R. S. 90° S. C. AC 29° 50' on the Sines,
Will Reach from T. $\angle$ C 29° 55', to T. C. $\angle$ A 15° 58' on the Tangents.
The Extent from T. C. Hypo. AC 55° 01', to Tang. Perp. AD 25° 38' on the Tangents,
Will Reach from R. S. 90° 00' to S. C. $\angle$ BAD 43° 17' on the Sines.

Example II. to this Case.

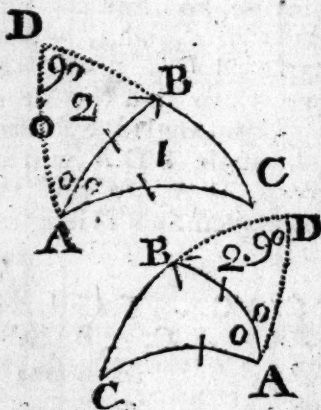
In the Right-angled Triangles ABC, Right-angled at D, letting fall the Perp. AD on the Side CB produced, *i. e.* without the Triangle; we have given in the second Triangle ABD the Hypoth. AB, and the $\angle$ ABD adjacent (the Compl. of the given $\angle$ ABC to a Semicircle) as before; to find the Perpend. AD.

In the first Triangle ACD will be given the Hypoth. AC, and Perpend. AD, to find the $\angle$ CAD, as before.

And the $\angle$ CAD less the $\angle$ BAD ($\angle$ BAD being found in like manner as in the Second Operation, in Nr. 1. preceding) is equal to the $\angle$ BAC required.

If Perp $\left\{ \begin{array}{l} \text{within} \\ \text{fall} \end{array} \right\}$ then BAD $\left\{ \begin{array}{l} \text{added to} \\ \text{taken from} \end{array} \right\}$ CAD, gives BAC.

CASE



CASE V. In the Oblique-Spherick Triangle ABC , Given two Angles ($BAC\ 148^{\circ}\ 59'$; $ACB\ 29^{\circ}\ 55'$) and a Side ($AB\ 60^{\circ}\ 10'$) opposite to one of them, to find the interjacent Side (AC)

1. This Triangle is projected as in Case the First (the same things being given;) and 2. measured as in Prob. 7. Case 1, or 3. of *Stereog.*

3. Having let fall the Perp. Bn , without the Triangle, the said Perp. Bn is found by Case 1. to be $26^{\circ}\ 33'$; and in the first Triangle BAn we have given the Hypoth. $BA\ 60^{\circ}\ 10'$, and $\angle BAn\ 31^{\circ}\ 01'$ (the Comp. of the Given $\angle BAC\ 148^{\circ}\ 59'$, to 180°) to find the Base An ; which found, in the second Triangle Cn , we have given the $\angle$ at $C\ 29^{\circ}\ 55'$, and Perpend. $Bn\ 26^{\circ}\ 33'$, to find the Base Cn ; and An subtracted from Cn , gives the Sides AC required. See the

Operation, by Case 1, 10, and 5, of Right-angled Spherick Triangles.

1. As Radius $S.\ 90^{\circ}$, Is to $S.$ Hypoth. $60^{\circ}\ 10'$; So is $S.\ \angle BAn\ 31^{\circ}\ 01'$, To the $S.$ Perp. $Bn\ 26^{\circ}\ 33'$.

2. As T.C. $BA\ 60^{\circ}\ 10'$	9.758517
Is to Radius $S.\ 90^{\circ}$	10.
So is S.C. $\angle BAn\ 31^{\circ}\ 01'$	9.933990

To the T. Base $An\ 56^{\circ}\ 12'$	10.174473
--------------------------------------	-----------

3. As Radius $S.\ 90^{\circ}$	10.
To T.C. $\angle C\ 29^{\circ}\ 55'$	10.240021
So is T. Perp. $Bn\ 26^{\circ}\ 33'$	9.698685

To S. Base $Cn\ 60^{\circ}\ 16'$	9.938706
----------------------------------	----------

Or Obtuse	119 44
Subt. Base An	56 12

Rem. the Side $AC\ 63\ 32$ Required.

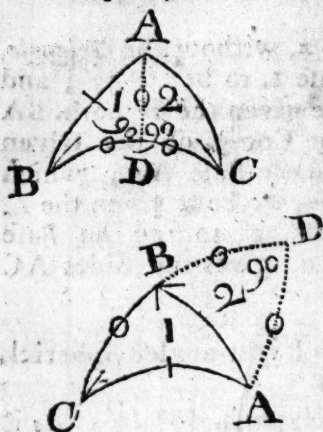
Note, In this Case. Cn is Obtuse, as is plain by the Figure to the First Case; therefore subtract the Answer in the third Operation from 180° , and you have the Base Cn .

IV. Instrumental, by Gunter's Scale.

The Extent from Radius $S.\ 90^{\circ}$, to $S.C\ \angle BAn\ 58\ 59'$ on the Sines,
Will reach from T.C. $BA\ 60\ 10'$, to T. Base An on the Tangents.

The Extent from T. $\perp$ C. $29^{\circ} 55'$ to T. Perp. $B \approx 26^{\circ} 33'$
on the Tangents,
Will reach from R. S. 9° to S. Base C $\approx 60^{\circ} 16'$ on the
Sines.

Example 2, and 3. Given the $\angle$ B and C, and Side Opposite to one of them, BA or CA, to find BC, *Fig. 1.* In the first Triangle let fall the Perpendicular AD within, and you have in the first Right-angled Triangle BDA, the Hypoth. BA, and $\perp$ at the Base DBA, given, to find the Base BD and the Perpen. DA, and in the Second Triangle CDA, then will be given the $\perp$ at the Base CD, and BD added to DC, gives BC required.



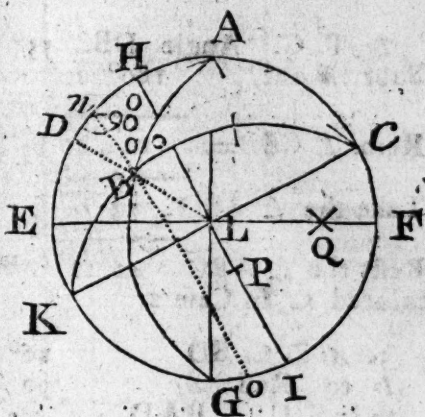
In *Figure II.* Given the Angles C and B ; and Side Opposite, (to B.) CA, to find CB.

In the first Triangle ACD, is given the Hypoth. AC, and Angle at the Base ACD, to find the Perpendicular AD (being let fall without the Triangle) and Base CD. In the Second Triangle ABD, (CD and AD being found) will be given the Perpend. AD and Angle, at the Base ABD (the Comp. to 180° of the given Angle CBA) to find the Base BD ; then BD subtracted from CD, remains CB ; the Sides required. See the 10th, and 5th, Cases of Right-angled Spherick-Triangles.

If Perp. $\left\{ \begin{array}{l} \text{within} \\ \text{fall} \end{array} \right\}$ then $\left\{ \begin{array}{l} \text{added to} \\ \text{BD} \end{array} \right\}$ DC, gives BC.

CASE 6. In the Oblique Spherick Triangle ABC Given, Two Angle (BAC $148^{\circ} 59'$ and ACB $29^{\circ} 55'$) and a Side Opposite (to one of them, as BC $116^{\circ} 36'$) to find the other $\angle$ ABC.

I. *Construction.* At any Point as C, (taken at pleasure) in the primitive Circle from an $\angle$ equal to the given $\angle$ (ACB $29^{\circ} 55'$) by *Prob. 2. Case 2. Stereog.* 2. find the Pole P of the Oblique Circle CBK and by the Help of it cut off the Arch of the Oblique Circle CB, = $116^{\circ} 36'$ by *Prob. 6. Case 3. of Stereog.* Or thus, Describe a Parallel to a Right Circle HI at $63^{\circ} 24'$ (the Comp. of $116^{\circ} 36'$ to 180°) distant from K to cut the Oblique Circle CBK in B.



3. On the Point B, (by *Prob. 3. of Stereog.*) draw an Oblique Circle ABG to make at the Primitive an $\angle$ equal to the Given $\angle$ CAC $148^{\circ} 59'$; that is, because the $\angle$ is Obtuse, make it with its Comp. to 180° viz. $31^{\circ} 01'$, of the Tangents from the Center L, and the Secant of the same $31^{\circ} 01'$ from the given Point B will intersect, each other in the Point Q; then with the Dist. or Radius QG = QA describe the Oblique Circle ABG, and ABC is the Triangle required.

4. Let fall the Perpend. BD without the Triangle, by drawing through the Center L and B, the Right Circle $\angle$ BD, and DBC is the first Triangle, in which is given the Hypoth. CB, and $\angle$ at the Base DCB, to find the Perpend. BD, and $\angle$ at the Perpend. DBC, by *Case 1. and 14, R. A. Sp. Triangles.* In the Second Triangle ABD, will be given, the $\angle$ B A D and Perpend. BD to find the $\angle$ D B A, and the $\angle$ D B A, taken from the $\angle$ D B C gives the $\angle$ A B C, required. See the

II. *Operation,* by *Case 1, 14, and 12. of Right-angl'd Spherical Triangles.*

1. As R S.	$90^{\circ} 00'$	10.
Is to S. Hypo. CB	$116^{\circ} 36'$	9.951412
So is S. $\angle$ B C D	$29^{\circ} 55'$	9.697874
To the Perp. BD	$26^{\circ} 29'$	9.649286
as in the last Case.		To
PART II.	H 2	To

2. As T. C. Angle C.	29° 55'	10.240021
To to Radius S	90	10.
So is S. C. CB	63° 24' Acute	9.651044
To T. C. Angle DBC	75° 34'	9.411023
Subt. from	180° 00'	

Rem. $\angle CBD = 114\ 26$

Take the $\angle ABD = 73\ 14$

Rests the $\angle ABC = 41\ 12$ W. W. Required and it's measured as, in Case 2.

3. As S. C. BD	26° 21'	9.951854
Is to Rad. S.	90 00	10.
So is S. C. $\angle BAD$	31 01	9.932990
To S. $\angle ABD$	73 14	9.981136

The Perpendicular BD, and $\angle ABD$ is Measured as in Case 1. and 3. of R. A. Spherick Triangles.

III. Instrumental, by Gunter's Scale.

The Extent from Rad. S. 90 to S. CB 63° 24' Acute on the Sines,

Will reach from S. $\angle C$ 29° 55' to S. Perpend. BD 26° 29' on the Sines.

The Extent from R. S. 90° to S. C. CB 26° 36' on the Sines,

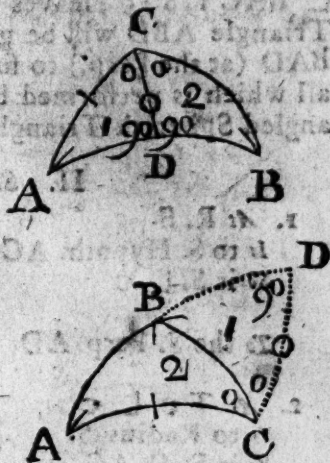
Will Reach from T. $\angle C$ 29° 55' to T. C. $\angle DBC$ 75° 34' on the Tangents

The Extent from S. C. BD 63° 31' to Rad. S. 90° on the Sines,

Will Reach from S. C. $\angle BAD$ 58° 59' to S. ABD 73° 14' on the Sines.

Example 2. and 3.

Given, the Angles A and B and Side AC Opposite to find the $\angle$ ACB, at the Perpendicular, being let fall within in the First; and without in the Second Figure which I leave the Learner's Exercise to Operate.



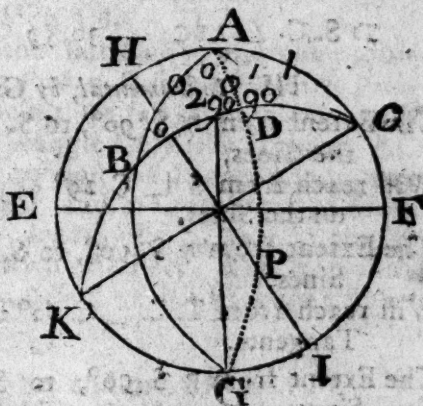
If Per. $\left\{ \begin{array}{l} \text{within} \\ \text{fall} \end{array} \right\}$ $\left\{ \begin{array}{l} \text{without} \\ \text{fall} \end{array} \right\}$ DCB $\left\{ \begin{array}{l} \text{Added to} \\ \text{Subt. from} \end{array} \right\}$ ACD, gives ACB.

CASE VII. In the Oblique Spherick Triangle ABC given two Angles, ($BAC 148^{\circ} 59'$, and $ACB 29^{\circ} 55'$) and interjacent Side ($AC 63 32'$) to find the Opposite $\angle$ ABC.)

1. Construction. P.C. 2. Make the $\angle$ BAC at the Primitive equal to $148 59'$, as in Case 1.

2. Make AC (the Given Side) on the Primitive, equal to $63 32'$, from your Line of Chords.

3. Make at the Pt. C, in the Primitive, an Angle ACB, equal to $29 55'$ (the other given Angle) as in Case 1. the two Oblique Circles ABG and CBK intersect each other in the Point B, and form the Triangle ABC required.



4 Let fall the Perpendicular AD within the Triangle (as in Case the Second;) so shall CDA be the first Triangle, in which is given the Hypoth. AC $63 32'$, and
PRAT H. H 3 $\angle$ at

$\angle$ at the Base ACD, to find the Perpendicular AD, and $\angle$ at the Perpendicular DAC, taken from the Given $\angle$ BAC $148^\circ 59'$, leaves the $\angle$ BAD; then in the second Triangle ABD will be given the Perpend. AD, and $\angle$ BAD (at the Perp.) to find the other $\angle$ ABC required; all which is performed by Case 1, 14, and 11, of Right-angled Spherick Triangles.

II. See the Operation.

1. As R. S.	90°	10.
Is to S. Hypoth. AC	$63^\circ 32'$	9.951917
So is S. $\angle$ C	$29^\circ 55'$	9.697874
To the S. Perp. AD		$26^\circ 31'$
		9.649791

2. As T. C. $\angle$ C	$29^\circ 55'$	10.240011
Is to Radius S.	$90^\circ 00'$	10.
So is S. C. AC	$63^\circ 32'$	9.649020
To T. C. $\angle$ CAD		$75^\circ 37'$
Subt. from $\angle$ BAC	$148^\circ 59'$	9.108999

Rem. the $\angle$ BAD = $73^\circ 22'$ is measured as in Case the Second.

3. As Radius S.	90°	10.
Is to S. C. AD	$26^\circ 31'$	9.951708
So is S. $\angle$ BAD	$73^\circ 22'$	9.981436
To S. C. $\angle$ ABC		$30^\circ 59'$
		9.953164

III. Instrumental, by Gunter's Scale.

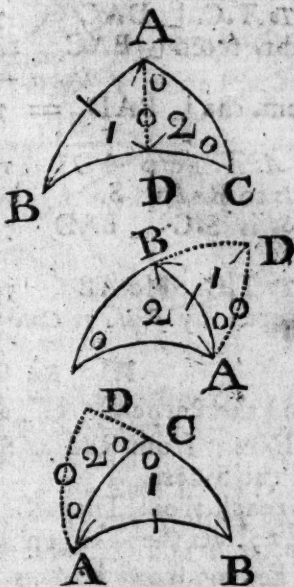
The Extent from R. S. 90° , to S. Hypoth. AC $63^\circ 32'$ on the Sines,
Will reach from S $\angle$ C $29^\circ 55'$, to S. Perp. AD $26^\circ 31'$ on the Sines.

The Extent from R. S. 90° , to S. C. AC $63^\circ 28'$ on the Sines,
Will reach from T. C. $\angle$ C $60^\circ 05'$, to T. $75^\circ 37'$ on the Tangents.

The Extent from R. S. 90° , to S. C. A $63^\circ 29'$ on the Sines,
Will reach from S. $\angle$ BAD $73^\circ 22'$, to $59^\circ 01'$ S. C. $\angle$ A BC on the Sines.

Examples 2, 3, and 4.

Given the *Angles* A and B, and *Side* between them AB (the Hypothenuse) to find the Perpendicular AD, and $\angle$ B AD in the first Triangle BDA (as before;) if the Perpendicular falls within, then the $\angle$ BDA subtracted from the Given $\angle$ A, gives the $\angle$ DA C; but if without, then BAD added to the Given $\angle$ A, gives the $\angle$ DAC; but if the greatest Side be given, you must subtract the $\angle$ BAC given, from the $\angle$ BAD (when found) and you have DAC in the second Triangle, and the Perpend. AD given, to find the other $\angle$ ACD; which, if required Obtuse (as in the third Figure above) must be subtracted from 180° , and you have the $\angle$ sought.



CASE VIII. In the Oblique Spherick Triangle ABC, given two *Angles* ($\angle$ BAC $148^\circ 59'$, and $\angle$ ACB $29^\circ 55'$) and the *Side* interjacent, or between them (AC $63^\circ 32'$) to find the other opposite Side AB.

I. This is projected as the last Case 7, and the required Side AB, measured by Prob. 7. Case 3, of *Stereog. Problems*) the Perpend. AD being let fall within, you will have given, in the First Triangle CDA the Hypoth. AC $63^\circ 32'$ and $\angle$ C $29^\circ 55'$, to find the Perpend. AD, and $\angle$ DAC (as in the last Case,) and DAC subtracted from the given $\angle$ BAC $148^\circ 59'$, leaves the $\angle$ BAD in the second Triangle; which, with the Perp. AD, you may easily find the Side AB, by Case 9. of Right-angled Spherick Triangles. See the

II. Operation, by Case 1, 11, and 9. of R.A.S.T.

1. As Radius S. 90° .. S. Hypoth. AC $63^\circ 32'$:: S. $\angle$ C $29^\circ 55'$.. S. Perp. AD $26^\circ 31'$, as in the last Case.

2. As T.C. $\perp$ C	29° 55'	10.240021
Is to Rad. S.	90 00	10.
So is S.C. AC	63 32	9.649020
To T.C. $\perp$ DAC	75 37	9.408999
Subt. from $\perp$ BAC	148 59	
Rem. the $\perp$ BAD =	73 22	
3. As T. Perp AD	26 31	9.698053
Is to Radius S.	90 00	10.
So is S.C. $\perp$ BAD	73 22	9.475303
To T.C. Side AB	59 06	9.777250
Measured by Prob. 7. Case 3. Stereog.		

III. By Gunter's Scale.

Find the Perpend. AD, as in the last Case.

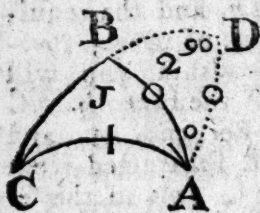
The Extent from Radius S. 90° , to S.C. AC $26^\circ 28'$ on the Sines,

Will reach from Tang. C. $\perp$ C $60^\circ 05'$, to T.C. $\perp$ A $14^\circ 23'$ on the Tangents.

The Extent from Radius S. 90° , to S.C. $\perp$ BAD $16^\circ 38'$ on the Sines,

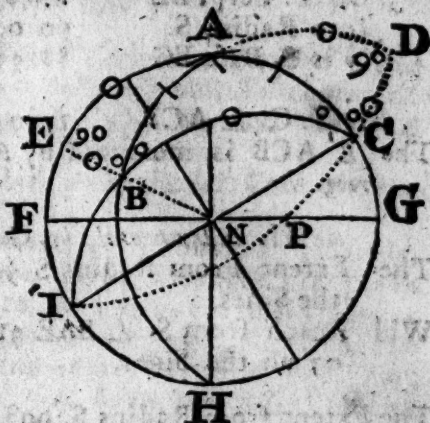
Will Reach from T. Perp. AD $26^\circ 31'$, to T.C. Side AB $30^\circ 54'$ on the Tangents.

Example 2. Given the Angles A and C, and Side AC, to find the Side AB, the Perpend. AD, being let fall without the Triangle. In the first Triangle ACD is given Angle C, and the Hypotenuse AC, to find the Perpendicular AD, and $\perp$ DAC, as before, and the $\perp$ DAC made less by the $\perp$ A, is the $\perp$ BAD in the second Triangle ABD; with which, and the Perpendicular AD, you may find the Hypotenuse or Side AB, by Case 9. of Right-angled Spherick Triangles.



CASE IX. In the Oblique Spherick Triangle ABC there are Given, the two Sides (AB $34^\circ 59'$, AC $80^\circ 10'$) and the contained $\perp$ (BAC $148^\circ 59'$, or acute $31^\circ 01'$) to find one of the opposite $\perp$ s, as C or B.

I. Construction, P.C.Q. Make the Side AC on the Primitive, equal to $60^{\circ} 10'$ from the Line of Chords, and the $\angle BAC$ at the Primitive equal to $148^{\circ} 59'$, as in *Case 1.* and also the Side AB on the Oblique Circle ABH equal to $34^{\circ} 59'$, as in *Case 2.* 2. Draw the Right Circle CNI, and thro' the Points CB and I draw the Oblique Circle CBI, and the Triangle ABC is made. 3. Let fall



the Perpendicular BE, or CD, without the Triangle ABC, and you'll have given in (each of) the first Triangles ABE (or ACD) the Hypothenufe AB $34^{\circ} 59'$ (or Hypothenufe AC $60^{\circ} 10'$) and $\angle$ at the Base BAE (or CAD $31^{\circ} 01'$ (the Complement of the given $\angle BAC$ to 180°)) to find the Perpendicular BE (or CD) and Base AE (or AD) and AE added to AC, gives EC, the Base in the second Triangle CEB; which, with the Perpendicular BE, you may find the $\angle$ at the Base BCE required, by *Cases 1, 10, and 6, of Right-angled Spherick Triangles.* Or AD added to AB, gives BD, the Base in the second Triangle CEB; which, with the Perpendicular CD given, the $\angle$ at the Base DBC, &c. may be found by the foresaid Cases. See the Operation.

1. As Radius S.	90° 00'	10.
Is to the Hypoth. AB	34° 59'	9.758411
So is the S. $\angle$ BAE	31° 01'	9.712049
To the S. Perp. BE	17° 03'	9.470460
2. As T.C. AB	34° 59'	10.155043
Is to Radius S.	90° 00'	10.
So is S.C. $\angle$ BAE	31° 01'	9.922990
To T. Base AE	30° 57'	9.777948
Add the Base or	60° 10'	
Side AC given		
Sum is the Base EC	91° 07'	

3. As T. Perp. BE	17° 03'	9.486693
Is to Radius S.	90 00	10.
So is S. Base EG	88 53	9.999917
To T. C. $\angle$ ACB	17 03	10.513224
The $\angle$ ACB is measured by <i>Probl. 8. Variety 4. of Stee</i>		
<i>reog.</i>		

III. Instrumental, by Gunter's Scale.

The Extent From Radius S 90° , to S. AB $34^\circ 59'$ on the Sines,

Will Reach from S. $\angle$ BAE $31^\circ 01'$, to S. Perp. BE $17^\circ 03'$ on the Sines.

The Extent from Radius S. 90° , to S.C. $\angle$ BAE $58^\circ 59'$ on the Sines,

Will Reach from T. AB $34^\circ 59'$, to T. Base AE $30^\circ 57'$ on the Tangents.

The Extent from Radius S. 90° , to S. EC $88^\circ 53'$ undiscernible, on the Sines and Tangents,

Will reach from T. BE $17^\circ 03'$, to T. $\angle$ C $17^\circ 03'$ on the Sines and Tangents.

Example 2. Given AC and EC, and the contained $\angle$ ACB, to find the $\angle$ BAC, in the first Triangle CDB, the Hypothenuse BC, and $\angle$ at the Base DCB given, and the Perp. BD, and Base DC required.



If the Perpendicular (BD) falls within the Triangle, then DC taken from AC, leaves AD; but if the Perpendicular (BD) falls without, then AC taken from DC, leaves AD; which being obtained, you have the Perpendicular BD, and Base AD given, to find the $\angle$ at the Base BAD (by the 1st, 10th, and 6th Cases of Right-angled Spherick Triangles.

Note, If the $\angle$ A be required Obtuse, then it's the Complement of the $\angle$ BAD to 180° . Or thus:

If DC $\begin{cases} \text{lesser} \\ \text{be} \end{cases}$ than AC the $\angle$ $\begin{cases} \text{similar} \\ \text{at A} \end{cases}$ to the $\angle$ $\begin{cases} \text{at C} \\ \text{dissimil.} \end{cases}$

Oblique Spherick Triangles.

123

CASE X.

In the Oblique Spherick Triangle ABC are given two Sides ($AB\ 34^\circ\ 59'$, $AC\ 60^\circ\ 10'$) and the contained $\angle\ BAC\ 148^\circ\ 59'$, as in the last Case) to find the third Side (BC .)

This Triangle is projected, as in the last Case, and the Side BC measured, as in Case I.

Let fall the Perpendicular CD without the Triangle, as before; and then in the First Triangle are given the Hypotenuse $AC\ 60^\circ\ 10'$, and $\angle$ at the Base $CAD\ 31^\circ\ 01'$ (the Complement of the given Angle BAC to 180 Deg.) to find the Perpendicular CD , and Base AD ; which added to $AB\ 34^\circ\ 59'$ you have the Base BD and Perpendicular CD , to find the Hypotenuse (or Side required BC ;) all which may be performed by Case 1, 10, and 8, of Right-angled Spherick Triangles.

Or, if you will, you may, as in the last Case, let fall the Perpendicular BE without, and so find the Perpendicular BE , and Base AE , and AE added to AC , gives EC ; which, with the Perpendicular BE (being found) you may find BC , as above.

The Operation.

1. As Radius S.	$90^\circ\ 00'$	10.
Is to Hypoth. AC	$60\ 10$	9.933258
So is S. $\angle CAD$	$31\ 01$	9.712049
To the Perp. CD	$26\ 33$	9.650307
2. As T.C. AC	$60\ 10$	9.758517
Is to Radius S.	$90\ 00$	10.
So is S.C. $\angle DAC$	$31\ 01$	9.932990
To the Base AD	$56\ 12$	10.174473
Add AB	$34\ 59$	
Sum is = Base BD	$91\ 11$	Or, $88^\circ\ 49'$ acute.
3. As Radius S.	$90\ 10$	10.
Is to S.C. BD	$88\ 49$	8.314954
So is S.C. CD	$26\ 33$	9.952231
To S.C. BC	$88\ 57$	8.267185
Subt. from	$180\ 00$	
Or	$91\ 03$	obtuse.
Or		

III.

III. Instrumental, by Gunter's Scale.

The Extent from Radius 90° , to S. AC $60^\circ 10'$ on the Sines,

Will reach from S. $\angle$ CAD $31^\circ 01'$, to S. Perp. CD $26^\circ 33'$ on the Sines.

The Extent from Radius S. 90° , to S.C. $\angle$ DAC $80^\circ 59'$ on the Sines.

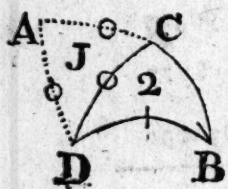
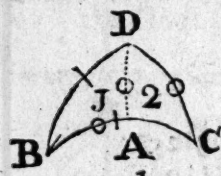
Will reach from T.C. AC $60^\circ 10'$, to T. Base AD $56^\circ 12'$ on the Tangents.

The Extent from Radius S. 90° , to S.C. BD $63^\circ 37'$ on the Sines,

Will reach from S.C. CD $01^\circ 11'$, to S.C. BC $01^\circ 03'$ on the Sines.

Example 2. BD and BC, and contained Angle DBC given, to find DC.

Find the Perpend. AD, and Base BA; as before. If the Perpend. falls within, then take BA from BC, and you have AC; but if the Perpend. falls without, then BC taken from BA, leaves AC the Base; which with the Perpend. AD, find the (Hypoth. or) required Side DC, as before. *q.eri.*

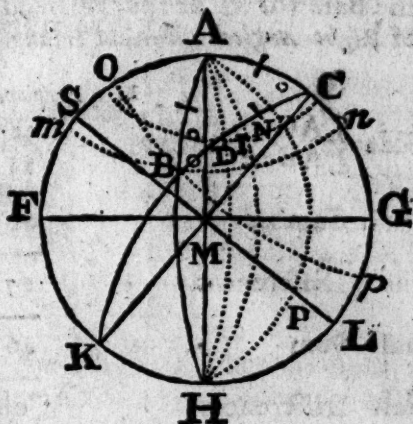


CASE II. In the Oblique Spherical Triangle, ABC are given three Sides (BA $55^\circ 30'$ AC $38^\circ 28'$ BC $67^\circ 30'$) to find an Angle ABC, or ACB, or BAC.

Oblique Spherick Triangles.

177

Construction. 1. PCQ. Make AC on the Primitive equal to $38^{\circ} 28'$ (the given Side AC) and draw the Right Circle CK, and at Right $\angle$ s to it, the Right Circle SL. 2. at $55^{\circ} 30'$ (the Side AB) distant from the Pole A, of the Right Circle FG, and describe the Parallel *mn* (by *Prob. 9. Case 3. Stereog.*) 3. Describe the Parallel *po* (by the foresaid *Prob.*) at $67^{\circ} 30'$ (the given Side CB) distant from the Pole C. These two Parallels cut each other in the Point B. 4. Through the Points ABH draw the Oblique Circle ABH, also through the Points CB and K draw the Oblique Circle CBK and the Triangle BAC is finished. 5. Let fall the Perpend. AI upon the Base BC. and make AD equal to AC; and BI equal to IC, equal to half the Base; then shall BD be equal to the Difference of the Segments of the Base, BC (made Perp. AI.) to find which is by the following Analogy, and is the same. (with the Prefixing only the word *Tangent* to each Line) as *Axiom 3. of Plain Trigonometry Oblique.*



Analogy or Proportion.

As the Tang. of the Base, (the Side on which the Perp. falls.)

Is to the Tang. of the Sum of the other two Sides;

So is the Tangent, of the Difference of those Sides,

To the Tang. of the Difference of the Segments of the Base.

Or thus; becase the Halves are as the Wholes in Proportion, Therefore,

As T. half the Base

Is to T. half the Sum of the Sides

So is T. half the Difference of the Sides.

To the T. half the Diff. of the Segments of the Base.

Then half the Base added to half the Differences of the Segments, gives the greater Base.

And half the Base lessend by half the Differences of the Segments gives the Lesser Base, in the Two Right-angled Triangles; then you'll have given the Hypothenuse.

thenuse, and Base in either Triangle, to find the *Angle* at the Base (or $\angle$ at the Perpendicular) by *Case 13*, and 22 of *Right Angled Spherical Triangles*. See the Work.

1. Preparation.

Side $\begin{cases} AC \\ AB \end{cases} =$	$38^{\circ} 28'$ $55 \quad 30$
their Sum =	$93 \quad 58$
their Differences	$17 \quad 02$
half Sum	$46 \quad 59$
half Differences	$08 \quad 31$

2. Operation.

As T. half Base BC =	$33^{\circ} 45'$	9.8248926
Is to T. half Sum Side	$46 \quad 59$	10.0300909
So is T. half Diff. Side	$08 \quad 31$	9.1753622
To T. half Diff. Segments =	$13 \quad 30$	19.2054531
Add. half Base =	$33 \quad 45$	9.3805605
Sum is greater Base BN =	$47 \quad 15$	
the Diff. is the lesser Base BN =	$20 \quad 15$	
3. As Radius	$90^{\circ} 0'$	10.
Is to T. C. BA	$55 \quad 30$	9.837134
So is T. BN	$47 \quad 15$	10.034144
To S. C. $\angle$ ABC	$41 \quad 59$	9.871278
Measured, as in Case 2.		
4. As Radius	$90^{\circ} 00'$	10.
Is to T. C. AC =	$38 \quad 28$	10.099913
So is T. CN =	$20 \quad 15$	9.566932
To S. C. $\angle$ ACB =	$62 \quad 20$	9.666845
Measured, as in Case 9.		

You may find the $\angle$ s at the Perpend. viz. BAN, and CAN, by *Case 2*. of *R.A.S.T.* (having their Hypothenuses BA, and AC, Bases given or found as before) or by

by the Rule of Opposite Sides and $\angle$ s; which added together gives the $\angle$ BAC, but more readily thus, by the Proportion of Opposite Sides and $\angle$ s.

As S. Side AC	38° 28	9.7938;2
Is to Sine of it's Op. $\angle$ ABC	41 59	9.825370
So is S. of the Side BC	67 30	9.964615
To S. of its Op. $\angle$ BAC	52 c6	19.790985
Subtr.	180 00	9.897153
Or Obtuse	127 55	

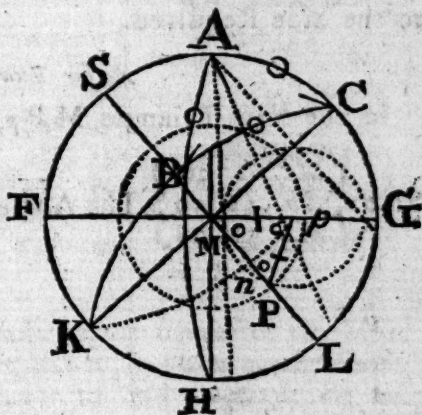
Or having, found any one of the $\angle$ s in the Oblique Triangle (ABC,) the Other $\angle$ s may easily be found by the Proportion of Opposite Sides and $\angle$ s, which, I think, is Obvious to any Capacity, by the preceding Figure it self. For having found the $\angle$ at B. You may say,

As the Sine, of the Side AC
Is to the Sine of the Angle B;
So is the Sine of the Side AB
To the Sine of the Angle C.

2 And, As the Sine of the Side AC
Is to the Sine of the Angle B;
So is the Sine of the Side BC,
To the Sine of the Angle A, as above.

CASE XII. In the Oblique - Spherick Triangle ABC There are Given the three Angles (BAC $127^{\circ} 54'$; ACB $62^{\circ} 20'$, ABC $41^{\circ} 59'$.) find the Sides (AB, AC, BC.)

This is Projected, as in the 11th, or Last Prob. of Stereography; the Distance of the Poles MP is equal to the $\angle$ ACB, and Mp is equal to the Complement to 180° , of the Angle BAC, and drawing a great Circle through the two Points or Poles P, p, by Prob. 4. of Stereog. the Arch Pp, is equal to the Angle



ABC;

ABC ; therefore in the Operation of this, instead of the given Triangle ABC, in which the three Angles are known, we Resolve another Triangle M P p, which is projected from the Poles of the Circles that constitute the Given Triangle (See the 20th Defin. of Spherical Triangles in Page 62.) and which is most conveniently formed by those Poles which are within the Primitive Circle.

In this New Triangle (M P p,) if all the Angles of the Given Triangle (ABC) or two of them be acute ; two Sides are equal to the two Lesser Angles of the Given Triangle ; and the third Side is the Complement of the greater Angle to 180° and two $\angle$ s in this Triangle, are equal to the two Lesser Sides in the given Triangle ; and the third $\angle$, the Complement of the greater Side, to 180° : If two of the Given Angles be Obtuse, two Sides of the New Triangle are Complements of the Obtuse Angles, to 180° ; and the third Side is the Complement of the acute $\angle$ to 180° .

If the three given Angles be Obtuse, then the Sides in this Triangle are the Complements of those Angles to 180° and whether all the Given Angles or two only be Obtuse, the Angles of the New Triangle are Complements of the corresponding Sides in the given Triangle to 180° Degrees.

This Conversion of Angles into Sides being made, and the Perpendicular let fall from the greatest Angle (P) upon the Base, (MP) and the other Preparations, as in the preceding Case, the Solution of this Case will be in all respects the same as in that ; to find the Segment of the Base, &c. the three Sides in the New Triangle, being given to find any Angle Corresponding to the Side Required.

As for Example.

In the New Triangle M P p, there is Given the

Side $\left\{ \begin{array}{l} MP \ 62^\circ \ 20' \\ Pp \ 41 \ 59 \\ Mp \ 52 \ 06 \end{array} \right\}$ The Angle p M P &c. Required.

I. Prep.

the
An
Ar
PA

1. Prep. and Operation.

Side	$\begin{cases} MP \\ PP \end{cases}$	$\begin{matrix} 52^{\circ} 06' \\ 41^{\circ} 59' \end{matrix}$
Their Sum		<u>94 05</u>
Their Diff.		<u>10 07</u>
Half Sum		<u>47 05</u>
Half Diff.		<u>05 03</u>
MP		<u>62 28</u>
Half Base =		<u>31 10</u>

2. As T. Half Base	$31^{\circ} 10'$	9.781631
Is to T. Half Sum Sides	$47^{\circ} 05'$	10.031611
So is T. Half Diff. Sides	$05^{\circ} 03'$	<u>8.946295</u>

To T. Half Diff. Segm.	$08^{\circ} 55'$	<u>18.977906</u>
------------------------	------------------	------------------

Add Half Base	$31^{\circ} 10'$	<u>9.196275</u>
---------------	------------------	-----------------

Sum is the greater Base $40^{\circ} 05' = M^{\circ}$

Diff. is the lesser Base, $12^{\circ} 15' = P^{\circ}$

3. As Radius	$90^{\circ} 00'$	10.
Is to T. M°	$40^{\circ} 05'$	9.925096
So is T. C. $M^{\circ} P$	$52^{\circ} 06'$	<u>9.891247</u>

To S. C. $\perp$ P M° $49^{\circ} 05'$ 9.816343
Equal to the Side AC

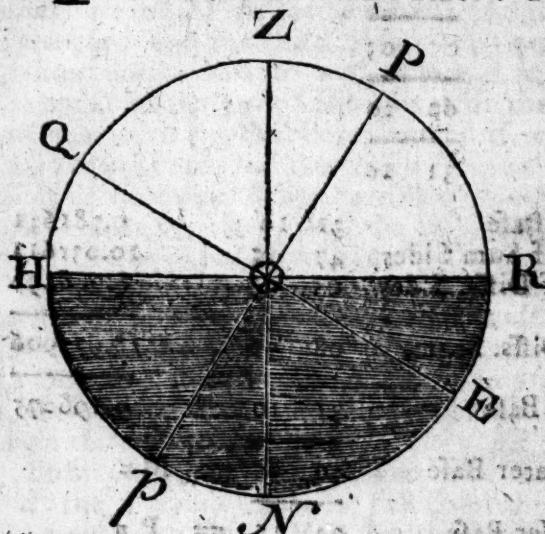
4. As Radius =	$90^{\circ} 00'$	10.
Is to T. P° =	$22^{\circ} 15'$	9.611841
& is T. C. $P^{\circ} P$ =	$41^{\circ} 59'$	<u>10.045817</u>

To S. C. $\perp$ M $P^{\circ} P$ = $62^{\circ} 58'$ 9.657658
Equal to the Side BC.

But we shall shew a readier Way hereafter to solve these two last Cases, (having the three Sides or the three Angles given) at one Operation to find an Angle, by an Axiom, or General Law.

PROBLEMS in Astronomy; relating to the
Sun and Fixed Stars.

1. **T**HE Elevation of the Pole above the Horizon, is the Latitude of the Place: For making Z the



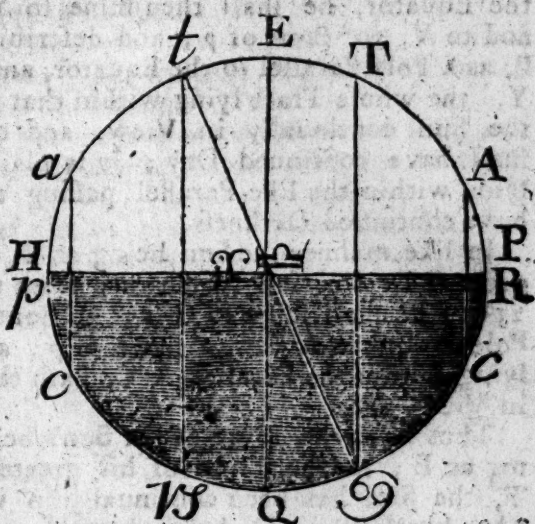
Place, whose Horizon is HR, Q E the Equator, and Pp the Poles, QZ is the Latitude, PR the Elevation of the Pole; and it is plain, $ZR = 90^\circ = PQ$ and $ZR - PZ = PQ - PZ$ *i. e.* $PR = QZ$. Q. E. D.

When the Sun is on the Equator (*i. e.* at either Point, where the Ecliptick cuts the Equator) the Days and Nights are equal all the World over. For 'tis plain $QO = QE$ that is, half the Equator, is in Darkness, the other in Light. Hence the 1° of Υ and $\varnothing$ are called, the Equinoctial Points.

Of the various Positions of the SPHERE.

2. A Right Sphere is when the Equator is at Right Angles with the Horizon.

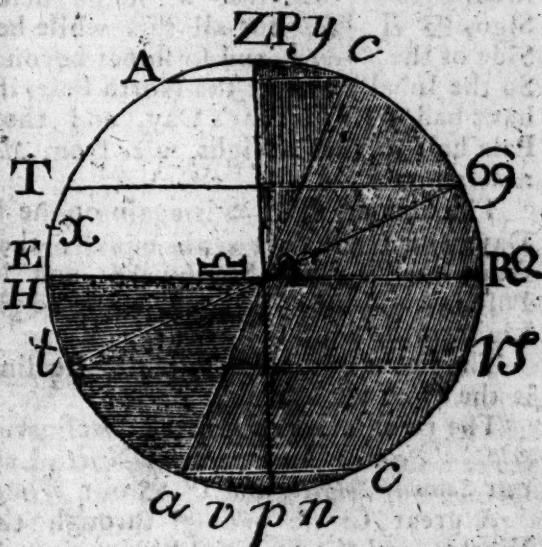
This Position of the Sphere they have that live at the Equator. The Poles of the World coincide with the North and South Points of their Horizon, and their Days and Nights are equal all the Year over: For the Equator and all its Parallels are bisected equally by the Horizon.



3. A Parallel Sphere is when the Equator E Q coincides with the Horizon H R, and consequently all its Parallels are Parallel to the Horizon.

This Position of the Sphere is proper to those that live at the Poles, and so the Poles of the World coincide with their Zenith and Nadir.

Seeing the Sun can at once illuminate but half the Globe, supposing him on the Equator at E, he then shines precisely to both Poles, and the Equator E Q and all its Parallels, T S, & W S, &c. have just one half in Light, and the other half in Darkness; that is, the Days and Nights are every where equal.



Supposing now the Sun at X, viz. 10° North from the Equator, he shall then shine to Y, 10° beyond P, and to V, 10° short of p; and describing a Circle upon P, as a Pole Parallel to the Equator, and passing through Y, the whole Tract lying within that Circle shall have the Sun continually in View, and consequently they shall have continued Day; it is plain, that the Tract lying within the like Parallel passing through V, shall have continued Darknes.

In like manner the Sun being at T, the Point of his greatest Declination Northward from the Equator, viz. $23^{\circ} 30'$ he shall shine to C, $23^{\circ} 30'$ Southward from the Pole P, and to A $23^{\circ} 30'$ short of p; and all that Tract lying within the Parallel A C, hath the Sun constantly in View, &c.

Thus from the Time of the Sun's being on the Equator at E; to the Time of his greatest Declination at T, the Sun has been continually in view of P, and p has been still in Darknes, that is for the space of three Months, answering to three Signs $\Upsilon \Theta \Pi$; for the first Degree of Θ is the Point of the Sun's greatest Declination Northward.

The Pole P has the Sun still in view during the following three Months, while he describes the three Signs $\Theta \Lambda \Upsilon$; for all this while he is on the North Side of the Equator, and so shines beyond its North Pole. So the Inhabitants of the North Pole, if there were any, have had half a Year's Day, and those of the South Pole half a Year's Night, viz. from March 10th, to September 13th.

The Sun at 1° of π is again on the Equator, and the Days and Nights are again equal; thence he declines from the Equator Southward, and leaves the North Pole in continued Darknes, while he gives half a Year's Light to the South.

The Point of the Sun's greatest Declination Southward is the 1° of ν .

The two Points of greatest Declination are called the Solstitial Points; and we have North Latitude, $1^{\circ} \Theta$ is our Summer-Solstice, and $1^{\circ} \nu$ our Winter-Solstice.

A great Circle passing through the Poles of the World, and the Solstitial Points, is called the Solstitial Colure.

A great Circle drawn thro' the Poles of the World, and the Equinoctial Points, is called the Equinoctial Colure.

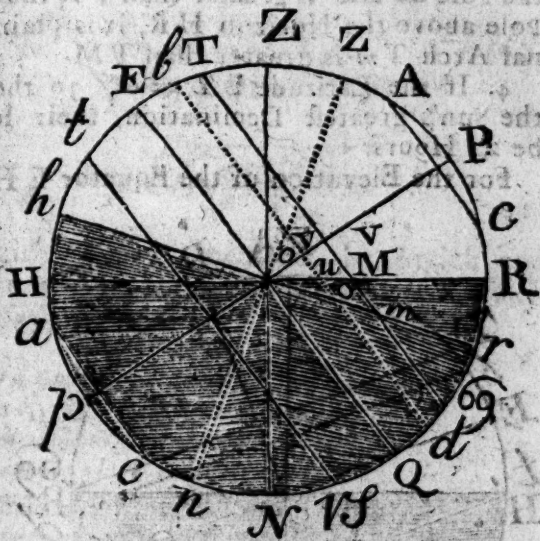
A Ctr.

A Circle drawn thro' $1^{\circ} \odot$ and Parallel to the Equator, is called the *Tropick of Cancer*; for the Sun at that Point begins to return to the Equator.

A Parallel to the Equator drawn thro' $1^{\circ} \vee S$, is called the *Tropick of Capricorn*; for the Sun at that Point again begins to return to the Equator.

4. An Oblique Sphere is when the Equator $E Q$, and all its Parallels make Oblique Angles with the Horizon $H R$ or $h r$.

This Position of the Sphere, they have that live betwixt the Equator; and the Poles (either in a North Latitude, as we our selves, or in a South Latitude) the Pole nearest to the Zenith is always elevated above the Horizon, and the other deprest below it; the Elevation or Depression of the Poles is always equal to the Latitude of the Place. Hence,



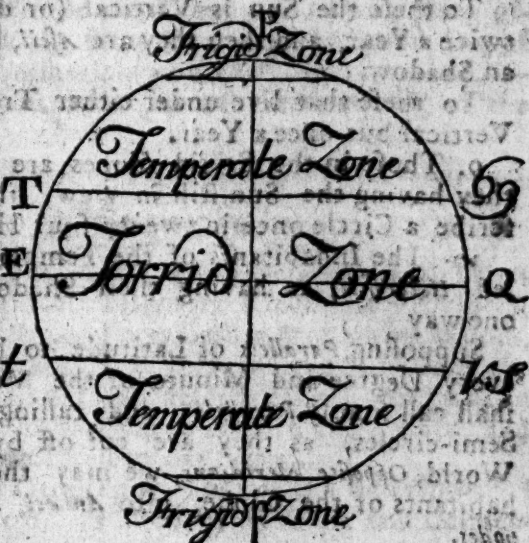
The Distance between the poles of the World, and those of the Horizon (*i. e.* the Zenith and Nadir) is equal to the Co-Latitude of the Place; likewise the elevation of the Equator, above the Horizon on one side, and its depression below it on the other side, equal to the Co-Latitude.

1. Let the Sun be on the Equator at E , whatever the Horizon be, whether $H R$ or $h r$, still the Semi-diurnal Arch $E O = 90^{\circ} = Q Q$ the Semi-nocturnal Arch; that is, the Sun being in the Equator, the Days and Nights are equal all the World over. Make now the Horizon $H R$.

2. Supposing the Sun at b , then is the Semi-diurnal Arch $b u$ greater than $b y$, but $b y$ and $E O$ are similar Arches, then the Day when the Sun is at b , is longer than when at E . Again, the Sun being at T the Solsti-

The two Tropicks, with the polar Circles, divide the Globe

into five Zones. 5. That lying betwixt the two Tropicks, is call'd the Torrid (or Burning) Zone, as being expos'd to the direct Stroke of the Sun's Rays, and consequently to the most violent Heats.



The two Frigid Zones lie between the Arctick and Antartick Circles, and their respective Poles: Of which Ovid thus speaks, *Metam.* 1.

*Utque dua dextra Cælum, totidemque sinistra
Parte servant Zone, quinta est ardentior, illis:
Sic unus inclusum numero distinxit eodem
Cura Dei, totidemque plaga tellure premuntur:
Quarum quæ medea est, non est habitabilis æstu:
Nix regit alba duas: totidem inter utramque locavit,
Temperiemque dedit mistâ cum Frigore flammâ.*

6. The two Zones lying within the Polars, are called the Frigid (or Frozen) Zones: the Stroke of the Sun's Rays being there very Oblique, and consequently faint and weak.

7. The two Zones lying between the Tropicks and Polars, are called Temperate Zones, as partaking equally of the Heats of the Torrid, and Colds of the Frigid Zones.

The Inhabitants of the several Zones take their Names from the various Projections of their Shadows, when the Sun is on their Meridians (or at Noon.)

8. The Inhabitants of the Torrid Zone are called *Amphiscii*, as having their Shadows projected, sometimes North, sometimes South, according as the Sun is on the South or North Side of their Zenith.

To these the Sun is Vertical (or directly over-head) twice a Year, and then they are *Ascii*, having no Meridian Shadow.

To these that live under either Tropick, the Sun is Vertical but once a Year.

9. Those in the Frigid Zones are called *Periscii*, for they having the Sun still in view, their Shadows describe a Circle once in twenty four Hours.

10. The Inhabitants of the Temperate Zones are called *Heteroscii*, as having their Shadows projected only one way

Supposing *Parallels* of Latitude to be drawn through every Degree and Minute of the Meridian, which we shall call simply *Parallels*; and calling the two opposite Semi-circles, as they are cut off by the Axis of the World, *Opposite Meridians*, we may then divide the Inhabitants of the Globe, into *Antæci*, *Periæci*, and *Antipodes*.

Antæci have the same Meridian, but opposite *Parallels*; and consequently they have the same Days and Nights, but opposite Seasons, and opposite *Poles* elevated.

12. *Periæci* have the same *Parallel*, but opposite Meridians; they have the same Seasons, and the same *Pole* elevated; but opposite Days and Nights.

13. *Antipodes* have opposite *Parallels* and opposite Meridians; they have opposite Seasons, opposite Days and Nights, and opposite *Poles* elevated.

14. *Climates* are little Zones terminated by two *Parallels* of Latitude, so taken as the longest Day at the greater Latitude is half an hour more than the longest Day at the lesser.

The longest day at the *Equator* is twelve Hours, and at either *Pole* it is twentyfour Hours; the Number of *Climates* betwixt the *Equator* and either *Poles* is twenty four, and their Number betwixt the two *Poles*, 48.

To find by a Map where the *Antæci*, *Periæci*, and *Antipodes* of any given Place are.

1. Note the Latitude of any Place given, and find on the same Meridian, the Point having the same Degree of Latitude, but on the contrary Side of the *Equator*; which said Point will be the *Antæci* of the place given.

2. To find the *Periæci*, Note the Longitude of the given place, and count 180° difference of Longitude in the same *Parallel*, which Point will be the *Periæci* of the place given.

3. Note

3. Note the Latitude and Longitude of any place given, and then seek the same Degree of Latitude, but of the contrary denomination, and 180° difference in Longitude, it gives you the Point of the *Antipodes*, of the Place given.

Of Geography.

Defn. **G**eography is an Art Mathematical, which shews how the Situations of Kingdoms, Provinces, Cities, Towns, Villages, Ports, Castles, Mountains, Woods, Hayens, Rivers, Creeks, &c. being on the Surface of the Terrestrial Globe, may be describ'd, and design'd, in Commensuration Analogical to Nature and Verity, and most aptly be represented to our View.

Ptolemy saith of Geography. That it is a description of all the known Earth, imitated by Writing and Delination, with all other things belonging thereunto. *Vide Gordon's Geograph. Grammar, &c. Concerning which Ovid saith, Metam. Lib. 2.*

*Terra, viros, urbesque gerit, frugesque, ferasque,
Fluminaque; hæc super est Cæli fulgentis imago*

In English thus:

The Earth Men, Towns, Fruits, Beasts and Rivers bears,
And over these are plac'd the Heavenly Spheres.

Of Geographical Definitions.

Because the Right and Left parts of the Heaven are frequently mention'd, we shall here observe, that Philosophers and Geographers called the East the Right, and the West the Left parts; the former, because the heavenly Bodies apparently mov'd from East to West; the latter, because in observing the *Polar* Altitude, the East was on their Right Hand.

2. Astronomers reckon'd West the Right, and East the Left parts: For looking Southward, in observing the Motions of the Stars, they had the West on their Right, and East on their Left Hand.

3. Augurs, looking towards the Rising-Sun, reckon'd South the Right, and North the Left parts.

4. Poets looking towards the Setting-Sun, reckoned North the Right, and South the Left parts; as appears from *Ovid, Metam. Lib. 2.*



OF PRACTICAL ASTRONOMY.

When the Sun is on the Equator, (i. e. at either Point where the Ecliptick cuts the Equator) the Days and Nights are equal all the World: For 'tis plain $Q\odot = \odot E$, that is, Half the Equator is in Darkness, the other in Light. Hence the first Degree of γ and π are called the *Equinoctial Points*.

Spherical Trigonometry Applied in Problems of Astronomy.

PROB. I. *The Day of the Month given, to find the Sun's Place in the Ecliptick.*

Keepe in Mind the Days on which the Sun enters the several Signs, which are these;

γ	δ	π	ζ	α	μ
Mar. 10.	Ap. 9.	May. 10.	June 11.	July 12.	Aug. 12.
π	μ	α	ζ	δ	γ
Sp. 12.	Oct. 12.	Nov. 12.	Dec. 11.	Jan. 9.	Feb. 8.

RULE. If at the Day given, the Sun be in the Sign proper to the given Month, subtract the Day the Sun enter'd that Sign, from the Day given, and multiply the Remainder into $59^{\circ} 8''$, $19'''$, $43'''$, $47'''$, $21'''$, $29'''$, the Sun's Motion in the Ecliptick in one Day, (but $59^{\circ} 8''$ may suffice for common use;) so have you the Degrees and Minutes of that Month's Sign the Sun is in.

Examp. 1. I demand the Sun's Place on the 25th. of August? $25 - 12 = 13 + 59^{\circ} 8'' = 12^{\circ} 58' 44''$ of μ . See the Operation.

19

$$\begin{array}{r}
 59' : 8'' \\
 \hline
 5^\circ : 54' : 48'' \\
 \hline
 2 \\
 \hline
 11 : 59 : 36 \\
 + 9 : 08 \\
 \hline
 \end{array}$$

Multiply by the Ratio of 12 Days i. e. 6 and 2, and add to the Product 59' 8" for the odd Days.

☉'s Place 12 : 59 : 44 of ♊

If the Sun be in a Sign different from the proper Sign of the Month; then from the Number of Days contained in the preceding Month, subtract the Day of the Sun's entering into that Month's Sign; to the Remainder add the Day of the Month given; that Sum multiplid into 59' 8" exhibits the Degree the Sun is in.

Examp. 2. August the 6th, the Sun is in ♊ $24^\circ 38' 21''$ (for July 31 Days — 12 Days in August the Sun enters ♊) = 19; which added to the given Day of the Month 6 = 25 Days. See the Operation.

$$\begin{array}{r}
 21 - 12 + 6 = 25 \text{ Days. } 59' : 8'' \\
 \hline
 25 \\
 \hline
 25 : 53 : 04 \\
 \hline
 23 : 39 : 12 \\
 + 9 : 08 \\
 \hline
 \end{array}$$

Multiply by the Ratio of 24, viz. 8 and 2, and to the Product add 59' 08" for the odd Day.

The Sun's Place 24 : 38 : 20 of ♊

Lemma 2. The Sun's Place given to find the nearest Equinoctial Point.

RULE. Divided the Heliptick into 4 Quadrants or Quarters thus;

1. ♈. ♉. ♊. ♋. } While the Sun is in the first and
2. ♌. ♍. ♎. ♏. } last Quadrants, ♈ is the nearest
3. ♐. ♑. ♒. ♓. } Equinoctial Point; and when in
4. ♈. ♉. ♊. ♋. } the middle two, ♎.

In the first Quadrant, the Sun's Longitude is the Distance from the nearest Equinoctial Point.

In

In the $\left\{ \begin{array}{l} 2. \text{ } 2r. \text{ } 180^\circ - \text{Sun's Longitude} \\ 3. \text{ } 2r. \text{ } \text{Sun's Longitude} - 180^\circ \\ 4. \text{ } 2r. \text{ } 360^\circ - \text{Sun's Longitude} \end{array} \right\}$ is the Distance.

PROB. 2. The Sun's greatest, and least Meridian Altitude (taken at Noon on the longest and shortest Days of the Year, at London) given, to find the Sun's greatest Declination, and the Latitude of the Place?

Examp. Suppose that with a Quadrant, Forestaff, or other Instrument You had Observ'd on

$\left. \begin{array}{l} \text{June} \\ \text{Decem.} \end{array} \right\} \text{the 11th. } \left\{ \begin{array}{l} \text{Sun Merid. Alt.} \\ \text{to be} \end{array} \right\} \left\{ \begin{array}{l} 61^\circ : 58' \\ 14 : 58 \end{array} \right.$

Their Diff. is the Dist. of the Tropicks $521 : 11$
 $47 : 00$

The Half of which is the $\odot$'s greatest Declin $= 23 : 30$

Merid. $\left\{ \begin{array}{l} 61^\circ : 58' \end{array} \right\}$ taken from $\left\{ \begin{array}{l} 28^\circ : 02' \end{array} \right\}$
Alt. $\left\{ \begin{array}{l} 14 : 58 \end{array} \right\}$ 90° gives $\left\{ \begin{array}{l} 75^\circ : 02' \end{array} \right\}$ $\odot$'s Zenith

Distance on $\left\{ \begin{array}{l} \text{June} \\ \text{December} \end{array} \right\}$ 11th.

To Sun's Zenith Distance $S^\circ 28^\circ : 02'$
Add Sun's Declination $N^\circ 23 : 30$ June 11th.

Sum is the Lat. of London $= 51 : 32$ North

From Sun's Zenith Distance $75^\circ : 02$ S. December
Take Sun's Declination $23 : 30$ S. 11th.

Rems the Lat. of London $= 51 : 32$ N $^\circ$.

PROB. 3. Given the Sun's Place and greatest Declination, to find his present Declination, and Right Ascension?

Examp. The Sun in $27^\circ : 30'$ of Taurus, and his greatest Declination $23 : 30$ N $^\circ$. what is his present Declination and right Ascension?

2. Lay off 90° off the Chords, from V to M and N , to cut the other Diameter or Equinoctial AEQ in p (which is called, the Pole of the Oblique Circle, or Meridian NBS).

4. Lay a Ruler over P and T , to cut the *Primitive* in d ; so shall Ed ($= BT$) be equal to the Chord of the *Sun's* present Declination, in this Case $19^\circ 39' N^\circ$.

5. The Right Ascension VB (and all Semidiameters from the Center V) is measured by a Line of Half Tangents; so that you'll find $VB = \text{Half Tangent } 55^\circ 13'$, the *Sun's* Right Ascension from V .

Calculation Logarithmetical, or Arithmetical.

1. $As R. S. \odot \text{ Long.} : : S. \odot \text{ Dec.} .. S. \odot \text{ pref. Decl.}$
 $— S. 90^\circ — S. 57^\circ 30' — S. 19^\circ 39' \text{ North};$
 because the *Sun* is in a Northern Sign.

2. $As R. S. C. \odot \text{ grt. Decl.} : : T. \odot \text{ Longitude} .. T. \odot \text{ Rt. Ascen.}$
 $— S. 90^\circ — S. 66^\circ 30' — T. 57^\circ 30' — T. 55^\circ 13'$
 from V , the *Sun* being then in the first Quadrant of the Ecliptick.

Instrumental, by Gunter's Scale.

The Extnr from $R. S. 90^\circ$, to $S. 57^\circ 30' \odot$'s Longit. on the Sines,
 Reaches from $T. \odot$'s gr. Decl. $23^\circ 30'$, to $\odot$'s pr. Decl. $19^\circ 39'$ on the Sines.

The Extent from $R. S. 90^\circ$, to $S. \odot$'s gr. Decl. $66^\circ 30'$ on the Sines,
 Reaches from $T. \odot$'s Long. $57^\circ \frac{1}{2}$, to $T. \odot$'s Rt. Ascen. $75^\circ 13'$ on the Tangents.

N. B. In the First Quadrant VB is the Rt. Ascen.

In $\left\{ \begin{array}{l} 2 \\ 3 \\ 4 \end{array} \right\} \text{ Qr. } \left\{ \begin{array}{l} 180 - VB \\ 180 + VB \\ 360 - VB \end{array} \right\}$ is the Right Ascension.

By this Problem may the *Tables* of the *Sun's* Declination and Right Ascension be calculated, the *Sun's* Place in the Ecliptick given, his greatest Declination, according to the exact Observations of the ingenious Mr. *Flamsteed*, is $23^\circ 29'$.

By the Reverse of this Problem may the two next succeeding Problems be solved, *viz.*

PROB. IV.

Given } Sun's present Declination $19^{\circ} 39'$ North.
 } Sun's Right Ascension $55^{\circ} 13'$ from Υ .

Required the Sun's place, and greatest Declination?

This Triangle is made by Case 6. of Right-angled Spherical Triangles, and measured and solved by Case 3. and 2. of the same,

Projection. (See the last Scheme.)

1. Make ΥB = Half Tangent Sun's Right Ascension $55^{\circ} 13'$ (the Circle being first described and quarter'd, as before directed.)

2. Through the Points NBS, describe the Meridian, or Oblique Circle NBS, and find the Pole thereof, as directed in the preceding Problem.

3. Make AE = Chord of $19^{\circ} 13'$, the Sun's Declination; and laying a Ruler over d and p , to cut NdS in T . Lastly, through T , and the Center Υ , draw the Ecliptick Line $ETVC$, and it is done.

N.B. The required Sides and Angles of the following Problems are measured by Prob. 7, and 8, of Stereographic Problems.

Calculation Logarithmetical.

1. As R :: S. C. $\odot$'s pr. Dec. :: S. C. $\odot$'s Rt. Asc.
 S. C. $\odot$'s Place or Long. As $S 90^{\circ} 39'$:: $S 70 : 21 :: S : 34 : 47$:: $S 57 : 30$, from Υ . Or $27 : 30$ in $\oslash$ is the Sun's place required.

2. As T . $\odot$'s pr. Decl. :: R S. $\odot$'s Rt. Ascen. :: T . C. of $\odot$'s grt. Decl. $T. 19 : 39$ — $S. 90^{\circ}$ — $S 55 : 13$, $T T 66 : 30$; which subtracted from 90 , gives $23 : 30$ which was required.

Operation Instrumental.

The Extent from R $S. 90$ to S. C. $\odot$'s pr. Dec. $70 : 21$ on the Sines,

Will reach from S. C. $\odot$'s Rt. Ascen. $34 : 47$ to S. C. $\odot$'s Long. $32 : 30$ on the Sines.

The Extent from S. $\odot$'s Rt. Ascen. $55 : 13$ to R $S. 90$ on the Sines,

Will reach from $T. 19 : 39$ to $T. 23 : 30$ on the Tangents.

P R O B. V.

Given } present } Decl. $\begin{cases} 19^{\circ} : 39' \\ 23 : 30 \end{cases}$ } North.
 Sun's } greatest }

Reqd. } Longitude } Constructed and measur'd by Case
 Rt. Ascen. } 3. of Rt. *A. Sp. Triangles.*

Projected thus : (See the last Scheme.)

1. The primitive Circle drawn and quarter'd. make $\overline{AE} = \text{Chord } 23^{\circ} 30'$; and draw the Ecliptick $\overline{EVC}$.

2. Make Nd and $Ne = \text{Com. Decl. viz. } 70^{\circ} 21'$ from the Chords; then with the Tangent of $70^{\circ} 21'$, and one Foot in Ed and e describe two Arches to intersect each other above N . and with the same distance, one Foot in the said Intersection, describe the Parallel of the Sun's (diurnal Course or) Declination dTe , to cut the Ecliptick Line $\overline{VE}$ in T .

3. Through the three points NTS , describe a Circle, or Meridian $NTBS$, as it is done. Then it is solved by Case 3, and 5, of *Right-angled Spherick Triangles.*

Logarithmically.

1. As $S. \odot$'s greatest Decl. .. $S. \odot$'s pref. Decl.

So is $R. S. \odot$'s Long. from V $57^{\circ} 30'$.

2. As $R. T. \odot$'s pref. Decl. : : $T.C. \odot$'s greatest Declin. .. $S. \odot$'s Rt. Ascension from V $55^{\circ} 13'$.

Instrumental, by Gunter's Scale.

The Extent from $S. \odot$'s greatest Decl. $23^{\circ} 30'$, to $S. \odot$'s pref. Decl. $19^{\circ} 39'$ on the Sines,

Will reach from $R. S. 90^{\circ}$, to $S. \odot$'s Long. $57^{\circ} 30'$ on the Sines.

The Extent From $T.C. \odot$'s pref. Decl. $70^{\circ} 21'$, to $T.C. \odot$'s greatest Decl. $66^{\circ} 30'$ on the Tangents,

Will Reach from $R. S. 90^{\circ}$, to $S. \odot$'s Rt. Ascen. $55^{\circ} 13'$ on the Sines.

For the Instrumental Operations of the following Problems, see the Cases of *Right-angled Spherick Triangles* referred to in the said Problems.

P R O B. VI.

Sun's Longitude $57^{\circ} 30'$, and Right Ascension $55^{\circ} 13'$ given,

To find his } present } Declination.
 } greatest }

This

This is made or projected thus: 1. Make γB = the Half T. Sun's Right Ascension $55^{\circ} 13'$; and through N, B, and S, describe the Meridian NTBS. 2. With the Half Tangent of the Sun's Longitude $57^{\circ} 30'$, and one Foot in γ , with the other cross the Meridian last drawn in T. Lastiy, through T and γ draw the Ecliptick Line $E\gamma C$ and it's done. The Sun's present Declination is measured as in *Prob. 3d*, of *Astron.* foregoing, and his greatest Declination = ΔE is measured on the Line of Chords. Calculation for the Sun's present Declination per *Case 7th*, of Right-angled Spherick Triangles, viz.

1. *As* S. C. Sun's Rt. Ascen. .. $R ::$ S.C. Sun's Long.
.. S. C. Sun's pref. Declin.
2. *As* $R ::$ T. $\odot$'s Long. $::$ T. C. $\odot$'s Rt. Ascen. ..
S. C. Sun's gr. Declin. as per *Case 12*, of *Right-angled Spherick Triangles*.

PROB. 7. If the Sun's Longitude and present Declination, were given; to find the Sun's greatest Declination, and Right Ascension. The Triangle is made, measured and solved by *Case 2d*. of *R. A. S. T.*

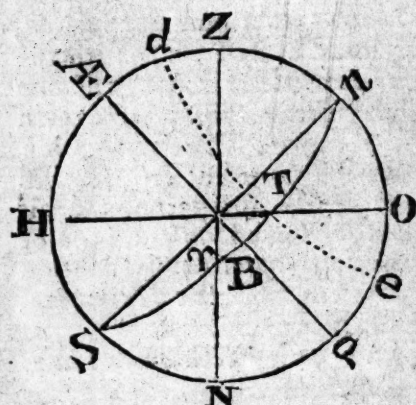
The Proportions are,

1. *As* S. $\odot$'s Long. .. $R ::$ S $\odot$'s pref. Decl. ..
S. $\odot$'s great. Declin.
2. *As* S. C. $\odot$'s pref. Decl. .. $R ::$ S.C. $\odot$'s Long;
.. S. C. $\odot$'s Right Ascension.

PROB. 8. The Latitude of a Place, ($43^{\circ} : 20' N^{\circ}$) and the Sun's Declination ($19^{\circ} : 39' N^{\circ}$) given; to find the Sun's Amplitude, and Ascensional Difference, which is the Time of the Sun's Rising and Setting from the Hour of 6.

PROJECTION.

1. The Circle being described (with a Chord of 60°) and quartered, make Z. the Zenith, N the Nadir, and H γ O the Horizon; H the South, O the North, and γ the East or West Points thereof.
2. Make On and Z Δ , each equal to the Chord of the Lat. $43^{\circ} : 20'$, and draw the Axis $n \gamma S$, and the Equinoctial $\Delta \gamma Q$.



3. Because the Declination given is North, describe a Parallel of the Sun's Declination, $70^{\circ} : 21'$ from the North Pole as was directed in *Prob. 5.* foregoing, and it will cut the Horizon HO in T .

4. Through the Points T . and S . describe the Meridian Circle TS . which

cuts the Equinoctial EQ in B ; so shall VT be the Sun's Amplitude from the East and West; and VB the Ascensional Difference, and both of them are measured from the Center V by a Line of Half Tangents: But to determine their just Quantities by Calculation, Logarithmetical, See the 3d. and 5th. Cases of *Right-angled Spherick Triangles*, viz. in the *Right-angled Spherical Triangle* V Right-angled at B , is given the $\angle TVB$, the Comp. of the Lat. and BT the Sun's Declination, to find VT , the Sun's Amplitude, and VB , the Ascensional Difference, *i. e.*

Logarithmetical.

As *S. C. Lat.* S . Sun's Decl. $\therefore R$. .. *S. Sun's Amp.*
— North. $S. 46 : 40$ — $S. 19^{\circ} : 39'$ — $S. 90^{\circ} — 27^{\circ} : 32'$.

That is, $E 27^{\circ} : 32' N.$ and $W. 27^{\circ} 32' N.$ which turn'd into Points of the Compass, gives nearest,

$E. N. E.$ Half $N.$ } at Sun { Rising
And $W. N. W.$ Half $N.$ } at Sun { Setting.

By this Problem may the Tables of Amplitude be Calculated.

Note, If the Sun's Declination be Northerly, the Amplitude is to the Northward of the East or West at Sun-rising, or Setting. If the Declination be Southerly, to the Southward.

As R . .. T . Lat. $\therefore T$ Sun's Declin. .. S . Sun's
Ascen. Difference, — $T. 45^{\circ} — T. 43^{\circ} : 20'$ — $T. 19 : 39'$
— $S. .. 19^{\circ} : 41'$.

PROB.

P R O B. IX.

The Ascensional Difference given or found, as in the last Problem, to find the time of the Sun's rising or setting, with the Length of the Day or Night.

Rule 1. If the Latitude and Declination be both North, or both South, the Ascensional Difference added to 6 Hours, gives the time of Sun-setting; and subtracted therefrom, Sun-rising. And contrary, if one be North, and the other South, the Ascensional Difference added to 6 Hours, gives the time of Sun-rising; and subtracted therefrom, the time of Sun-setting.

Example. Let the Latitude be $43^{\circ} 20' N^{\circ}$. and the Sun's Declination $19^{\circ} 19' N$. and the Ascensional Difference (as in the last Problem) $19^{\circ} 41'$ to find as above.

O P E R A T I O N.

5) $19^{\circ} 41' 00''$ } Divide the Ratio of 15, viz.
 $3^{\circ} 36' 12''$ } 5, and 3.

Hours $1^{\circ} 18' 44''$ Sun's Ascens. Difference.
 Add $6^{\circ} 00' 00''$

Hours $7^{\circ} 18' 44''$ Time of Sun-setting.

Setting sub. $4^{\circ} 41' 16''$ Time of Sun-rising.

☉ set doub. $= 14^{\circ} 37' 28''$ The Length of the Day.

☉ ris. doub. $9^{\circ} 22' 32''$ The Length of the Night.

Or, in general, you may turn Degrees of Motion into Time, by the Table hereunder.

Allow $\left\{ \begin{matrix} 15^{\circ} 00' \\ 1^{\circ} 00' \\ 0^{\circ} 15' \end{matrix} \right\}$ for $\left\{ \begin{matrix} 1^h 0' \\ 0^h 4' \\ 0^h 1' \end{matrix} \right\}$ of Time

Rule 2. Because the Hour of Sun-setting shews the time elapsed between Noon, and the End of the Day, and which is therefore equal to half the Length of the Day: Therefore if the Hour of Sun-setting be doubled, it gives the Length of the Day, or Number of Hours the Sun is visible above the Horizon.

148 *The Doctrine of the Sphere : Or,*

Again, because the Hour of Sun-rising shews the time elapsed between the time of the Sun-rising and Midnight; and which is therefore equal to half the Length of the Night; if the Hour of Sun-rising be doubled, it gives the Length of the Night.

By this Problem may the Tables of the Sun-rising or setting, and Length of the Day or Night be calculated.

Also in the foregoing Scheme or Triangle ∇BT may the two next succeeding Problems be solved.

P R O B. X.

The Sun's true Amplitude by Observation ($27^{\circ} 32'$ North) and the Sun's Declination ($19^{\circ} 39'$ North) also given; to find the Latitude of the Place?

Projection. See the last Scheme.

1. The Circle described and quartered as formerly; make also N the North Pole, S the South Pole, and at Right Angles thereto ÆQ for the Æquinoctial .

2. Describe the Parallel of the Sun's Declination $d e$, $70^{\circ} 21'$ distant from the North Pole N, as was directed in Prob. 5. or 8.

3. With the Half Tangent of the Sun's Amplitude $27^{\circ} 32'$, and one Foot in the Center ∇ , cross the Parallel of Declination in T. And lastly, through ∇ and T, draw the Horizontal Line HO; which crosses at Right Angles with the prime Vertical Line ZN; so shall Z be the Zenith, N the Nadir, and ∇BT the Triangle. For the Solution of which, see Case 2, of *Right-angled Spherical Triangles*.

As $S. \odot \text{ Amp.} \dots R :: S. \odot \text{ Decl.} \dots S. \text{ of the Comp. Lat.}$

— $S. 27^{\circ} 32' = S. 90^{\circ} S. - 19^{\circ} 39' = S. 46^{\circ} 40'$, whose Comp. to 90° is $43^{\circ} 20'$, the Lat. North required.

P R O B. XI.

The Latitude of a Place, and the Sun's Declination given; to find the Oblique Ascension or Descension.

Rule 1. Find the Right Ascension by Prob. 3. and the Ascensional Difference by Prob. 8.

2. When the Latitude and Declination are both North, or both South, the Ascensional Difference added to, and subtracted from the Right Ascension, the first is the Oblique Descension and the latter is the Oblique Ascension.

5. When

3. When the Latitude and Declination is one North, and the other South, add and subtract as before; the first is the Oblique Ascension, and the latter is the Oblique Descension.

Note, When you cannot subtract, add 360 Degrees to the Right Ascension, and then subtract; also when you have added, if the Sum exceed 360 Degrees, subtract 360 Degrees therefrom, the Remainder is the Thing required.

EXAMPLE.

Latit. $43^{\circ} : 20' \text{ N}$ and Sun's Declination $23^{\circ} : 30'.$

Let the Right Ascension be $55^{\circ} : 13'$ by *Prob. 3*
and the Ascensional Difference $19 : 41$ by *Prob. 8*.

Right Ascension	55 : 13
-----------------	---------

Ascensional Difference 19 : 41

Added, is the Obl. Descension = 74 : 54

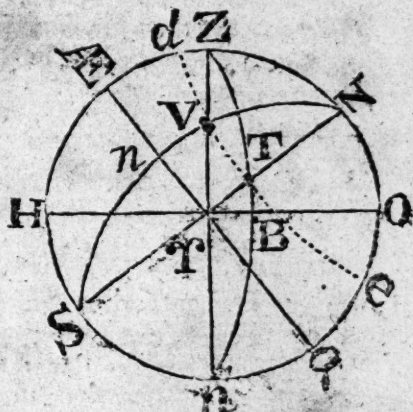
Subt. is the Obl. Ascension = 35 : 34

PROB. XII. The Latit. of a Place (suppose $35^{\circ} : 32'$ N^o.) and the Sun's Declination (admit $23^{\circ} : 30'$ N^o.) given, to find the Sun's Altitude (Height) and Azimuth at the Hour of 6.

Projection.

1. Having described the Circle and quartered (which to avoid needless Repetitions let be signified in the following Problems by P. C. Q.)

2. Set off the Lat. and draw the Parallel of Declination, as directed in *Prob. 8th*, and 5th, the Parallel of Declination cuts the *Axis V T S* in *T*.



3. Draw through **Z** the Zenith-Point, the Point **T**, and the Nadir **n**, an Azimuth Circle **Z T B n**, it will form the Right-angled Spherical Triangle **T B T**. Right-angled at **B**, the Axis **N T S** is the 6-a-Clock.

PART II. K 3 Hours

Hour-Circle, and B T the Sun's Altitude, and γ B on the Horizon, the Sun's Azimuth from East or West, at the Hour of 6 Each of which may be measured on the Scale of Half Tangents as in *Prob. 3*.

Solution. For the Sun's Altitude, it will be by the Rule of Opposite Sides and Angles, *viz.*

As $R \dots S. \odot$ Declin. $;$ $S. \text{ Lat.} \dots S. \odot$ Alt. at 6.

— $S. 60^{\circ} 30' - S. 23^{\circ} 30' - S. 35^{\circ} 32' - S. 13^{\circ} 24'.$

For the Sun's Azimuth, by Case 10. of *Right-angled Spherick Triangles.*

As T.C. $\odot$ Decl. $\dots R ; ; S.C. \text{ Lat.} \dots T. \odot$ Azim. at 6.

— $T. 66^{\circ} 30' - S. 90^{\circ} - 54^{\circ} 28' - T. 19^{\circ} 28'$ East Northerly, if in the Morning; or West Northerly, if in the Evening: Wherefore the Sun's Azimuth $19^{\circ} 28'$ subtracted from 90° , gives the Azimuth from the North $70^{\circ} 32'$ East, or near E.N.E. $\frac{1}{4}$ E. in the Morning.

Again, the Sun's Azimuth $19^{\circ} 28'$ found above, added to 90° , gives his Azimuth from the South $109^{\circ} 22'$ towards the East, if it be in the Morning; but if Observation had been in the Evening, his Azimuth would have been W.N.W. $\frac{1}{4}$ W.

Note, If the Declination be contrary to the Latitude; that is, if one be North, and the other South, then instead of the Altitude before found, it gives the Depression, and shews how many degrees the Sun is below the Horizon at the Hour of 6.

P R O B. XIII.

The Latitude (suppose $35^{\circ} 32'$ North,) and Sun's Altitude at the Hour of 6, ($13^{\circ} 24'$) given; to find his Declination and Azimuth.

Projection. (See the last Scheme.)

P.C.Q. And the Latitude set off, as before, draw (by *Prob. 3*.) a Parallel of Altitude al , at $76^{\circ} 36'$ from the Zenith Z, to cut the Axis N γ S in T. And lastly, through Z, T, and N, describe the Azimuth-Circle ZBN, and it is done. The Declination may be found, as in the 3d Case of *Right-angled Spherick Triangles*, or by the Rule of Opposite Sides and Angles; that is,

As S. Lat. $\dots S. \odot$ Alt. $;$ $R \dots S. \odot$ Declin. required.

1 — $S. 35^{\circ} 32' - S. 13^{\circ} 24' - 90^{\circ} S. 23^{\circ} 30' \gamma T.$

2 For the Sun's Azimuth, by Case 5, of *Right-angled Spherick Triangles.*

As R .. T.C. Lat. : : T. $\odot$ Alt. .. S. $\odot$ Azim. at 6 h.
 — T. 45° .. T. $54^\circ 28'$ — T. $13^\circ 24'$ — S. $19^\circ 28'$ γB required.

P R O B. XIV.

Given the Latitude ($35^\circ 32'$) and Sun's Azimuth at the Hour of 6 ($19^\circ 28'$ East Northerly) to find his Declination and Altitude.

Projection. (Vide last Scheme.)

P.C.Q. And the last Latitude set off as before; make γB equal to the Sun's Azimuth $19^\circ 28'$ (upon the Horizon) from the Half Tangents, and describe the Azim. Circle $Z B n$, as directed in the last Problem, and it is done.

For Solution, see Case 9, and 4, of Right-angled Spherical Triangles.

1. As T. $\odot$ Azim. .. R : : S.C. Lat. T.C. $\odot$ Declin.
 — T. $19^\circ 21'$ — S. 90° — S. $54^\circ 28'$ — T. $66^\circ 30'$ N.T. — 90° = γT .
2. As T.C. Lat. .. R : : S. $\odot$ Azim. T. $\odot$ Altit.
 — T. $54^\circ 28'$ — S. 90° — S. $19^\circ 28'$ — T. $13^\circ 24'$ B.T.

P R O B. XV.

The Sun's Declination ($23^\circ 30'$ N.) and his Altitude ($13^\circ 24'$ A. M. or in the Forenoon) given; to find the Latitude and Sun's Azimuth at the Hour of 6.

Projection. (See the last Scheme.) P.C.Q. Draw the Parallel of Altitude, a, l , at $76^\circ 36'$ from the Zenith Z , as directed in Prob. 3, and 13.

2. With the Half Tangent of $23^\circ 30'$ and one Foot in the Center γ ; with the other cross the Parallel of Altitude, a, l , in T .

3. Draw through γT the Axis $N T \gamma S$ and at Right Angles thereto the $\text{Æquinoctial Line } \text{Æ} \gamma Q$. Lastly, through the Points Z, T and n describe the Azimuth Circle $Z T B n$, and it will form the Triangle $\gamma B T$.

The Proportions for Solution are as in Case 2, and 7, of Right-angled Spherick Triangles, viz.

1. As s. $\odot$ Decl. R : : s. $\odot$ Altit. .. s. Latit. the $\angle B \gamma T$.
 — s. .. $23^\circ 30'$ s. 90° — s. $13^\circ 24'$ — s. $35^\circ 32'$ reqd.
2. As s.c. $\odot$ Alt. .. R : : s.c. $\odot$ Decl. .. s.c. $\odot$ Azim. γB .
 — s. — $76^\circ 36'$ — s. 90° — s. $66^\circ 30'$ — s. $70^\circ 32'$ reqd.

P R O B. XVI.

The Sun's Declination ($23^{\circ} 30'$ North) and his Azimuth at the Hour of 6 ($19^{\circ} 28'$ East Northerly) given; to find the Latitude, and Sun's Altitude.

Projection. (See the last Scheme.)

P.C.Q. 1. With HVO , the Horizon, and ZVn , the prime Vertical. 2. Make VB on the Horizon equal to the Half Tangent of $19^{\circ} 28'$, the Sun's Azimuth; and through the Points ZB and n , draw the Azimuth-Circle $ZTBn$; and then in the Triangle VBT , the Angle TVB is the Latitude, and BT the Sun's Altitude; which are solved and measured, as in Case 13, and 2, of *Right-angled Spherick Triangles*.

1. As $R :: T \odot$ Azim. $:: T.C. \odot$ Declin. $.. S.C.$ Lat.
 $— S. 90^{\circ} — T — 19^{\circ} 28' — T. 66^{\circ} 30' — S. 54^{\circ} 28'.$

2. As $S.C. \odot$ Azim. $.. R :: S.C. \odot$ Decl. $.. S.C. \odot$ Alt.
 $— 70^{\circ} 32' — S. 90^{\circ} — S. 66^{\circ} 30' — 76^{\circ} 36'.$

P R O B. XVII.

Given the Sun's Azimuth at 6. ($19^{\circ} 28'$ W. Northerly) and his Altitude ($13^{\circ} 24'$ P.M.) to find the Latitude and his Declination?

Projection. (See the last Scheme.)

1. P.C.Q. Make VB on the Horizon equal to the Half Tangent $19^{\circ} 28'$, the Sun's Azimuth, and draw through the Points ZBn the Azimuth-Circle, as before. Then,

2. Describe a Parallel of Altitude *a. l.* $76^{\circ} 36'$ distant from the Zenith Z , as in Prob. 13. foregoing; which will cut the Azimuth-Circle in T .

Lastly, through V and T draw the Axis, or 6-a-clock Hour-Line $NTVS$, and at Right Angles to it the Equinoctial $AEVQ$, and it is done. VBT is the Triangle, in which the Angle TVB is the Lat. and VT the Sun's Declination; therefore by the 6th and 8th Cases of *Right-angled Spherick Triangles*,

1. As $T. \odot$ Alt. $.. R :: S. \odot$ Azim. $.. T.C.$ Lat. $\angle T$
 $VB.$

$— T. 13^{\circ} 24' — S. 90^{\circ} — S. 19^{\circ} 28' — T. 54^{\circ} 28'$

2. As $R :: S.C. \odot$ Alt. $.. S.C. \odot$ Azim. $.. S.C. \odot$ Decl.
 $— S. — 90^{\circ} — S. 76^{\circ} 36' — S. 70^{\circ} 32' — S. 66^{\circ} 30'.$

PROB.

P R O B. XVIII.

The Latitude of the Place, suppose ($35^{\circ}32'$ North) and the Sun's Declination (as before, $23^{\circ}30'$ North) given; to find his Altitude, and the time when he is due East and West.

Note, When the Sun is either East or West, he is then upon the prime Vertical $Z\gamma n$.

Projection, (as in the last Scheme, Prob. 12.)

P.C.Q. Set off the Latitude, and draw the Axis $N\gamma S$; also the $\text{\AA}$ equinoctial $\text{\AA}$, and the Parallel of Declination, to cut the prime Vertical, or first Azimuth in V ; then through N, V , and S , draw the Meridian NVS , cutting the $\text{\AA}$ equinoctial at Right Angles in the Point n ; so that in the Right-angled Triangle γnV will be given the Angle $V\gamma n$, the Latitude, and the Sun's Declination nV , to find his Height γV , and Hour from 6. γr (if East, after 6. in the Forenoon; but when West before 6. Afternoon;) each of which may be measured on the Half Tangents.

Proportions for Solution, as by Case 5, and 3, of *Right-angled Spherick Triangles*.

1. As $S. Lat. \dots S. \odot Decl. : : R. \dots S. \odot Alt. E. or W.$

— $S. 35^{\circ}32'—S. 23^{\circ}30'—S. 90^{\circ}—S. 43^{\circ}19'$.

2. As $R. \dots T. C. Lat. : : T. \odot Decl. \dots S. Hour from 6.$

— $S. 90^{\circ}—T. 54^{\circ}48'—T. 23^{\circ}30'—S. 37^{\circ}25'$; which reduced into time, is 2h. 29m. 40 sec. and shews how long after 6 in the Morning it is, e're the Sun be due East, and how long before 6 in the Evening the Sun comes to the West.

Wherefore to 90 Deg. or

Add the Time above

Sum is the time $\odot$'s due East, Morn.

Again,

From 90 Degrees, or

Subtract the said Time

Remains the time $\odot$ is due West in
the Afternoon.

h. ' "

6 00 00

2 29 40

8 29 40

6 00 00

2 29 40

3 30 10

Note, If the Latitude and Declination given, had been one North; and the other South, the Sun would be under the Horizon, and the Question of no Use.

PROB.

P R O B. XIX.

The Sun East, the Latitude. ($25^{\circ} : 32^{\circ} \text{ N}^{\circ}$.) and Sun's Altitude ($43^{\circ} : 19'$ A. M.) given; to find his Declination and the Hour of the Day.

Projection. (In the aforefaid Scheme.)

1. P. P. Q. and Lat. Set off also the Equinoctial drawn.

2. Make $\gamma V =$ the $\odot$'s Altitude from the Half Tangents, and through the Points N, V, and S, describe the Meridian $N \propto S$, which cuts the Equinoctial in the Point n at Right Angles, and constitutes the Triangle $\gamma n V$, as before; in which you have given the Angle $V \gamma n$ the Latitude, and γV the Altitude, to find $n V$ the $\odot$'s Declination, and γn the Hour from 6.

Proportions, by Case 1, and 10, of *Right-angled Spherick Triangles*.

1. As $R \propto S$. $\odot$ Alt. : : S. Lat. .. S. $\odot$ Decl.

—S. 90° deg —S. 43° deg. 19 m. —S. 35° deg. 32 m. —S. 23° deg. 30 m. North.

2. As T.C. $\odot$ Alt. .. R : : S.C. Lat. .. T. Hour from 6, when E.

—T. 46° deg. 41 m. —S. 90° deg. —S. 54° deg. 28 m. —T. 37° deg. 30 m. — or $2 \text{ h. } 29' 40''$.

P R O B. XX.

The Sun West, the Latitude (35° deg. 32 m. North,) and Hour of the day ($2 \text{ h. } 29' 40''$ P. M. or 37° deg. 30 m. from 6.) given; to find his Altitude and Declination?

Projection (in the same Scheme.)

P.C.Q. Latitude set off, and Equinoctial drawn, as before, make $\gamma n =$ Half Tangent of 37° deg. 30 m. and through the Points N $\propto$ S describe the Meridian, as in the last Problem, $\gamma V = \odot$ Alt. and $n V$ the Declination, are found by Case 9, and 4, of *Right-angled Spherick Triangles*.

P R O P O R T I O N S.

1. As T. Hour .. R : : S.C. Lat. .. T. C. $\odot$ Altit. when W.

—T. 37° deg. 30 m. —S. 90° deg. —S. 54° deg. 28 m. —T. 46° deg. 41 m.

2. As T.C. Lat. $\therefore R$: S. Hour .. T. $\odot$ Decl. North.
 — T. 54 deg. 28 m. — S. 90 deg. — S. 37 deg. 30 m. —
 T. 23 deg. 30 m.

P R O B. XXI.

The $\odot$ East, his Declination (23 deg. 30 m. North) and Altitude (43 deg. 19 m. A. M.) given; to find the Latitude and Hour of the day.

Projection. (See the foregoing Scheme.)

P.C.Q. And noted with $N\gamma S$ the Axis, and $\mathcal{E}\gamma Q$ the Equinoctial.

2. Describe the Parallel of Declination (66 deg. 30 m.) distant from N the North Pole, as before taught.

3. With the Half Tangent of $\odot$'s Altitude = 43 deg. 19 m. and one Foot in γ , with the other cross the Parallel of Declination in V.

4. Through V, and the Center γ draw the prime Vertical $Z\gamma n$, and at Right Angles to it, the Horizon $H\gamma O$. Lastly, through the Points N, V, and S, describe the Meridian Circle NnS , as before; so shall γnV be the Triangle. $\gamma V = \odot$ Alt. and $nV = \odot$ Declin. given, to find the Angle $V\gamma n$, and Base γn , by Case 2, and 7, of Right-angled Spherick Triangles, i. e.

1. As S. $\odot$ Alt. $\therefore R$: S. $\odot$ Decl. .. S. Lat. North.
 — S. 43 deg. 19 m. — S. 90 deg. — S. 23 deg. 30 m. — S. 35 deg. 32 m.

2. As S.C. $\odot$ Decl. $\therefore R$: S.C. $\odot$ Alt. .. S.C. Hour from 6, A.M.

— S. 66 deg. 30 m. — S. 90 deg. — S. 46 deg. 41 m. — S. 52 deg. 30 m. Comp. is 37 deg. 30 m. or 2h. 29' 40".

P R O B. XXII.

The $\odot$ West, his Declination (23 deg. 30 m. N.) and Hour of the day (3h. 30' 20", or 2h. 29' 40" from 6 = 37 deg. 30 m.) given; to find the Latitude, and $\odot$'s Altitude?

Projection. P.C.Q. and noted as before, with the Axis $N\gamma S$, and Equinoctial $\mathcal{E}\gamma Q$, make $\gamma n =$ Half Tangent 37 deg. 30 m. and through the Points NnS draw the Meridian NnS , as before.

2. Describe from N the Parallel of Declination, as in the last Problem, which cuts the Meridian last drawn in V. Lastly, through V, and the Center γ , draw the prime Vertical $Z\gamma n$, and at Right Angles thereto, the Horizon $H\gamma O$.

In

156 *The Doctrine of the Sphere : Or,*

In the Triangle $\gamma n V$ is given γn Base, and $n V$ Perpend. to find the Angle $V \gamma n$ the Latitude, and γV the $\odot$'s Altitude, by Case 6, and 8, of *Right-angled Spherick Triangles*; that is,

1. *As* T. $\odot$ Decl. $.. R : : S.$ Hour $.. T.C.$ Lat. North.
 $-T. 23^{\circ} 30' - S. 90^{\circ} - S. 37^{\circ} 30' - T. 54^{\circ} 28'.$ Comp. is $35^{\circ} 32'.$
2. *As* $R .. S.C. \odot$ Decl. $.. S.C.$ Hour $.. S.C. \odot$ Altit. P.M.
 $S. 90^{\circ} - S. 66^{\circ} 30' - S. 52^{\circ} 30' - S. 46^{\circ} 41'.$ Comp. is $43^{\circ} 19'.$

P R O B. XXIII.

The Sun East, his Altitude (43 deg. 19 m. A. M.) and Hour of the Day, as before (2h. 29' 40" from 6h. = 37 deg. 30 m.) given; to find the Latitude, and $\odot$'s Declination?

Projection. P.C.Q. With the Axis $N \gamma S.$ and Equinoctial $\gamma E \gamma Q$ make $\gamma n = 37$ deg. 30 m. from the Half Tangents, and describe the Meridian $N n S.$

2. With the Half Tangent of 43 deg. 19 m. and one Foot in the Center γ , with the other cross the Meridian $N n S$ in $V.$ Lastly, through V , and γ the Center, draw the prime Vertical $Z V \gamma n$, and at Right Angles to it, the Horizon $H \gamma O$; then in the Triangle $\gamma n V$ is given the Hypotenuse $\gamma V =$ Altit. 43 deg. 19 m. and Base $\gamma n =$ Hour from 6; to find the Angle at the Base $V \gamma n =$ Latit. and Perpend. $n V = \odot$'s Declination, by Case 13, and 7, of *Right-angled Spherick Triangles.* Proportions are,

1. *As* $R .. T.$ Hour $.. T.C. \odot$ Alt. $.. S.C.$ Lat. North.
 $-S. 90$ deg. $T. 37$ deg. 30 m. $-T.C. 46$ deg. 41 m.
 $-S. 54$ deg. 28 m. Comp. is 35 deg. 32 m.
2. *As* $S.C.$ Hour $.. R : : S.C.$ Alt. $.. S.C. \odot$ Declin.
 $-S. 52$ deg. 30 m -90 deg. $-S. 46$ deg. 41 m.
 $-S. 66$ deg. 30 m. Comp. is 23 deg. 30 m.

Note, In the three last Problems the Latitude may be either North or South, and truly answer the Questions.

P R O B. XXIV.

The $\odot$ in the Equator, or Equinoctial, the Latitude of a Place, and the Altitude given; to find the Azimuth and Hour of the Day?

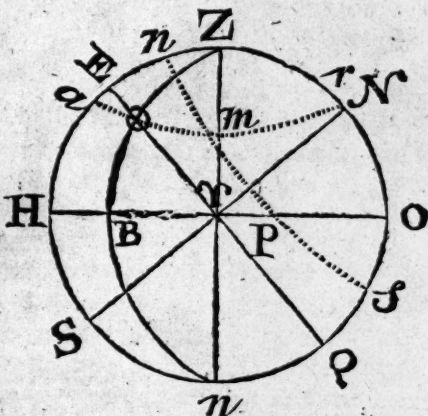
N.B. When the ☉ hath no declination, he is then said to be in the Equator, which is twice in a Year, viz. about the 9th of March, and the 12th of September.

Example. The $\begin{cases} \text{Latitude} & 35^{\circ} 32' \text{ North.} \\ \text{☉'s Altitude} & 43^{\circ} 19' \text{ A.M. or Morn.} \end{cases}$

Explanation of the Scheme.

In the Right-angled Spherical Triangle $\gamma B \odot$ is to be noted,

1. $\gamma B \odot$ the Right Angle.
2. The Angle $B \gamma \odot$ Comp. Latitude.
3. The Hypotenuse $\gamma \odot$ the Hour from 6.
4. The Perpendicular $B \odot$ the Sun's Altitude.
5. The Base γB the Sun's Azimuth from the East or West; but,
6. BH his Azimuth from the South in Nth.



Latitude; but from the North in South Latitude Easterly in the Forenoon, and Westerly in the Afternoon.

Projection Stereographical.

1. P.C.Q. Make Z the Zenith, n the Nadir, $H \gamma O$ the Horizon. 2. Set off the Lat. $ON = 35^{\circ} 32' m$; and at Right Angles thereto the Equinoctial $E \gamma Q$. 3. Describe a Parallel to the Horizon at $46^{\circ} 41' m$. (the Comp. Altitudes) distant from the Zenith Z (after the same manner as was directed to draw the Parallel of Declination in Prob. 3.) to cut the Equinoctial $E \gamma Q$ in $\odot$. Lastly, through the Points Z $\odot$ and n describe the Azimuth-Circle $Z \odot B n$, and 'tis done.

In the Right-angled Spherick Triangle $\gamma B \odot$ Right-angled at B, is given, the Angle $B \gamma \odot = \text{Comp. Lat. } 54^{\circ} 28' m$ and $\odot$'s Altitude $B \odot = 43^{\circ} 19' m$. to find the Base $\gamma B = \odot$ Azim. and Hypoth. $\gamma \odot = \text{Hour of the Day from 6-a-Clock}$, when the $\odot$ is in the Equinoctial; by Case 5, and 3, of Right-angled Spherick Tri-

158 *The Doctrine of the Sphere : Or,*

1. *As* $\mathcal{R} \dots T. C. \text{ Com. Lat.} :: T. \odot \text{ Altit.} \dots S. \text{ Hour}$
from 6.

— $S. 90 \text{ deg.} \dots T. 35 \text{ deg. } 32 \text{ m.} \dots T. \dots 43 \text{ deg. } 19 \text{ m.} \dots S. 42 \text{ deg. } 20 \text{ m.} = \gamma B.$

2. *As* $S. \text{ of Com. Latit.} \dots \mathcal{R} :: S. \odot \text{ Altit.} \dots S. \text{ Hour}$
from 6.

— $S. \dots 35 \text{ deg. } 32 \text{ m.} \dots 90 \text{ deg.} \dots S. 43 \text{ deg. } 19 \text{ m.} \dots S. 49 \text{ deg. } 44 \text{ m. Comp. } 40 \text{ deg. } 16 \text{ m. from Noon:}$
Or, $2\text{h } 41' 4''$ from Noon; that is, $9\text{h. } 18' 56''$ in the Morning.

In the abovesaid Triangle $\gamma B \odot$ we shall resolve five more Problems, *viz.*

P R O B. XXV:

The $\odot$ in the Equinox, his Azimuth from the East ($42 \text{ deg. } 20 \text{ m.}$) and Altitude ($43 \text{ deg. } 19 \text{ m. A.M.}$) given; to find the Latitude and hour of the Day?

Projection (as in the foregoing Scheme.)

1. P.C.Q. And noted with Z, π , and HO; also the Parallel of $43 \text{ deg. } 19 \text{ m.}$ Altitude drawn as in the last Problem. Then,

2. Make $\gamma B = 42 \text{ deg. } 20 \text{ m.}$ ($\odot$'s Azimuth) from the half Tangents, and through the Points Z, B, and π , describe the Azimuth-Circle $ZB\pi$, which cuts the Parallel of Altitude in $\odot$.

Lastly, through $\odot$ and γ the Center, draw the Equinoctial $E \odot \gamma Q$, and at Right Angles thereto the Axis $N \gamma S$, and it's done. Then in the Triangle $\gamma B \odot$ is given the Base $\gamma B = 42 \text{ deg. } 20 \text{ m.}$ and Perpendicular $B \odot = 43 \text{ deg. } 19 \text{ m.}$ given, to find the Angle at the Base $B \gamma \odot = \text{Comp. Lat.}$ and Hypothenufe $\gamma \odot$, the Hour from 6, by Case 6, and 8, of *Right-angled Spherick Triangles*, i. e.

1. *As* $T. \odot \text{ Alt.} \dots \mathcal{R} :: S. \odot \text{ Azim.} \dots T. C. \text{ Comp. Lat.}$
i. e. the Lat. North.

— $T. 43 \text{ deg. } 19 \text{ m.} \dots S. 90 \text{ deg.} \dots S. 42 \text{ deg. } 20 \text{ m.}$
 $T. 35 \text{ deg. } 32 \text{ m.}$

2. *As* $\mathcal{R} \dots S.C. \odot \text{ Alt.} :: S.C. \odot \text{ Azim.} \dots S.C. \text{ Hour}$
from 6.

— $S. 90 \text{ deg.} \dots S. 46 \text{ deg. } 41 \text{ m.} \dots S. 47 \text{ deg. } 40 \text{ m.} \dots S. 40 \text{ deg. } 16 \text{ m. or } 2\text{h. } 41' 4''$ from Noon; or $9\text{h. } 18' 56''$ in the Morning.

PROB.

PROB. XXVI.

The Sun in the Equator his Azimuth (42 deg. 20 m) and Hour of the Day (from 6 a-Clock A. M. 2h. 18' 56" or 49 deg. 44 m.) given, to find the Latitude and ☉'s Altitude.

Projection. P. C. Q. Make $\gamma B = 42 \text{ deg. } 20 \text{ m.}$ from the half Tangents and describe the Azimuth-Circle ZBn , as before.

1. With the Half Tangent of 49 deg. 44 m. one foot in the Center γ cross the Azimuth-Circle ZBn in ☉. Lastly, through ☉ and γ draw the Equinoctial $E\gamma Q$ and at Right Angles thereunto the Axis $N\gamma S$; and it's done.

In the Triangle $\gamma B\odot$ is given the Base, $\gamma B = 42 \text{ deg. } 21 \text{ m.}$ and Hypoth. $\gamma\odot = 49 \text{ deg. } 44 \text{ m.}$ to find the Angle at the Base $B\gamma\odot$; and Perp. $B\gamma$ by Case 15, and 7, of *Right-angled Spherick Triangles*.

1. $A\gamma R \therefore T\odot$ Azimuth $\therefore T.C.$ Hour $\therefore S.C.$ Comp. Lat.

— S. 90 deg, — T. 42 deg. 20 m. — T. 40 deg. 16 m. — S. 35 deg. 32 m. the Lat. N.

2. $A\gamma R S.C\odot$ Azimuth $\therefore R \therefore S.C.$ hour $\therefore S.C.\odot$ Altitude.

— S. 47 deg. 40 m. — S. 90 deg. — S. 40 deg. 16 m. — S. 46 deg. 41 m. Comp. = 43 deg. 19 m.

PROB. XXVII.

The Sun in the Equator, the Hour of the Day from 6, A. M. 3h : 18' : 56. or (49 : 44') and his Altitude (43 : 19') given; to find the Latitude and Sun's Azimuth.

Projection. (See the last Scheme.)

P.C.Q. And Parallel of Altitude drawn; 2. With the Half Tangents of the Hour from 6, 43 : 19' one Foot in the Center γ , with the other cut the said Parallel of Altitude in ☉, and draw the Equinoctial, and Axis and Azimuth Circle as directed in the last Problem; then in the Triangle $\gamma B\odot$ is given the Hypoth. $\gamma\odot = 49h : 44'$ and Perpendicular $BC = 43 : 19'$ to find the Angle at the Base $B\gamma\odot = \text{Comp. Lat.}$ and Base γB . by Case 2, and 7, of *Right-angled Spherick Triangle*.

1. $A\gamma$

158 *The Doctrine of the Sphere : Or,*

1. *As* $\mathbb{R} \dots T. C. \text{Com. Lat.} :: T. \odot \text{Alt.} \dots S. \text{Hour}$
from 6.

— $S. 90 \text{ deg.} \text{ — } T. 35 \text{ deg. } 32 \text{ m.} \text{ — } T. \text{ — } 43 \text{ deg. } 19 \text{ m.} \text{ — } S. 42 \text{ deg. } 20 \text{ m.} = \gamma B.$

2. *As* $S. \text{ of Com. Latit.} \dots \mathbb{R} :: S. \odot \text{Alt.} \dots S. \text{Hour}$
from 6.

— $S. \text{ — } 35 \text{ deg. } 32 \text{ m.} \text{ — } 90 \text{ deg.} \text{ — } S. 43 \text{ deg. } 19 \text{ m.} \text{ — } S. 49 \text{ deg. } 44 \text{ m. Comp. } 40 \text{ deg. } 16 \text{ m. from Noon:}$
Or, $2\text{h. } 41' 4''$ from Noon; that is, $9\text{h. } 18' 56''$ in the Morning.

In the abovesaid Triangle $\gamma B \odot$ we shall resolve five more Problems, *viz.*

P R O B. XXV:

The $\odot$ in the Equinox, his Azimuth from the East ($42 \text{ deg. } 20 \text{ m.}$) and Altitude ($43 \text{ deg. } 19 \text{ m. A.M.}$) given; to find the Latitude and hour of the Day?

Projection (as in the foregoing Scheme.)

1. P.C.Q. And noted with Z, π , and HO; also the Parallel of $43 \text{ deg. } 19 \text{ m.}$ Altitude drawn as in the last Problem. Then,

2. Make $\gamma B = 42 \text{ deg. } 20 \text{ m.}$ ($\odot$'s Azimuth) from the half Tangents, and through the Points Z, B, and π , describe the Azimuth-Circle $ZB\pi$, which cuts the Parallel of Altitude in $\odot$.

Lastly, through $\odot$ and γ the Center, draw the Equinoctial $E \odot \gamma Q$, and at Right Angles thereto the Axis $N \gamma S$, and it's done. Then in the Triangle $\gamma B \odot$ is given the Base $\gamma B = 42 \text{ deg. } 20 \text{ m.}$ and Perpendicular $B \odot = 43 \text{ deg. } 19 \text{ m.}$ given, to find the Angle at the Base $B \gamma \odot = \text{Comp. Lat.}$ and Hypothenufe $\gamma \odot$, the Hour from 6, by Case 6, and 8, of *Right-angled Spherick Triangles*, i. e.

1. *As* $T. \odot \text{Alt.} \dots \mathbb{R} :: S. \odot \text{Azim.} \dots T. C. \text{Comp. Lat.}$
i. e. the Lat. North.

— $T. 43 \text{ deg. } 19 \text{ m.} \text{ — } S. 90 \text{ deg.} \text{ — } S. 42 \text{ deg. } 20 \text{ m. } T. 35 \text{ deg. } 32 \text{ m.}$

2. *As* $\mathbb{R} \dots S. C. \odot \text{Alt.} :: S. C. \odot \text{Azim.} \dots S. C. \text{Hour}$
from 6.

— $S. 90 \text{ deg.} \text{ — } S. 46 \text{ deg. } 41 \text{ m.} \text{ — } S. 47 \text{ deg. } 40 \text{ m.} \text{ — } S. 40 \text{ deg. } 16 \text{ m. or } 2\text{h. } 41' 4'' \text{ from Noon; or } 9\text{h. } 18' 56'' \text{ in the Morning.}$

PROB.

PROB. XXVI.

The Sun in the Equator his Azimuth (42 deg. 20 m.) and Hour of the Day (from 6 a-Clock A. M. 2h. 18' 56' or 49 deg. 44 m.) given, to find the Latitude and ☉'s Altitude.

Projection. P. C. Q. Make $\gamma B = 42 \text{ deg. } 20 \text{ m.}$ from the half Tangents and describe the Azimuth-Circle $Z B n$, as before.

2. With the Half Tangent of 49 deg. 44 m. one foot in the Center γ cross the Azimuth-Circle $Z B n$ in ☉. Lastly, through ☉ and γ draw the Equinoctial $E \gamma Q$ and at Right Angles thereunto the Axis $N \gamma S$; and it's done.

In the Triangle $\gamma B \odot$ is given the Base, $\gamma B = 42 \text{ deg. } 21 \text{ m.}$ and Hypoth. $\gamma \odot = 49 \text{ deg. } 44 \text{ m.}$ to find the Angle at the Base $B \gamma \odot$; and Perp. $B \gamma$ by Case 15, and 7, of *Right-angled Spherick Triangles*.

1. As $R \propto \therefore T \odot$ Azimuth $\therefore T. C.$ Hour $\propto S. C.$ Comp. Lat.

— S. 90 deg. — T. 42 deg. 20 m. — T. 40 deg. 16 m. — S. 35 deg. 32 m. the Lat. N.

2. As $R \propto S. C \odot$ Azimuth $\propto R \therefore S. C.$ hour $\propto S. C. \odot$ Altitude.

— S. 47 deg. 40 m. — S. 90 deg. — S. 40 deg. 16 m. — S. 46 deg. 41 m. Comp. $= 43 \text{ deg. } 19 \text{ m.}$

PROB. XXVII.

The Sun in the Equator, the Hour of the Day from 6, A. M. 3h : 18' : 56. or (49 : 44') and his Altitude (43 : 19') given; to find the Latitude and Sun's Azimuth.

Projection. (See the last Scheme.)

P.C.Q. And Parallel of Altitude drawn; 2. With the Half Tangents of the Hour from 6, 43 : 19' one Foot in the Center γ , with the other cut the said Parallel of Altitude in ☉, and draw the Equinoctial, and Axis and Azimuth Circle as directed in the last Problem; then in the Triangle $\gamma B \odot$ is given the Hypoth. $\gamma \odot = 49 \text{ h} : 44'$ and Perpendicular $B \odot = 43 : 19'$ to find the Angle at the Base $B \gamma \odot = \text{Comp. Lat.}$ and Base γB . by Case 2, and 7, of *Right-angled Spherick Triangle*.

1. As

1. *As* S. Hour .. $\mathcal{R} :: S \odot$ Altitude .. S. of the Comp. Lat.

— S. $49^{\circ} : 44'$ — S 90° — S. $54 : 28'$ Comp. is $35 : 32'$ the Lat.

2. *As* S. C. $\odot$ Alt. .. $\mathcal{R} :: S. C.$ Hour .. S. C. $\odot$ Azim.

— S. $46^{\circ} : 41'$ — S 90° — S. $40^{\circ} : 16'$ — S. $47^{\circ} : 40'$ Comp. is $42^{\circ} : 20'$, the Sun's Azimuth Required.

P R O B. XXVIII.

The Sun in the Equinox, the Lat. ($35 : 32'$ N.) and Angle of the Sun's Position *i. e.* the Angle the Azim. Circle makes with the Equinoctial ($53^{\circ} : 07'$) given, to find the Azimuth and Hour of the Day, from 6 a-Clock, A. M.

Projection. See the last Scheme.

P. C. Q. The Latitude Set off and also the Equinoctial drawn, as before.

2. Describe a Parallel Circle, 53 deg. 07 m. distant from N. the North Pole (after the same manner as was directed in Prob. 5, foregoing, to describe the Parallel of the $\odot$'s Declination) this Parallel will cut the Horizon in the Point P.

3. Measure the Distance γ P. on the half Tangents, and subtract it from 90 deg. and set off the half Tangent of the Remainder from γ to B. Lastly, through the Points Z B γ , describe the Azimuth-Circle Z $\odot$ B $\odot$, as before taught, and 'twill form the Triangle γ B $\odot$, in which you have given the Angles B γ $\odot$ 54 deg. 28 m. and γ $\odot$ B 64 deg. 33 m. to find the Base γ B = $\odot$ Azimuth γ $\odot$ Hypoth = Hour from 6 a-Clock; By Case 16. and 15, of *Right-angled Spherick Triangles*, *i. e.*

1. *As* S. Comp. Lat. .. $\mathcal{R} :: S. C. \angle \odot$ Posit. .. S. C. $\odot$ Azimuth.

— S. 54 deg. 28 m. — S. 90 deg. — S. 36 deg. 53 m. — S. 47 deg. 40 m. Comp. = 42 deg. 20 m.

2. *As* $\mathcal{R} :: T. C.$ Comp. Lat. : : T. C. $\angle \odot$ Posit. .. S. C. Hour from 6.

— S. 90° T. 35 deg. $32'$ m. — T. 36 deg. 53 m. — S. 40 deg. 16 m. or $2h. : 41' : 04''$ from Noon. *viz.* $9h. 18' : 56''$ A. M. or Morning.

P R O B. XXIX.

PROB. XXIX.

Given the same things as in the last Prob. to find the ☉'s Azimuth, and Altitude the ☉ being in the Equinoctial.

Projection. The same as the last Prob. the Operation, by Case 16.

1. *As* S. Comp. Lat. .. $\hat{R}$:: S. C. $\angle$ ☉ Posit. .. S. C. ☉'s Azimuth.

— S. 54 deg. 28 m. — S. 90 deg. — S. 36 deg. 53 m. — S. 47 deg. 40 m. Comp. = 42 deg. 20 m.

2. *As* S. Comp. Lat. ... $\hat{R}$:: S. $\angle$ ☉ Posit. ... S. C. ☉ Altitude.

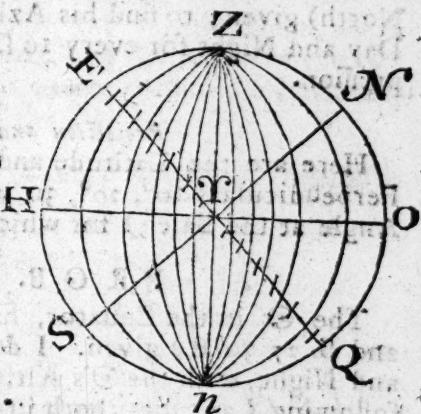
— S. 54 deg. 28 m. — S. 90 deg. — S. 53 deg. 07 m. — S. 46 deg. 41 m. Comp. = 43 deg. 19 m.

P R O B. XXX.

The ☉ in the Equator, the Lat. ($35^{\circ} 32'$ North) given; to find the ☉'s Azimuth and Altitude, and the Quantity of his Depression every hour of the Day and Night.

Projection. P. C. Q. Set off the Latitude, and draw the Equinoctial, as before directed.

2. Divide the Equinoctial into Hours; that is, set off the half Tangent of 15° , 30° , 45° , 60° , 75° , both ways, from the Center γ . Lastly, through each of these Points, and the Zenith Z , and Nadir N , draw the several Azimuth-Circles, and 'tis done; so shall you have Five



Triangles above the *Horizon*, and Five below it, to solve, in each of which is given the Hypothenufe, or hour from 6. 15° , 30° , &c. and the Angle at the Base $\angle \gamma H$, or $\angle \gamma Q$ (the Comp. Latitude) common to all the *Triangles*, to find the Bases (the ☉'s Azimuth) and Perpendiculars, (the ☉'s Altitude above, or Depression below the *Horizon* HO , by Case 10, and 1, of *Right-angled Spherick Triangles*.

P R O B. XXXI.

The ☉ in the Equator, the Latitude (45 00' North) given, to find the ☉'s Altitude, and the Quantity he is depress'd, the hour of the Day, and the Night at every 15 degrees of his Azimuth from the East or West.

Projection. (See the last Scheme.)

1. P.C.Q. Lat. set off, Equinoctial drawn as before. Then 2. Set off the half *Tangent* of 15°, 30°, 45°, 60°, 75°, from the Center γ both ways upon the *Horizon*, and draw the Azimuth-Circles from Z through each of the said Divisions; and then in the several *Triangles* are given the Bases 15°, 30°, 45, &c. from γ , and the Angle at the Base, the Comp. Latitude, to find the Perpendiculars, i.e. the several Altitudes or Depressions, also the Hypothenuses, which are the several hours of the Day and Night, by Case 4, and 9, of *Right-angled Spherick Triangles*.

P R O B. XXXII.

The ☉ in the Equator, the Latitude (15 deg. 21 m. North) given; to find his Azimuth, and the hour of the Day and Night for every 10 Degr. of Altitude and Depression.

Projection and Solution.

Here are the Latitude and Altitude given, viz. (the Perpendiculars 10°, 20°, 30, 40°, 50°, 60°, 70°, 80°, and Angle at the Base;) for which see Prob. 24, foregoing.

P R O B. XXXIII.

The ☉ in the Equator, his Azimuth (N. 27 30' W. and S. 27 30' E.) given. I demand the hour of the Day and Night, with the ☉'s Altitude and Depression, in the following Latitudes, both in the North and South Hemispheres, viz. 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°.

Here are the several Angles at the Base, and the Base also given, to find the several Hypothenuses and Perpendiculars. For Projection and Solution of which see Case 9, and 4, of *Right-angled Spherick Triangles*.

P R O B. XXXIV.

The ☉ in the Equator, his Azimuth North 40 degr. East, and South 40 degr. West. I demand the several Parallels of Latitude at every 10 Degrees of Altitude and

and Depression above and below the Horizon; with the hour of the Day and Night in each Latitude?

Here are given, the Bases, and several Perpendiculars (or Altitudes) $10^{\circ}, 20^{\circ}, 30^{\circ}, 40^{\circ}, 50^{\circ}, 60^{\circ}, 70^{\circ}, 80^{\circ}$, to find the several Angles at the Base, and the Hypothenuses; for which see Case 6, and 8, of *Right-angled Spherick Triangles*, or Prob. 17, foregoing.

PROB. XXXV.

The Sun in the Equator, his Azimuth, N. 70° deg. E. A.M. and S. 70° deg. W. P. M. given. I demand the several Latitudes, and $\odot$'s Altitude, the Quantity he is depressed under the Horizon, for every 56 Minutes of time (or 14 Degrees) from the Equator to the Poles of the World?

Here are given, the Bases and the several Hypothenuses (viz. $14^{\circ}, 28^{\circ}, 42^{\circ}, 56^{\circ}, 70^{\circ}$, and 84° .) to find the Angles at the Base, and the several Perpendiculars, which may be solved by Case 2, and 7, of *Right-angled Spherick Triangles*.

PROB. XXXVI.

The Sun in the Equator, in the North Hemisphere the Sun's Azimuth 45 Degrees, and the Angle of the $\odot$'s Position 58 degrees, 30 min. given, to find the Latitude, $\odot$'s Altitude, and hour of the Day?

Projection. P. C. Q. 1. Make γB = Sun's Azimuth 45 deg. from the half Tangents; and through the Points Z, B, and n , describe the Azimuth Circle ZBn.

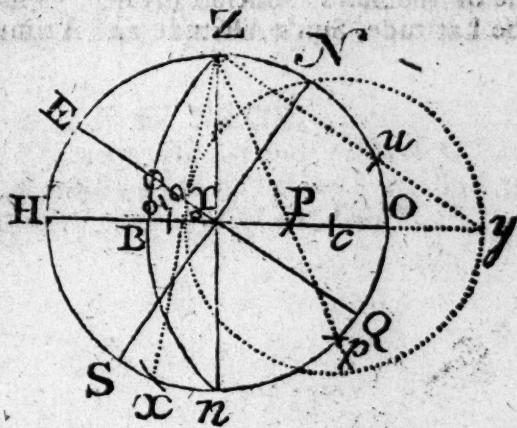
2. Make γP = Comp. γB the $\odot$'s Azim. (in this Case 45 deg.) from the half Tangents; so shall P be the Pole of the Oblique Circle ZBn.

3. Lay a Ruler over Z and P, to cut the primitive Circle in P.

PART II.

L 2

4. Set



4. Set off the Chord of $58^{\circ} 30'$ from p both ways from p to x and u on the primitive Circle, and draw the prick'd Lines $Z\gamma x$, and $Zu\gamma$, to cut the Horizontal Line HO in γ and y . Then on half the Distance γy , as at C as a Center describe the Circle $\gamma N \gamma P$, which cuts the Primitive in N , the Point representing the North Pole.

Lastly, Draw the Axis $N\gamma S$, and at Right Angles to it, the Equinoctial EQ , and it will form the Triangle $\gamma B \odot$, in which is given the Base γB 45° , and Angle at the Perpendicular $B \odot \gamma$ $58^{\circ} 30'$ to find the Angle at the Base $B\gamma \odot = \text{Comp. Lat.}$ the Perpendicular $B \odot$ the Sun's Altitude, and Hypothennuse $\gamma \odot$ the Hour of the Day from 6, by Case 12, 5, and 3, of Right angled Spherick Triangles.

P R O B. XXXVII.

The Sun in the Equinoctial, the Latitude ($50^{\circ} 10'$ North) and Sun's Angle of Position $65^{\circ} 8'$ demand his Azimuth, Altitude, and the hour of the Day?

For Projection, see Prob. 28, foregoing, and its Solution is by Case 16, and 15, of Right-angled Spherick Triangles.

P R O B. XXXVIII.

The Sun in the Equator, the hour of the Day $9^h \frac{1}{2}$ A.M. (or $3^h \frac{1}{2} = 52^{\circ} 30'$ from 6,) and the Angle of the Sun's Position ($61^{\circ} 58'$) given; to find the Latitude, Sun's Altitude and Azimuth.



PROB.

PROJECTION.

1. P.C.Q. Make $\gamma \odot$ = half Tangent, hour from 6, 53 deg. 30 m. and γp = to its Complement (or Pole) $37^{\circ} 30'$ from the said half Tangents; and thro' $N \text{ } p \text{ } S$ draw an Oblique (or Meridian) Circle $N \text{ } p \text{ } S$.

2. Since the Angle of the Position must be distant from the Pole of the Right Circle (or Equinoctial,) that is, from N or S $61^{\circ} 38'$, therefore set off the Chord of $61^{\circ} 58'$ from N to m , and from E draw the prick'd Line mE , to cut the Oblique (or Meridian) Circle, $S \text{ } p \text{ } N$ in P : So shall P be the Pole of the Required Oblique (or Azimuth) Circle.

3. Through p and γ the Center, draw the Horizontal Line $H \gamma O$, and at Right Angles thereto the prime Vertical $Z \gamma n$.

Lastly, Through the Points Z , $\odot$, and n , describe the Azimuth-Circle $Z B n$; so shall $\gamma B \odot$ be the required Triangle, in which are given, the Hypothenuse $\gamma \odot$ = $52^{\circ} 30'$, and Angle at the Perpendicular $\gamma \odot B$ = $61^{\circ} 58'$; to find the Angle $\odot \gamma B$ = Comp. Latitude, Perpendicular $B \odot$ = Sun's Altitude, and the Base γB = Sun's Azimuth, by Case 14, 9, and 1, of Right-angled Spherical Triangle.



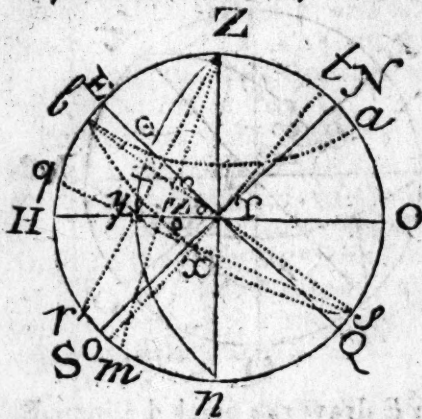
P R O B. XXXIX.

The Sun in the Equator, his Altitude (389) A.M. and the Sun's Angle of Position (58 30') given; to find the Latitude, his Azimuth, and hour of the Day?

PROJECTION.

P.C.Q. 1, Draw the Parallel of Altitude, as before taught.

2, From the Point, where the Parallel of Altitude



cuts the primitive Circle, as at *b*, thro' the Center *V* draw a Diameter *bs*, and cross it at Right Angles with the Diameter *os*.

3. Make oq equal to $\angle$ Position $58^{\circ} 30'$, and draw the Line qs , cutting the former Diameter os in x .

5, Make nm equal to hr , and draw the Line Zxm .

6, Thro' the three points Z, z, n draw the Oblique or Azimuth-Circle Zzn , to cut the parallel of Altitude in $\odot$.

7. Thro' Υ the Center, and $\odot$, draw the Equinoctial $E\odot\Upsilon\odot$; and at Right Angles thereto, the Axis $S\Upsilon N$; so shall $\Upsilon Z\odot$ be the Triangle to be solved; in which is given the Perpend. $= \odot$'s Alt. 38° , and Angle at the said Perpendicular $=$ Angle posit. $58^\circ 30'$, to find the Angle at the Base $\odot\Upsilon R =$ Comp. Lat. the Base $\Upsilon Z =$ Sun's Azimuth, and the Hypoth. $\Upsilon\odot =$ hour from 6, by Case 11, 4, and 9, of *Right-angled Spherick Triangles*.

PROB. XL.

The Sun's Declination ($15^{\circ} 10'$ North) and the Angle of the Sun's Meridian with the Horizon ($54^{\circ} 13'$) at his Rising, given; to find the Latitude, Amplitude, and the time of the Sun's rising and setting, with the Length of the Day and Night.

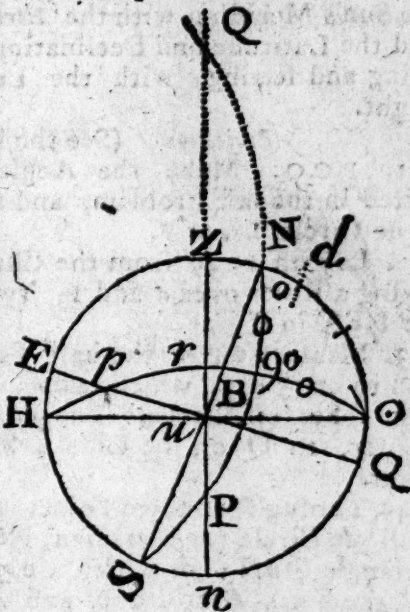
FRO-

PROJECTION

P.C.Q. 1. Make $\odot N$ = Comp. of the Declination $74^{\circ} 50'$ from the Chords, draw the Axis from N thro' the Center to S, and cross it with the Equinoctial EQ at Right Angles thereto.

2. At $\odot$ the Intersection of the Meridian, or primitive Circle; make an Angle of $4^{\circ} 13'$, by drawing the Oblique (Horizon $\odot r H$, and find the Pole thereof (by Prob. 2, Case 1, and Prob. 1, of Stereographick Problems) as P.

3. Thro' the Points N, P, and S, describe the Oblique Circle (or Meridian) NBPS. So shall P be the Pole thereof,



and it cuts the former Oblique Circle HB $\odot$ in B at Right Angles. Then in the Right-angled Spherick Triangle $\odot BN$ is given the Hypotenuse $\odot N$, the Comp. Sun's Declination $74^{\circ} 50'$, and the Angle at the Base B $\odot$ N (the Angle of the Sun's Meridian with the Horizon;) to find the Perpendicular BN, the Latitude, the Base $\odot B$ = Amplitude and Angle at the Perpendicular BN $\odot$ = hour from Noon, by Case 1, 10, and 14, of Right-angled Spherick Triangles.

Lat. is $51^{\circ} 30'$, Sun's Amplitude from the South $65^{\circ} 8'$ and the hour from 6, $19^{\circ} 37'$, or in Time, 1h, $19' 48''$.

Then from	—	—	—	6 00 00
Take	—	—	—	1 19 48

Remains the time of Sun-rising	4 40 12
--------------------------------	---------

And added, is time of Sun-setting	7 19 48
-----------------------------------	---------

Doble time of Sun-setting gives the	} 14 39 36
Length of the Day	

Doble time of Sun-rising gives the	} 9 20 24
Length of the Night	

PART II.

L 4

PROB.

P R O B. XLI.

The Sun's Amplitude (25 49' North) and the Angle of the Sun's Meridian with the *Horizon* (54 13') given; to find the Latitude and Declination, also the time of Sun-rising and setting; with the Length of the Day and Night.

Projection. (See the last Schems.)

1. P.C.Q. Make the Angle $\text{E}\odot\text{N} = 54\ 13'$, as directed in the last Problem, and find the Pole of the Oblique Circle, i.e. at P.

2. Lay off 25 49' from the Chords from $\odot$ to d ; and laying a Ruler over d and P, 'twill cut the Oblique Circle HB $\odot$ in B.

3. Measure u P on the half Tangents, which you will find to be 55 deg., which taken from 180 deg. rests 125 deg. Set off the half Tangent 125 deg. from u , the Center, to Q on the Line u Z produced for a third Point.

4. Through the three Points P, B, and Q draw the Oblique Circle (or Meridian) NBS, and it will form the Triangle $\odot\text{BN}$; in which are given, the Base $\odot\text{B}$ 25 49', the Sun's Amplitude, and Angle at the Base $\text{B}\odot\text{N}$ 54 deg. the Angle of the Sun's Meridian with the *Horizon*; to find the Perpendicular $\text{BN} = \text{Lat. } \odot\text{N}$, the Hypotenuse $= \text{Co. Declin.}$ and the Angle at the Perpendicular $\text{BN}\odot = \text{hour from Noon}$, by Case 4, 9, and 11, of *Right-angled Spherical Triangles*.

In the foregoing Triangle $\odot\text{BN}$, may three Problems more be solved, which I leave to the Industry of the young Student.

P R O B. XLII.

In any Latitude from the polar Circles, to the Poles of the World (i.e. from the Latitude 66 30', to Latitude 90 deg) to find how many days they have the Sun visible above the *Horizon*, and how many days Obscurity.

Example. I demand, how many days of the Year have they the Sun visible above the *Horizon*; and how many days Obscurity, in the Latitude of 75 30' North or South?

PROJECTION, on the Plain of the Equinoctial
CIRCLE.

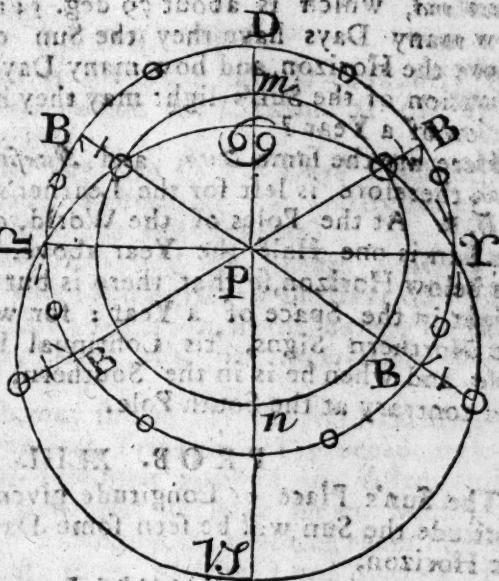
P.C.Q. 1. Make the Angle $B\gamma\odot$ at the Primitive=
Sun's greatest
Declinat. viz.
 $23^{\circ}30'$ by
drawing the
Oblique Circle
(or Ecliptick)
 $\gamma\odot\equiv\gamma\gamma$.

2. With the
half Tangent of
 $75^{\circ}30'$ describe
the Parallel to
the Primitive,
as $B\odot m\odot B$, to
cut the former
Oblique Circle
in the Point B
 $\odot$.

3. Thro' the
Center P , the
Pole, and the
Points $\odot$ and
 $\odot$ on both sides, draw the Meridians or Right Circles
 $B\odot P B$, and continue them both ways below to $\odot$ and
 $\odot$; so shall you have two Right-angled Triangles $B\odot$
 γ and $B\odot\equiv$ within the Plain of the Projection, and
two more mark'd $B\odot\equiv$ and $B\odot\gamma$ without; in either
of which you have given the Angle at the Base $B\gamma\odot$
(the Obliquity of the Ecliptick, or Sun's greatest Declina-
tion) $23^{\circ}30'$, to find the Bases γB . and $\equiv B$, that is,
the Sine-Complement of half the Visibility, or Sine-
Complement of half the Obscurity, counting for every
Degree a natural Day. The Solution by Case 4, of
Right-angled Spherick Triangles.

$A. B. \angle B. T.C. \odot \text{Decl.} :: T. \text{of Comp. Lat.} :: S. C.$
half Visib.

$-S. 90^{\circ} - T. \angle B\gamma \gamma 66^{\circ}30' - T. B\odot 14^{\circ}35' - S.C.$
 $36^{\circ}30'$. Complement to 90° is $53^{\circ}30'$; which doub-
led, is 107 Days, they have the Sun Night and Day Vi-
sible above the Horizon, and the same number of Days
Obscurity; and 107 days doubled, is 214 days; which
taken from 365 days, remain 151 days, that the Sun ri-
ses and sets before and after 107 continual Visibility.
Allo



Also 75 deg. 30 m. from 90 deg. rests 14 deg. 30 m. for the Sun's Declination, which answers to about April 18th, and August the 3d, Including just 107 Days. Also,

Example 2. In the Lat. of *Halcutt-Head-Land* in *Greenland*, which is about 79 deg. 54 m. N°. I demand how many Days have they the Sun constantly Visible above the Horizon, and how many Days of Obscurity or privation of the Sun's light may they have in the Revolution of a Year?

Here are the same *Data*, and *Quæsitæ* as in the last *Prob.* therefore is left for the Learner's Exercise.

N. B. At the Poles of the World, or Lat. of 90 deg. the Sun is one Half the Year above, and 'tother half the below Horizon, so that there is but one Day and one Night in the Space of a Year; for while the Sun is in the Northern Signs, 'tis Continual Day at the North Pole, and when he is in the Southern, Continual Night, and contrary at the South Pole.

PROB. XLIII.

The Sun's Place or Longitude given, to find in what Latitude the Sun will be seen some Days Visible above the Horizon.

EXAMPLE.

If the Sun be 4 deg. 15 m. in $\odot$. I demand in what Latitude the Sun will be Visible constantly above the Horizon, as also the Number of Days of Illumination and Obscuration?

(See the last Scheme).

Here is given the Angle $BT\odot$ 23 deg. 30 m. and $\gamma\odot$ the Sun's Longitude from γ 30 deg. + 4 deg. 15 m. to find the Base γB and Perpend. $B\odot$, by Case 10th, and 1st, of *Right-angled Spherick Triangles*.

PROB. XLIV.

The Sun's Declination Given, to find the Latitude that the Sun will be some days constantly visible above the Horizon?

EXAMPLE.

Let the Sun's Declination be 10 deg. 30 m. N°. I Demand the Latitude, and Number of Days, he will Constantly appear above the Horizon, in that Latitude; also the Days he will rise, and set, and how many Days of Privation, of the Sun's Appearance?

(See

(See the last Scheme.)

For Solution, See *Prob. 42.* foregoing, Accounting here the Comp. of the Declin. *i. e.* 19 deg, 50 m. for the Latitude there.

PROB. XLV.

By the Number of Days the Sun is constantly seen above the Horizon to find the Latitude of the Place, and Sun's Declination.

Example. Admit the Sun be 128 Days constantly above the Horizon; I demand the Latitude of the Place, &c. (See the last Scheme)

Here is given $\frac{1}{2} \frac{2}{3} = 64$ deg. the Comp. Base, or the Base $\gamma B = 26^\circ$ and (Sun's greatest Declination 23 deg. 30 m.) the $\angle B \gamma \odot 23$ deg. 30 m. at the Base to find the Perpend. $B \odot$ by Case 4th, of *Right-angled Spherick Triangles.*

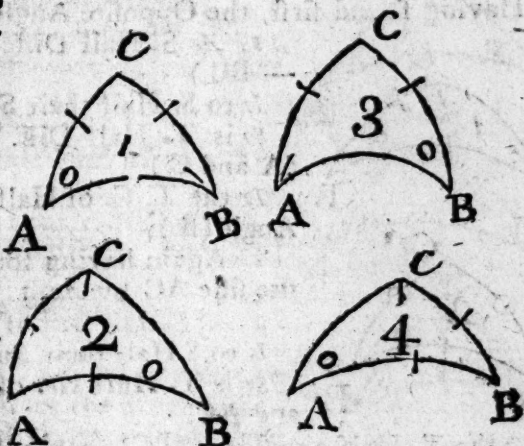
And thus much may suffice for the Application of a Right-angled Spherical Triangle. I proceed now to Oblique-Spherical-Triangles applied in Astronomical Problems: And although we have shewn before the Solution of these Triangles, by dividing them into two Right, and solving, them by the Lord *Napair's* Proposition; and also by Axioms for that purpose; which Axioms are also plainly demonstrated; Yet for the Learners benefit shall again repeat the said Axioms, in Order to the more expeditious Resolving the following Astronomical Problems.

Given Two sides and an Angle opposite to one of them; to find the Angle opposite to the

AXIOM. I.

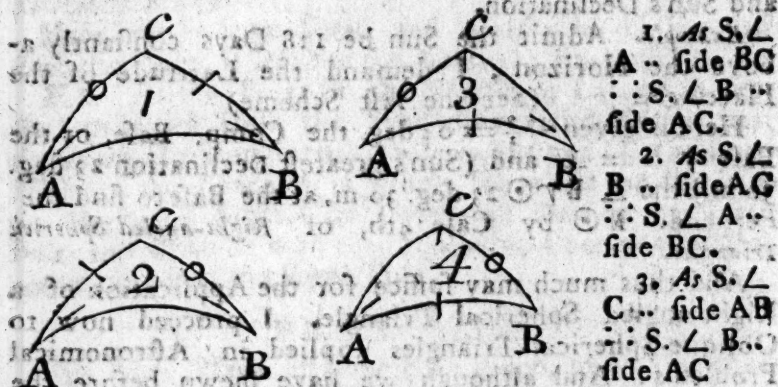
1. In all Spherical Triangles, the Sines of the sides are in direct Proportion to the Sines of their Opposite

Angles, and contrary. That is,



1. As S. side AC .. S. $\angle$ B :: S. side BC .. S. $\angle$ A.
2. As S. side AB .. S. $\angle$ C :: S. side AC .. S. $\angle$ B.
3. As S. side BC .. S. $\angle$ A :: S. side AC .. S. $\angle$ B.
4. As S. side AB .. S. $\angle$ C :: S. side BC .. S. $\angle$ A.

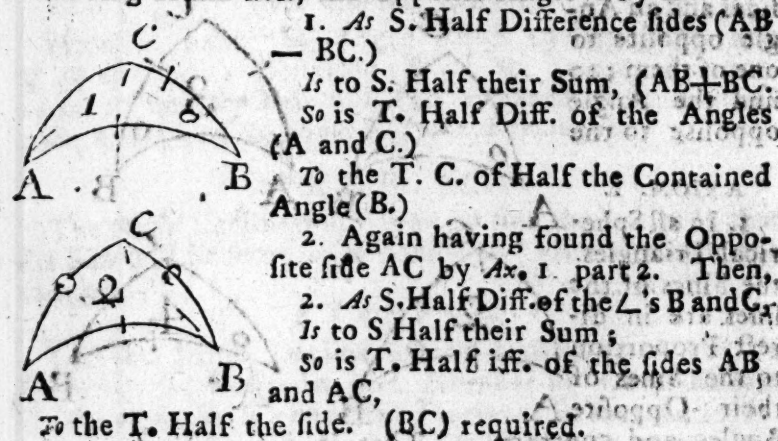
2. Again ; In all the Spherical Triangles, The Sines of their Angles are in Direct Proportion, to the Sines of the Opposite sides ; That is,



4. As S. $\angle$ C :: side AB :: S. $\angle$ A :: side BC.
- 3d. When No Opposite sides nor Angles are given, neither Two sides, and their Contained Angle ; neither Two Angles and Sides betwixt them ; Then,

AXIOM. II.

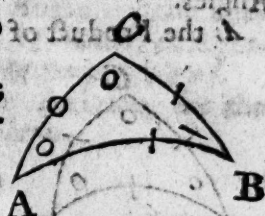
Having found first, the Opposite Angle C by Ax. 1.



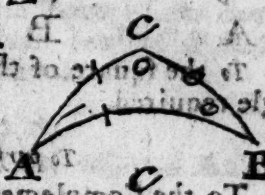
AXIOM III

When two Sides, and their contained Angle are given, say,

1. As the S. half Sum of any two sides containing an Angle,
Is to the S. half their Difference;
So is the T. C. of half the contained Angle,
To the T. of half the Diff. of the other two Angles.

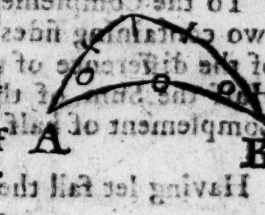


- Again,
2. As the S. C. of half the Sum of the two sides,
Is to the S. C. of half their Diff.
So is T. C. of half the contained Angle,



- To the T. of half the Sum of the other two Angles.

Then the half Difference added to the half Sum, gives the greater Angle; and subtracted from the half Sum, gives the lesser of the two Angles required.



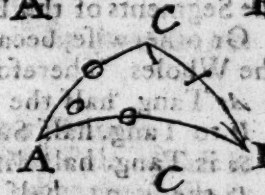
AXIOM IV

When two Angles and a Side betwixt them is given, say,

1. As S. half Sum of any two Angles,
Is to S. half their difference;
So is T. half the contained Side,
To T. half difference of the other two sides.

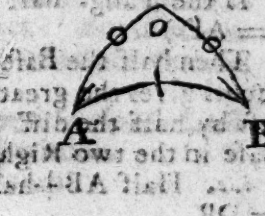


- Again,
2. As S. C. half Sum of two Angles,
Is to S. C. half their difference;
So is the T. of half the contained side,



- To the T. of half the Sum of the other two sides.

Then the half difference added to the half Sum, gives the greater side; and the half difference subtracted from the half Sum, gives the lesser side required.



Axiom V.

When the three sides are given, to find any of the Angles.

As the Product of the two sides containing an Angle required,

Is to the Square of Radius;

So is the Product of the Sines of the half Sum of the three sides, and of the difference of the opposite side (to the Angle required) from the half Sum.

To the square of the Sine-Complement of the half Angle required.

To perform this by Logarithms.

To the Complement Arithmetical of the Sines of the two containing sides, add the Sine of the half Sum, and of the difference of the opposite side from the half Sum: Half the Sum of these four Logarithms is the Sine-Complement of half the Angle required.

Or thus :

Having let fall the Perpendicular CD, and made ED = DB, AE shall be the difference of the Segments of the Base. Then,

As the Tangent of the Base (the side on which the Perpend. falls,)

Is to the Tangent of the Sum of the other two sides;

So is the Tangent of the diff. of those sides,

To the Tang. of the difference of the Segments of the Base.

Or otherwise, because the Halves are in proportion as the Wholes, Therefore,

As Tang. half the Base (= half AB,)

Is to Tang. half Sum of the sides (= half AC + BC,)

So is Tang. half diff. of the sides (= half AC - BC,)

To the Tang. half diff. of the Segments of the Base (= AE.)

Then half the Base added to half the diff. of the Segments gives the greater Base; and half the Base made less by half the diff. of the Segments, gives the lesser Base in the two Right-angled Triangles.

i. e. Half AB + half AE = AD, and half AB - half AE = DB.

Then

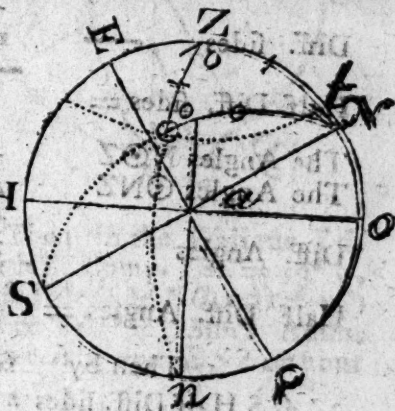
For a full demonstration of these Axioms, see the
Pages following the Projection of the Spheres.

PROB. L

The Latitude ($30^{\circ} 51'$ North) and hour of the day (3h. 15'= $48^{\circ} 45'$) with the $\odot$'s Altitude ($42^{\circ} 16'$ P.M.) given; to find his Azimuth and Declination?

PROJECTION.

4. Thro' the Points Z , $\odot$ and n , describe an Oblique or Azimuth-Circle as $Z\odot n$, and 'tis done.



Explanation of the Scheme.

In the Oblique spherick Triangle $Z\odot$ is to be no-

1. ZN, the Comp. of the Latitude.
2. Z☉, the Complement of the Sun's Altitude.
3. N☉, his distance from the elevated Pole, which is the Declination, added to 90 degrees, when the Latitude and Declination are of contrary Names: But if of one Name (as here in this Example) it is the Complement of the Declination.
4. ☉ZN, his Azimuth from the North in South Latitude; but from the South, in South Latitude.
5. ☉NZ, the Hour of the day at Noon.

Z, the

Z, the Zenith, z the Nadir, N the North Pole, S the South Pole, N & S the Axis, E & Q the Equinoctial, or first Azimuth of East and West.

The Operation, by Axiom 1.

1. As S. of Comp. Alt. .. S $\angle$ Hour : S.C. Lat.

As $\angle$ Position, ..

As S. side Z $\odot$ $48^{\circ} 50'$.. S $\angle$ ZNO $48^{\circ} 45'$ Side ZN

S. $\angle$ $59^{\circ} 09'$.. N $\odot$ Z 59°

Then to the Side ZN $59^{\circ} 09'$
(Add the) Side Z $\odot$ $48^{\circ} 50'$

Sum sides ————— 101 59

Half Sum sides ————— 50 59

Diff. sides ————— 10 19

Half Diff. sides = 5 09

The Angles N $\odot$ Z 59 00'

The Angles $\odot$ N Z 48 45'

Diff. Angles ————— 10 19

Half Diff. Angles = 5 07

Then by Axiom 2, Part 1.

2. As S Half Diff. sides .. S Half Sum, sides : T. Half Diff. Angles .. T. C. Half Cont. Angle.

— S. $5^{\circ} : 09'$ — S. $53^{\circ} : 59'$ — T. $5^{\circ} : 07'$ — T. $51^{\circ} : 07'$ which doubled, is $102^{\circ} : 14'$ the $\angle$ $\odot$ ZN, the Azimuth from the North, which subtract from $180^{\circ} : 00'$, Remains $77^{\circ} : 46'$, the Sun's Azimuth from the South; which turned into Points of the Compass is

7 Points from the South almost or near { E b. S. } A. M. { W b. S. } P. M.

By Axiom 1. Part 2.

3. As S. $\angle$ ZNO .. S. side Z $\odot$: S $\angle$ NZ $\odot$.. S. side N $\odot$.

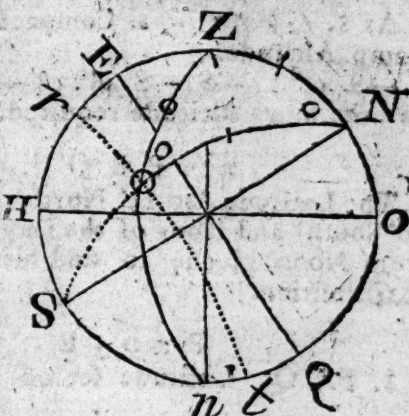
4. S. Hour $48^{\circ} 45'$ — S. C. Alt. $48^{\circ} 50'$ — S. Azim. $102^{\circ} 14'$ S. C. Declin. $78^{\circ} 06'$ — 90° = The Sun's Declin. $11^{\circ} 54'$ North.

PROB. II.

The Latitude ($30^{\circ} 51'$ North,) Sun's Declination ($20^{\circ} 30'$ South) and Azimuth $130^{\circ} 30'$ from the North) given, to find his Altitude, and hour of the day,

PROPORTIONS.

1. P.C.Q. Lat. set off, Equator drawn, as formerly,
2. Draw the Parallel of Declination; because South $69^{\circ} 30'$ from the S. Pole, as $\gamma \odot r$.
3. Make the Angle $\odot Z N =$ to $130^{\circ} 30'$ (by Prob. 1. Case 1. of Stereog. Prob.) by drawing the Azimuth-Circle $Z \odot n$ with the Secant of the Comp. thereof, i. e. $69^{\circ} 30'$, to cut the Parallel of Declin. in $\odot$.
4. Through $N, \odot$, and S , draw the Meridian Circle $N \odot S$, and 'tis done: Then in the Oblique Triangle $Z \odot N$ is given, the side NZ , Comp. Lat. $= 59^{\circ} 9'$, the Sun's dist. from the North Pole (side $N \odot = 110^{\circ} 30'$) and its opposite Angle $\odot Z N = 130^{\circ} 20'$, Sun's Azim. (to find the side $Z \odot =$ Comp. Alt. and $\odot N Z =$ hour from Noon.



Operation by Ax. I.

1. As s. $\odot$ Dist. from Pole.. s. $\odot$ Azimuth; $\therefore$ s. of Comp. Lat. .. s. $\angle$ Position.
 — s. $110^{\circ} \text{ deg. } 30 \text{ m.}$ — s. $130^{\circ} \text{ deg. } 30 \text{ m.}$ — s. $59^{\circ} \text{ deg. } 09 \text{ m.}$ — s. $44^{\circ} \text{ deg. } 11 \text{ m.}$

Then $59^{\circ} 09'$ added,

110 30

169 39 Sum sides.

84 49 $\frac{1}{2}$ Sum sides

50 21 diff. sides.

25 10 $\frac{1}{2}$ diff. sides.

130 30

44 11

86 19 diff. Angles.

43 09 $\frac{1}{2}$ diff. Angles.

PART II.

M

1. By
2. B $\frac{1}{2}$

2. By Axiom 2. Part I.

As S. half diff. sides .. s. half Sum sides :: T. half diff. Angles .. T.C. half Ang, $\odot$ NZ.

—S. $25^{\circ} 10'$ —S. $84^{\circ} 49'$ —T. $43^{\circ} 09'$ —T. $24^{\circ} 29'$ doubled is $48^{\circ} 58'$, and reduced to Time, is 3h. 15' 52" from Noon, or 8h. 44' 08" A.M. if the Azimuth was taken in the Morning.

3. By Axiom 1.

As s. $\angle$ Posit. .. s. Comp. Lat. :: s. Hour .. s. of Comp. Altitude.

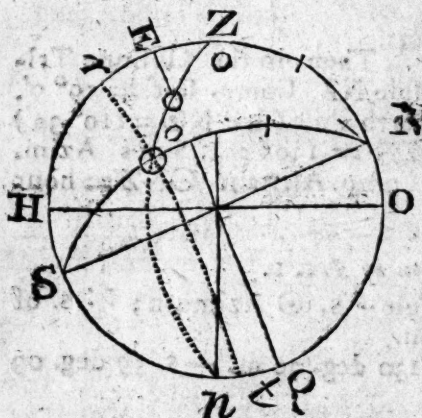
—S. $44^{\circ} 11'$ —S. —S. $48^{\circ} 58'$ —S. $68^{\circ} 18'$, Comp. is $21^{\circ} 42'$ the Sun's Altitude required.

P R O B. III.

The Latitude ($20^{\circ} 11'$ North) Sun's Declination ($23^{\circ} 30'$ South) and Hour of the Day (9h. 24' A.M. or 2h. 36' from Noon) given; to find his Azimuth, Altitude and $\angle$ Position.

P R O J E C T I O N.

1. P.C.Q. Latitude set off; and Equinoctial drawn (2h. 36' reduced to time is $= 39^{\circ}$.)



2. Make the Angle $\odot$ NZ equal to 39° , as in the first Problem hereof; by drawing the Oblique Circle or Meridian N $\odot$ S.

3. Draw the Parallel of Declination r ; $66^{\circ} 30'$ from the South Pole S, to cut the former Meridian Circle in $\odot$.

4. Through the Points Z, $\odot$, and n, describe

the Azimuth-Circle Z $\odot$ n, and 'tis done. Then in the Oblique Triangle N $\odot$ Z are given the two sides NZ = $69^{\circ} 49'$, N $\odot$ = $113^{\circ} 30'$ (i. e. the Comp. Lat. and Sun's Dist. from the North-Pole) to find the Angle, NZ $\odot$ and N $\odot$ Z (the Sun's Azim. from the North, and Angle of Position, by Axiom 3.)

PREPARATION.

N $\odot$ =	143° 30'	
NZ =	69 49	
Sum sides	183 19	Angle ZN $\odot$ = 39° 30'
Diff. sides	43 41	Half the Angle 19 45
Half Sum sides	91 39	90 00
Half Diff. sides	21 50	Comp. Half $\angle$ 70 15

1. As S Half Sum sides .. S Half Diff. sides :: T. C. Half Cont. Angle .. T. Diff. Angles.

— As S. — 91° 39' — S. 21° 50' — T. 70° 15' — T. 46° 01'.

2. As S. C. Half Sum sides .. S. C. Half Diff. sides :: T. C. Half Cont. Angle .. T. Half Sum Angles.

— As S. — 91° 36' — S. — 21° 50' — T. 70° 15' — T. 87° 03'.

Then { Half Sum $\angle$'s 87° 03'
Half Diff. $\angle$'s 46° 01'

Sum 133 04 grt. $\angle$ $\odot$ ZN the Sun's Azim. from the North

Diff. 41 02 Lesser. $\angle$ N $\odot$ Z = the Angle of the Sun's Position.

By Axiom. 1.

3. As S $\angle$ Posit. .. S. C. Lat. :: S. Hour .. S. C. Alt.

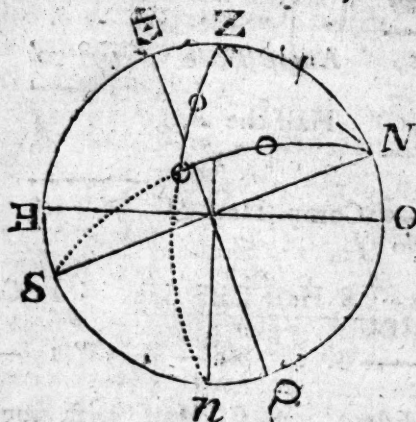
— S. 41° 02' — S. 69° 49' — S. 39° 30' — S. 39° 30' — S. 65° 25' Comp. 24° 35' = $\odot$'s Alt.

PROB. IV.

The Latit. (13° 10' N) Sun's Azim. 120° 30' from the North) and Hour of the Day (9h 36' A. M. or 2h 24' from Noon equal to 36°) given to find his Declination and Altitude.

PROJECTION.

i. P.C.Q. Lat. Set off, and Æquinoctial drawn.



2. Make the Angle $\odot\text{NZ} = 36^\circ$ and Angle $\odot\text{ZN} = 120^\circ 30'$ by drawing the two Obl. Circles, the Meridian $\text{N}\odot\text{S}$ and Azimuth $\text{Z}\odot\text{n}$, to intersect each other in $\odot$, and it's done. In the Triangle $\text{Z}\odot\text{N}$ is given, the two Angles $\text{ZN}\odot$ and $\odot\text{ZN}$ and side contained ZN to find the other two sides $\odot\text{N}$ and $\text{Z}\odot$ the

Sun's Dist. from the North Pole and Comp. of the Sun's Altitude. Therefore, By Axiom 4,

PREPARATION.

$\angle \odot\text{ZN}$	$120^\circ 30'$	side ZN Comp. Lat. =	$76^\circ 50'$
$\angle \odot\text{NZ}$	$36^\circ 00'$		
Sum	<u>$156^\circ 30'$</u>	Half the side =	$38^\circ 25'$

Half Sum	<u>$78^\circ 15'$</u>
----------	----------------------------------

Diff.	<u>$84^\circ 30'$</u>	Let Z. signifie Sum, X Difference,
		= equal to, + more, - less, L

Half Diff	<u>$42^\circ 15'$</u>	Angle, Sd. sides, $\angle$'s Angles.
-----------	----------------------------------	---------------------------------------

Then As S Half Z $\angle$'s .. S Half X $\angle$'s :: T. Half Sd. .. T Half X. other two sides.

$\text{S. } 78^\circ 15'$	$\text{S. } 42^\circ 15'$	$\text{T. } 38^\circ 25'$	$\text{T. } 28^\circ 35'$
---------------------------	---------------------------	---------------------------	---------------------------

2. As S. C Half Z. $\angle$'s .. S. C. Half X. $\angle$'s :: T. Half Sd. .. T. Half Z other two sides.

$\text{S. } 71^\circ 45'$	$\text{S. } 47^\circ 45'$	$\text{T. } 38^\circ 25'$	$\text{T. } 66^\circ 24'$
---------------------------	---------------------------	---------------------------	---------------------------

Half Z sides	<u>$66^\circ 24'$</u>
--------------	----------------------------------

Half X sides	<u>$28^\circ 35'$</u>
--------------	----------------------------------

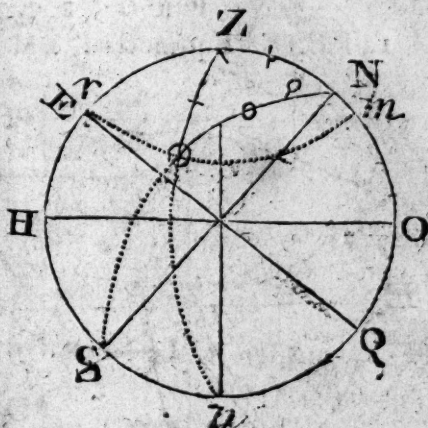
Sum	<u>$94^\circ 59'$</u>	greater side = Sun's Dist. from
		the Elevated Pole.

Diff.	<u>$37^\circ 49'$</u>	Lesser side = Comp. Sun's Alt.
		From

PROB. V.

Projection. 1. P.C.Q. Lat. Set off and *Æquinoctial*
Drawn.

3. Draw a Parallel
 $40^{\circ} 20' = \text{Comp. Alt.}$
 Dist. from Z, as $r m$, to
 cut the Azimuth Cir-
 cle Z $\odot n$ in $\odot$. Lastlly,
 thro' the Points N. $\odot$
 and S. draw the Meri-
 dian Circle N $\odot$ S. and
 'tis done.



In the Oblique-angled Triangle $N\odot Z$ are given the two sides $Z\odot$ $40^{\circ} 20'$ and $ZN = 38^{\circ} 28'$ (Comp. Lat.) with the Contained Angle $\odot NZ$ and sides $N\odot$ by Axiom 3.

PREPARATION.

$40^{\circ} 20' \pm 38^{\circ} 28' = \begin{cases} 78^{\circ} 48' \\ 91 \quad 52 \end{cases} \begin{cases} \text{Half of} \\ \text{each is} \end{cases} \begin{cases} 39^{\circ} 24' \frac{1}{2} \text{ fds.} \\ 0^{\circ} 56' \text{ x fds.} \end{cases}$
 Again the $\angle$ given $119^{\circ} 44'$ its Half is $= 59^{\circ} 52'$.

Again the $\angle$ given $119^{\circ} 44'$ its Half is $= 59^{\circ} 52'$.

1. Δ S Half Z Sds, S Half X. sides :: T, C. Half Cont.
 $\angle$.. T. Half X. Angles.

— S — $39^{\circ} 24'$ — S — $0^{\circ} 56'$ — T. $30^{\circ} 08'$ T. $00^{\circ} 51'$.

2. As S.C. Half Z sides .. S. C. Half X sides : : T. C.
Half Cont. $\angle$.. T Half Z Angles.

— S. 50° 36' — S. 89° 04' — T. 30° 08' — T.

Then Half Z 36 deg. 54 m. + Half X 0 deg. 51 m. = 37 deg. 45 m. $\angle$ of the hour from Noon; which reduced to Time is 2h. 31' from Noon, or 9h 29' $\therefore$ A. M.

3. *As* S. hour .. S. C, $\odot$ Alt. :: S. $\odot$ Azim. .. S. of
Comp. Declination.

— S. 37 deg. 45 m. — S. 40 deg. 20' — S. 119 deg. 44' — S. 66 deg. 38 min. Comp. 23 deg. 22 m. the Declination.

Then by Ax. 2. Pt. 2.

—S. 16° 59' — S. 83° 59' — T. 5° 09' .. T. 15° 36',
Doubled is 31° 12'.

PROB. VII.

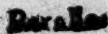
The Sun's Altitude ($31^{\circ} 25'$) his Declination ($23^{\circ} 30' S$) and Hour of the Day ($2^h 45'$ P. M. or $41^h 15'$) given; to find his Azimuth and Latitude.

P R O J E C T I O N.

1. P.C.Q. Axis and Equinoctial, drawn also the Angle of the Hour described by the Meridian NQS, as in the last Problem.

2. Draw the Parallel of Declination $\epsilon = 66^{\circ} 30'$ from the Sth. Pole, to cut the Meridian NQS in O.

3. By Prob. 9.
Case 3, of Stereog.
Problems describe
from © (as a
Pole of an Obli-
que Circle) the
Plane H.



Parallel Circle $S \cap Z$. to cut the Primitive, or General Meridian $58 \text{ deg. } 35 \text{ m.}$ in Z . the Zenith; then thro' the Center a , draw the Prime Vertical $Z a n$, and at Right-Angles thereto the Horizon $H a O$.

4. Lastly, thro' the points $Z \odot$ and n , draw the Azimuth Circle $Z \odot n$; and 'tis done.

Then in the Triangle $Z \odot N$ is given $N \odot =$ Sun's Distance from the North or Elevated Pole $Z \odot =$ Comp. Alt. and its Opposite $\angle \odot N Z$, the Hour from Noon $= 41 \text{ deg. } 15 \text{ m.}$ to find the $\angle \odot Z N =$ Sun's Azim. side $NZ =$ Comp. Lat.

Therefore by Axiom 1.

$90 \text{ deg.} - 31 \text{ deg. } 25 \text{ m.} = 58 \text{ deg. } 35 \text{ m.}$ Co. Alt. $90 \text{ deg.} + 23 \text{ deg. } 30 \text{ m.} = 113 \text{ deg. } 30 \text{ m.}$

$1 \text{ As s.c. Alt.} :: \text{S.Hour} :: s. \odot \text{ Dist. from N. Pole}$
 $'' s. \odot \text{ Azim. from the North.}$

$— s. 58 \text{ deg. } 35 \text{ m.} — s. — 41 \text{ deg. } 15 \text{ m.} — s.$
 $113 \text{ deg. } 30 \text{ m.} — s. 45 \text{ deg. } 05 \text{ m. Comp. to } 180 \text{ m.} =$
 $134 \text{ deg. } 55 \text{ m.}$

Now $\angle NZ \odot$	$134^{\circ} 55'$	side $N \odot$	$113^{\circ} 30'$
$\angle ONZ$	$41^{\circ} 15'$	side $N \odot$	$58^{\circ} 35'$
Z	$176^{\circ} 10'$	X	$54^{\circ} 55'$
Half Z	$88^{\circ} 05'$	Half X	$27^{\circ} 22'$
X	$93^{\circ} 40'$		
Half X	$46^{\circ} 50'$		

Then by Axiom 2d, Part 2d.

2. As s. Half $X \angle$'s $:: s.$ Half $Z \angle$'s $:: T$ Half X . sides $:: T$ Half sides $Z N =$ Comp. Lat.

$— s 46 \text{ deg. } 50 \text{ m.} — s — 88 \text{ deg. } 05 \text{ m.} — T. 27 \text{ deg. } 22 \text{ m.} — T. 37 \text{ deg. } 21 \text{ m. Doubled is } 74 \text{ deg. } 42 \text{ m.}$

Which taken from 90 deg. rest $15 \text{ deg. } 18 \text{ m.}$ for the Latitude North.

P R O B. VIII.

The Altitude ($29 \text{ deg. } 30 \text{ m. A.M.}$) Azimuth (130 deg. from the North) and Declination ($19 \text{ deg. } 12 \text{ m. N}$) given; to find the Latitude and Hour of the Day.

11.4.6

me Ver-

Then thro' N and the Center a draw

Right Angles thereto the Equino-

the three Points N, $\odot$, and S, de-
 $N \odot S$, and it is done.

angle $Z \odot N$ is given $N \odot = \text{Comp.}$
 $R \odot = \text{Comp.}$ Alt. 60 deg. 30 m. and

eg. Sun's Azimuth, to find the $\angle$
Therefore by *Axiom 1*,

m. : S.C. © Alt. .. S. hour

deg.—S. 60 deg. 30 m.—S.
1. 1. 9h. 00' 24" A. M. or

1000

1951

Side $\begin{cases} N\odot = 70^{\circ} 48' \\ Z\odot = 60^{\circ} 30' \end{cases}$

$$\begin{array}{r} 25 \\ \times 12 \\ \hline 50 \\ 250 \\ \hline 300 \end{array}$$

Half X	10	10
--------	----	----

41111 X 5 19

786
198

$\therefore S. \text{ Half } X \perp s, S. \text{ Half } Z \perp s \therefore T. \text{ Half } X \text{ sides } \perp T. \text{ Half side } ZN = \text{Gomp. Lat.}$

—S. 42 deg. 33 m. — S. 87 deg. 27 m. — T. 5 deg. 19 m. — T. 7 deg. 50 m. Doubled, is 14 deg. 40 m.

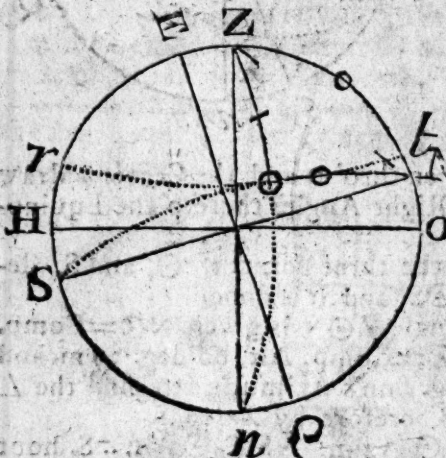
The Complement to 90 deg. is = 74 deg. 20 m. the Latitude North.

PROB. IX.

The Sun's Altitude ($24^{\circ} 30'$) Azimuth (75° deg. from the North) and hour of the Day ($4^{\text{h.}} 56'$ P.M.) given; to find his Declination and the Latitude of the Place.

PROJECTION.

1. P. C. & Parallel of Latitude drawn 65 deg. 30 m.
distant from Z the
Zenith.



2. Make the Angle $\odot ZN = 75^\circ$ the Sun's Azimuth, by drawing the Azim. Circle $Z\odot n$, which cuts the Parallel of Altitude rs in $\odot$.

3. On the Point $\odot$ describe an Oblique Circle, or Meridian $N\odot S$, to make with the Primitive or Meridian an Angle $= 69$

deg. (= h. 36', and it is done.

In the Oblique Triangle $Z\odot N$ is given the Angle $\odot ZN = 75$ deg. Angle $Z\odot N = 69$ deg. and its opposite side $\odot Z$ equal 65 deg. 30m. to find the other opposite side $\odot N$ equal Comp. Declin. and side ZN equal Comp. Latitude.

Therefore by *Axiom 1. Part 2.*

1. *A*. S. Hour 69 deg., S.C. Alt. 65 deg. 30 m. : : S.
 © Azim. 75 deg. 00 m. S. of Comp. Decl.

And 90 deg. — 70 deg. 18 m. equal 19 deg. 42 m. the
Sun's Declination North. Then,

$\angle \odot ZN$	75°
$\angle \odot NZ$	69
Z	144
Half Z	72
X	6
Half X	3

$N \odot$	70° 18'
$Z \odot$	65 30
X	4 48
Half X	2 24

} Sds.

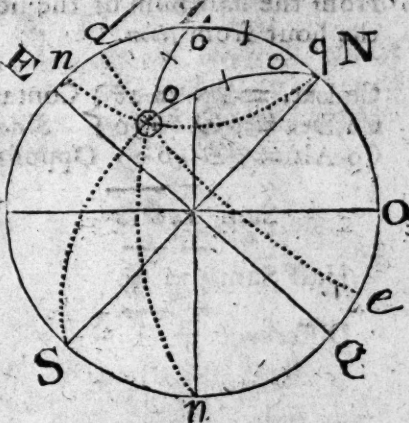
Then by Axiom 2, Part 2,
 2. S. half X Angles .. S. half Z Angles; $\therefore$ Half X
 sides .. T. half Sd. $ZN = \text{Comp. Lat.}$
 — S. 3 deg. — S. 72 deg. — T. 2 deg. 24 m. — T. 37 deg.
 18'. Doubled, is 74 deg. 6 m. Whose Complement to
 90 deg. is 15 deg. 24 m. the Latitude North.

P R O B. X.

The Latit. (51 deg. 32 m. N) Sun's Declination (23 deg. 30 m. N.) And his Altitude (49 deg. 40 m. A.M.) Given, to find his Azimuth and Hour of the Day.

P R O J E C T I O N.

1. P.C.Q. Lat. Set off Equinoctial, and Axis drawn.
2. Describe a Parallel of Declination 66 deg. 30 m. from N, the North Pole, and a Parallel of Altitude 40 deg 20 m. from the Zenith Z. these two Parallels intersect each other in $\odot$.
3. Thro' the Points N, $\odot$, and S, draw the Meridian $N \odot S$, and thro' the Points $Z \odot$ and n the Azimuth Circle $Z \odot n$, and it's done.



In the Ob. Triangle $Z \odot N$, are given the 3 sides $NZ =$
 Comp. Lat. 38 deg. 28 m. $Z \odot = \text{Comp. Alt. 40 deg.}$
 20 m. And $N \odot = \text{Comp. Declin. 66 deg. 30 m.}$ to find
 the Angles $\odot ZN$ the Azim. from the North (in North
 Lat.) but from the South in South Lat. $\odot NZ$ the Hour
 from Noon, and $N \odot Z$ the Angle of the Sun's Position.

Therefore

Therefore by Axiom 5, 1. [For the Azimuth,

Comp. Lat. Z⊙	= 38° 28'	{ Contain- ed sides } Opposite side	Co. Ar.
Comp. Alt. Z⊙	= 40 20		S. 0.20617
Comp. Declin. N⊙	= 66 30		S. 0.18894

Z. sides	145 18	S. 9.97977
		S. 9.02991

Half Z sides	72 39	
--------------	-------	--

Diff. Op. side } from Half z. }	6 09	19.40479
------------------------------------	------	----------

S. C. equal 59° 44'

Doubled, is	119 28	⊙ Azimuth from the N.
Subt. from	180 00	

Rem. 60 32 ⊙ Azimuth from the S.
Which turn'd into Points of the Compass is 5 Pts. $\frac{1}{4}$ 1 deg. 28 m.

So that { S. E. b. E. $\frac{1}{4}$ E. 1° 28' Ely. } if { A. M.
the Sun is { S. W. b. W. $\frac{1}{4}$ W. 1 28 Wly. } if { P. M.

Secondly, To find the Hour of the Day, the Operation will be the same; only you must take the Comp. Altit. from the half Sum of the sides; because 'tis opposite to the hour from Noon.

Co. Lat. = ZN	38 28	{ Contained Sides } Opposite side.	Co. Ar.
Co. Dec. = N⊙	66 50		S. 0.20617
Co. Alt. = Z⊙	40 20		S. 0.03761

Sum	145 18	
-----	--------	--

Half Sum	72 39	S. 9.97977
----------	-------	------------

Difference	32 19	S. 9.72802
------------	-------	------------

		19.951571
--	--	-----------

S. C. 18° 59'		9.975758
---------------	--	----------

Doubled is 37 58 or 2° 31' 52" from Noon.

Thirdly,

Thirdly, For the Angle of Position, the Operation is the same; only you must take the Comp. Lat. from the half Sum of the sides, by reason it is opposite to the Angle of Position.

Co. Decl. $66^{\circ} 30'$	} Cont. Sides	S.	Co. Ar.
Co. Alt. $40^{\circ} 20'$		S.	0.93761
Co. Lat $38^{\circ} 28'$			0.18894

z	145 18		
Half z	72 39	S.	9.97977
Diff.	34 11	S.	9.74961
			19.95593

S. C. $18^{\circ} 06'$
 Doubled, is $36^{\circ} 12'$, the Angle of Position.

But Note, Having found any one of the Angles by this Method, the other two may be found by *Axiom I.* or *Rule of opposite Sides and Angles.* For another Method by Logarithms, or Instrumental, see Case 11, of *Oblique Spherick Triangles* foregoing.

P R O B. XI.

The Sun's Azimuth ($119^{\circ} 28'$ from the North) the Hour from Noon ($2^h. 31' 52''$, or $37^{\circ} 38'$ m.) and Angle of Position ($36^{\circ} 12'$ m.) given; to find the Latitude, Declination, and Altitude of the Sun.

If two of the Given Angles be Obtuse, then Two sides of the New Triangle are Complements of the Obtuse Angles to 180° . and the third side is the Complement of the Acute Angle to 180° .

If the three given Angles be Obtuse, then the sides in this Triangle are the Complements of those Angles to 180° ; and whether all the Given Angles, or two only be Obtuse; the Angles of the New Triangle are Complements of the Corresponding sides in the given Triangle to 180° .

This Conversion of Angles into sides being made, there are given the three sides in the New Triangle $a P p$ viz. $a P = 37^\circ 58'$ $a p = 60^\circ 32'$ and $p P = 36^\circ 12'$ to find the Angles $P a p$ and $a P p$ and $a p P$, by Axiom 5, or after the same manner as in the last Case.

Operation. 1. for the Latitude				
sides	$a P$	$37^\circ 58'$	— S.	0.210981
	$a p$	$60^\circ 32'$	— S.	0.060159
	$p P$	$36^\circ 12'$	—	
	Z	$134^\circ 42'$	S.	9.965143
			S.	9.713726
Half	Z.	$67^\circ 21'$		
Diff.		$31^\circ 09'$	Z	19.950009
			Half Z	9.975004

by Axi 5.

$$S. C. = 19^\circ 14'$$

Doubled $= 38^\circ 28'$ equal Angle $p a P$ equal side QO equal side NZ , that is, the Comp, Lat. req. which taken from 90° remains $51^\circ 32'$ for the Latitude North,

2. For the Sun's Declination by Axiom 1. Part. 2.

As $S. \angle$ Posit. .. Comp. Lat. :: $S. \odot$ Azim. .. $S. C. \odot$ Declin. North.

— $S. 36^\circ 12'$ — $S. 38^\circ 28'$ — $S. 119^\circ 28'$ — $S. 66^\circ 30'$ Comp. to 90° is equal $23^\circ 30'$ the Sun's Declination North.

3. For the Sun's Altitude by the said Axiom 1. Part. 2.

As $S. \angle$ Posit. .. Comp. Lat. :: $S. Hour$.. $S. C. \odot$ Altitude.

— $S.$

— S. 36 deg. 12 m. — S. 38 deg. 28 m. — S.
37 deg. 58 m. — S. 40 deg. 23 m. Comp. to 90 deg. is
49 deg. 37 m. the Sun's Altitude.

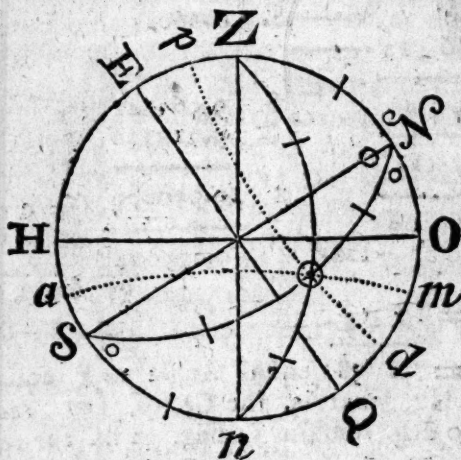
P R O B. XII.

The Lat. (36 deg. 36 m. N.) the Sun's Declination (21° 21' N.) (with the Sun's Depression 18° below the Horizon) Given ; to find the Length of the Crepusculum ; that is, the beginning and Ending of the Morning and Evening Twilight, with the time of its Continuance in both ?

Note. The Twilight is the Refraction of the Sun, which lasts till the Sun is 18° below the Horizon, after Sun's setting, and break of Day begins, when the Sun is 18 deg. below the Horizon before he Rises.

P R O J E C T I O N.

1. P.C.Q. Let. Set. off Axis, and Equinoctical.



Drawn as in *Prob.*
10. foregoing, Describe the Parallel of Declination $p d$ 68° 39m. Comp Declin. Dist. from N the North Pole.

2. Draw a Parallel of the Sun's Depression $a m$, 18° below the Horizon, or 72° from n the Nadir 'twill cut the Parallel of Declination $p d$ in $\odot$.

3. Thro' the Points $Z \odot$ and n Draw the Azimuth Circle $Z \odot n$.

4. Thro' the Points $N, \odot$, and S , draw the Meridian Circle $N \odot S$, and its done, then in the Triangle $S n \odot$, is given $S \odot$ equal Sun's Declination added to 90 deg. viz. 111 deg. 21 m. Sun's Dist. from the South Pole. $S n$ equal Comp. Lat. 53 deg. 24 m. And $n \odot$ equal Comp. Sun's Depression equal 72 deg. to find the Angle $n S \odot$ the Time of Dawning from Midnight to Morning, whose Comp. to 12 Hours gives the end of the Evening-Twilight.

There

Therefore by *Axiom 5.* or *2d Operation* to *Prob. 10.*
foregoing,

$$\begin{array}{lcl} \text{Cont. Side} & \left\{ \begin{array}{l} S \odot = 111^{\circ} 21' \\ S. = 53 \ 24 \end{array} \right. & \begin{array}{l} S. 0.030875 \\ S. 0.095382 \end{array} \\ \text{Opp. Side} & n \odot = 72 \ 00 & \end{array}$$

$$Z \quad 236 \ 45$$

$$\text{Half } Z \quad 118 \ 22 \quad S. 9.944446$$

$$X \quad 46 \ 22 \quad S. 9.859601$$

$$Z. 19.930304$$

$$\text{Half } Z. 9.965152$$

$S. C. 22^{\circ} 39'$ doubled, is $45^{\circ} 18'$, the Angle
 $\odot S.$ Or in Time $3h. 01' 12''$ from Midn.
 Which taken from $12 \ 00 \ 00$

Rem. End of Twilight = $8 \ 58 \ 48$ in the Even-
 In Lat. $36^{\circ} 36' N.$ and Decl. $21^{\circ} 21' N.$ the Sun sets at } $7 \ 10 \ 00$ Twil. beg.

Length of the Twilight } $1 \ 48 \ 48$
 After Sun-setting

Also subtr. from Sun-rising $4 \ 50 \ 00$

Rem. Beg. of Day-dawning $3 \ 01 \ 22$ in the Morn.

Note, When the Latitude and Sun's Declination are both of one Name, that if you subtract 18° from the Comp. Lat. and the Remainder be greater than the Sun's Declination, there will be a beginning of Day-dawning in the Morning, and Ending of the Evening-Twilight. But if the Remainder be less than the Sun's Declination, the Twilight lasts all Night. So that in the Latitude of London $51^{\circ} 32'$, you'll find that there will be no real Night, but Twilight, from May 12, to July 10.

PROB. XIII.

The Latitude ($51 \ 32'$ North) Sun's Declination ($23^{\circ} 30'$ North) and his Angle of Position ($36 \ 12'$) given; to find his Altitude and Azimuth, and hour of the day.

Angle {	$\odot Z N \ 119^{\circ} \ 28'$ $N \odot Z \ 36 \ 12$	}	Then by <i>Axiom</i> 2, and Part. 2,
X	$\hline 83 \ 16$		
Half X	$\hline 41 \ 38$		

As S. Half X. sides .. S. Half Z. sides :: T. Half X.
 Ls .. T. C. Half Cont. Angle.

— S. 14° — S. $52^{\circ} : 30'$. — T. $41^{\circ} : 38'$. — T. 71°
 04. Comp. $18^{\circ} : 56'$. which doubled, is $37^{\circ} : 52'$. Or, 2h.
 31' 28". — from Noon.

3. By *Axiom* 1, Part 2.

As S. Δ Posit. .. S. C. Lat. :: S. Hour .. S. C. Altit.
 — S. $36^{\circ} \ 12'$ — S. — $38^{\circ} \ 28'$ — S. — $37^{\circ} \ 52'$
 — S. $40^{\circ} \ 16'$ Comp. is $49^{\circ} \ 44'$ the Sun's Altitude
 required.

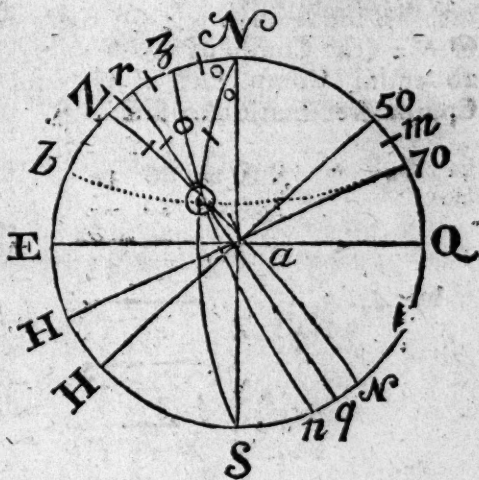
PROB. XIV.

The Sun's Declination ($23^{\circ} \ 33' \ N.$) and two different
 Latitudes (*viz.* $50^{\circ} \ N.$ and $70^{\circ} \ N.$) given; to find
 what Hour of the Day the Sun's Altitude is equal in
 both these Latitudes; and also what's the Sun's Altitude
 at that time.

PROJECTION.

1. P.C.Q. Make N. the North and S. the South
 Pole; set off the La-
 titudes given from
 N to 50, and from
 N to 70 and draw H
 50, for the Hori-
 zon to Lat. 50,
 and H 70, for the
 Horizon to Lat.
 70, by reason the
 two places have
 different Horizons.

2. Find the Mid-
 dle betwixt 50 and
 70 as *m*, and draw
r.a.q equally distant
 from each Hori-
 zon.



PART II.

N 4

3. Draw

3. Draw the Parallel of Declination b , 70° , distant from N , $66^\circ 30'$ to cut raq in $\odot$. Now seeing that 'tis evident the Sun is somewhere in the Line raq , because 'tis equally distant from both the Horizons; and it must be in the Parallel b 70° , because that is the Parallel of the Sun's Declination given; and therefore it must be at the Intersection at the Point $\odot$: Then set off 90° Chords from 50 both ways to Z and N , and you have the Zenith and Nadir to the Horizon H 50 ; also set off the Chord of 90° both ways from 70 to z and n , and it finds the Zenith and Nadir to the Latitude.

4. Draw the two great circles $Z\odot N$ and $z\odot n$, and they shall be Azimuths to each Horizon; the Arches $z\odot$ and $Z\odot$, (being equal) is the Comp. of the equal Altitudes in both Latitudes; and the Angle $\odot N$ the Hour from 6, when the Sun's Altitude is equal in both the Given Latitudes.

Operation 1. In the Quadrantal Triangle $\odot a N$ there are given (besides the Quadrantal sides $N a$) the sides $N\odot$ equal Comp. Declin. $96^\circ 30'$ and the Angles $\odot a N$ 30° (being the Comp. of the Middle between the Latitudes; which in this Case is 30° ;) for $70 + 50 = 120$ its Half $= 60$ and $90^\circ - 60^\circ = 30^\circ$ To find the Angle $a N \odot$, the hour after 6 A.M, or before 6 P.M. Therefore,

As B .. $T. \odot a N$, $30^\circ :: T.C, \odot N$ $23^\circ 30'$.. $S \angle \odot N a$, $14^\circ 31'$ or in Time gives $58'$ past 6 A.M. or 2 Minutes past 5 P.M.

2. In the Triangle $N z \odot$, there are given the Angles $\odot N z$ (the Comp. of $\odot N a$) $75^\circ 28'$ and the sides $N z$ 20 equal Comp. Lat. of 70° and $N\odot = 66^\circ 30'$ equal Comp. Declination to find $z \odot$.

$N\odot = 66^\circ 30'$	$\angle \odot N z = 75^\circ 28'$
$Nz = 20 \ 00$	
$Z \ 86 \ 30$	Half the $\angle \ 37 \ 44$
by Ax. 3. Half $Z \ 43 \ 15$	S.C. Half $\angle \ 52 \ 16$
$X \ 46 \ 30$	
Half $X \ 23 \ 15$	

As S. Half Z sides .. S Half X sides :: T.C. Half
Cont. $\angle$.. T. half X $\angle$'s.

— S, $43^{\circ} 15'$ — S — $23^{\circ} 15'$ — T, $52^{\circ} 16'$ —
T $36^{\circ} 40'$.

As S. C. $\frac{1}{2}$ Z sides .. S. C. $\frac{1}{2}$ X sides :: T. C. half
Cont. $\angle$.. T. half Z $\angle$'s.

— S, $46^{\circ} 45'$ — S, $66^{\circ} 45'$ — T, 52 deg. 16 m.
— T, 58 deg. 28 m.

Then, $\left\{ \begin{array}{l} \text{half Z } \angle \text{'s } 58^{\circ} 28' \\ \text{half X } \angle \text{'s } 36^{\circ} 40' \end{array} \right\}$

Z. greater $\angle$ N z $\odot$ 95 08

X. Lesser $\angle$ N $\odot$ z 21 48

Then by Axiom 1,
Part 2.

As S. $\angle$ N $\odot$ Z 21 deg. 48' m. :: S. sides z N 20 deg.
:: S $\angle$ $\odot$ N z 75 deg 28 m. .. S. sides Z $\odot$ 63 deg. 41 m.

Whose Comp. to 90 deg. 26 deg. 56 m. is the Altitude
at the Time required, and the Hour before found (2
Minutes past 5,) when the Altitude and Hour is the
same in the two given Latitudes; and 'tis needless to
repeat the Operation for the Triangle N $\odot$ Z, since it's
evident by Construction, that Z $\odot$ is equal to Z $\odot$.
However, we shall solve this Problem by another Me-
thod, viz. by finding the Time of the Day when the
Sun is in the prime Vertical, or East and West Azimuth
of the Middle Latitude, between 50 deg. and 70 deg.
(i. e. 60 deg.) for then is the Altitude equal in Places
under the same Meridian, and equally remote there-
from.

Therefore by Case 18, of *Right-angled Astronomy*, Pro-
portion 2d.

As R. .. T. C. Lat. :: T $\odot$ Declin. .. S. Hour
from 6.

— S. 90 deg. — T. 30 deg. — T. 23 deg. 30 m.

— S. 14 deg. 32 m. or in Time 58 m. as before.

P R O B. XV.

Upon the Northern Seas, where boisterous Storms did rise,
And darksome Clouds obscur'd the Azure Skies;
And from our Sight all Celestial Bodies skreen,
That Day or Night to us could not be seen;
During which Time, our Gallant Ship was toss'd
Midst Rocks, and Sands, each Minute cast for lost:

PART II.

N 3

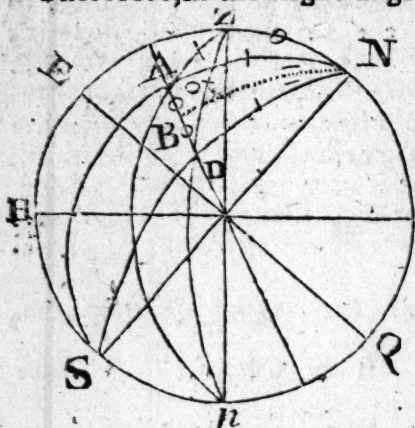
Where

Where every Billow did present a Grave,
 And Death Triumphant, rode on ev'ry Wave;
 When in the Height of Sorrow, Fear and Doubt,
 The Storm allay'd; (and to our Joy) brit SOL look'd out;
 Whose Altitude was took then in the Heav'n,
 And prov'd 3° and Minutes about Six (or Sev'n.)
 The Calm continu'd, and in that very Place,
 We stay'd there for full two Hour's Space;
 And then Bright *Phabus* smil'd on us once more,
 Whose Altitude again we did Explore;
 And found it 56° and Minutes thirty four. }
 By our Accounts, we found that very Day,
 Fell out exactly on the Tenth of May.
 Now (You) Young Artist's, whose Wits are ripe at Will,
 In this Distress, I pray exert your skill.
 And from what here is found, give me, I pray,
 The Latitude o'th' ship, and Hours o'th' Day.

SOLUTION.

On the Tenth of May the sun's Declination is 20 deg.
 14 m. N.

Therefore, in the Right-angl'd Triangle *ABN*, are given
 the Hypotenuse $AN = 69$ deg. 46 m. (the Comp.
 Declination) the Angle ANB equal to 15 deg half
 the Time between the
 Observations, to find the
 side AB . by Case 1,
Right-angled Spheric Tri-
angles



1. As R .. S. side AN
 $\therefore$ S. $\angle BNA$.. S. side
 AB .
 $\text{--- S. } 90^{\circ} \text{ --- } 69^{\circ} 46'$
 $\text{S. } 15^{\circ} \text{ --- S. } 14^{\circ} 03' \text{ Dou-}$

bled, is 28 deg. 06 m. equal side AD .

2. In the Oblique Triangle DNE , are given the Two
 sides DA 28 deg. 06 m. and DN 69 46', the Angle op-
 posite $DN \angle = 30^{\circ}$, to find the other opposite Angle DAN
 by *Axiom 1*, Part 2.

As S. side AD .. S. $\angle DNA$: : S. side DN .. S. $\angle D$
 AN .

$\text{--- S. } 28 06' \text{ --- S. } 30 \text{ deg --- S. } 69 46' \text{ --- S. } 84 54'$

3. In

3. In the Oblique Triangle DAZ , there are given Three sides $ZA = 33^{\circ} 06'$ (Comp. greater Alt.) $ZD = 46^{\circ} 54'$ (and $AD = 28^{\circ} 06'$, to find the Angle DAZ by *Axiom 5*.

$$\text{Side } \begin{cases} ZA = 33^{\circ} 26' & S. 0^{\circ} 58875 \\ DA = 28^{\circ} 06' & S. 0^{\circ} 326968 \\ ZD = 46^{\circ} 54' \end{cases}$$

$$Z \quad 108^{\circ} 26'$$

$$\text{Half } Z \quad 54^{\circ} 13' \quad S. 9.909146$$

$$X \quad 7^{\circ} 19' \quad S. 9.105009$$

$$Z \quad 19.599998$$

$$S.C. 50^{\circ} 51' \quad \text{Half } Z \quad 9.799999$$

Doubled, is $101^{\circ} 44' =$ the Angle DAZ .

$$\text{Subt. } \angle DAN \quad 84^{\circ} 54'$$

Remains $16^{\circ} 05'$ the Angle NAZ .

4. In the Oblique Triangle NAZ are given, the two Sides $NA = 69^{\circ} 46'$, $AZ = 33^{\circ} 26'$, and the Contained Angle $NAZ = 16^{\circ} 05'$ to find the Azimuth from N at second Observation. *viz.* $\angle AZN$ the Hour from *Noon* = $\angle ANZ$, and $ZN =$ Comp. Latitude, by *Axiom 3*.

$$\text{Side } \begin{cases} NA = 69^{\circ} 46' \\ ZA = 33^{\circ} 26' \end{cases} \quad \text{Angle } NAZ \quad 16^{\circ} 05'$$

$$Z \quad 103^{\circ} 12' \quad \text{Half the } \angle \quad 8^{\circ} 25'$$

$$\text{Half } Z \quad 51^{\circ} 36' \quad 90^{\circ} 00'$$

$$X \quad 36^{\circ} 20' \quad S.C. \text{ half Cont. } \angle \quad 81^{\circ} 35'$$

$$\text{Half } X \quad 18^{\circ} 10'$$

As $S.$ half Z Sds. $\therefore S.$ half X Sds. $\therefore T.C.$ half Cont. $\angle$
 $\therefore T.$ half $X \angle$ s.

$$S. 51^{\circ} 36' \quad S. 18^{\circ} 10' \quad T. 81^{\circ} 35' \quad T. 69^{\circ} 46'$$

2. As $S.C.$ half Z Sds. $\therefore S.C.$ half X Sds. $\therefore T.$ half Cont. $\angle \therefore T. Z \angle$ s.

$$\begin{array}{r} \text{---S. } 38^{\circ} 24' \text{---S. } 71^{\circ} 50' \text{---T. } 81^{\circ} 35' \text{---T. } 81^{\circ} 29' \\ \text{Add half X} \quad \quad \quad 69^{\circ} 36' \end{array}$$

$$\text{Sun's Azimuth from the North} = 151^{\circ} 05'$$

$$= \angle \text{AZN.}$$

$$\text{The Hour from Noon} = 11^{\circ} 53'$$

$$= \text{Time } 47^{\circ} 53''.$$

3. By *Axiom* 1.

As S. Hour.. S.C. Alt. : : S. $\angle$ Posit. ZAN.. S.C. Lat.

---S. 6 deg. 44 m. ---S. 33 deg. 26 m. ---S. 16 deg. 50 m. ---S. 33 deg. 17 m. Comp. is 51 deg. 43 m. the Latitude North required.

PROB. XVI.

Two unequal Altitudes of the Sun (as suppose 43 deg. 06 m. and 56 deg. 34 m.) and the difference of the correspondent Azimuths taken in one Day (which let be 39 deg. 16 m.) and the Sun's Declination (20 deg. 14 m. North) given ; to find the Latitude of the Place Nly.

See the last Scheme. First, In the Qblique Triangle DAZ are given, the two Sides ZA = (Comp. greater Alt.) 33 deg. 26 m. ZD = 46 deg. 54 m. (Comp. lesser Alt.) and the Contained Angle DZA = 39 deg. 16 m. diff. Azimuths; to find the Angle DAZ, and the third Side DA, by *Axiom* the Third.

Side	Z D 46 54'	Angle DZA	39° 16'
	Z A 33 26		
	Z 80 20	Half Angle	19 38
	Half Z 40 10		90 00
	X 13 28	S.C. Half $\angle$	70 22
	Half X 6 44		

As S. half Z sides.. S. half X sides : : T. C. half Cont. $\angle$.. T. X $\angle$ s.

---S. 40 deg. 10 m. ---S. 6 deg. 44 m. T. 70 deg. 22 m. ---T. 27 deg.

Again,

Again,

As S. C. half Z sides .. S. C. half X sides : : T. C. half Cont. $\angle$.. T. half Z $\angle$ s.

—S.C. 40 10'—S.C. 06 44'—T. C. 19 38'—T. 74 39'
To which add the half X before-found 27 00

Sum is the greater Angle DAZ equal to 101 39

2. By Axiom 1,

As S. $\angle$ DAZ .. S. DZ : : S. DZE .. S. AD.

—S. 101 39'—S. 46 54'—S. 79 16'—S. 28 09',
half 14 04' equal to AB.

2. In the Right-angled Triangle ANB, are given the Hypotenuse AN 69 46', the side AB equal to 14 04', and the Angle BAN is required. Therefore by Case 13, of Right-angled Spherical Triangles,

As R. .. T.C. AN : : T. AB .. S. C $\angle$ BAN.

—S. 90°—T.C. 69 46'—T. 14° 04'—S. 84 42', which sub.
from the Angle DAZ = 101 39

Remains the Angle NAZ = 16 57

3. In the Oblique Triangle NAZ are now given the two sides NA equal to 69° 46', ZA equal to 33° 26', with the contained Angle NAZ equal to 16° 57', to find the other Angles, and third side NZ equal to the Complement of the Lat. by Axiom the Third.

Side	NA 69 48'	}
	ZA 33 26'	
z	103 12	
Half z	51 36	
X	36 20	}
Half X	18 10	

Angle N A Z = 16° 57'

Half Angle = 8 28

As S. half z sides .. X sides : : T. C. Cont. Angle .. T. half X Angles.

—S. 51° 36'—S. 18° 10'—T.C. 8° 28'—T. 69 29'.

As S. C. half Z sides .. S. C. half X sides : : T. C. half Cont. Angle .. T. half z Angles.

—S.C. 51° 36'—S.C. 18 10'—T.C. 8 28'—T. 84 27'.

Then

224 *The Doctrine of the Sphere : Or,*

Then to the half Z $8^{\circ} 27'$
Add the half X $69 \ 29$

Sum is the Angle NZA $153 \ 56$

Difference is the Angle ANZ = $14 \ 58'$

Then by Axiom 1,

As S, Angle ANZ.. side ZA :: S. Angle NAZ.. side ZN = Comp. Lat.

— S. $14 \ 8$ — S. $33 \ 26'$ — S. $16 \ 57'$ — S. $38 \ 46'$,
whose Complement to 90° is $51 \ 14'$, the Latitude Nor-
therly. *W.W.R.*

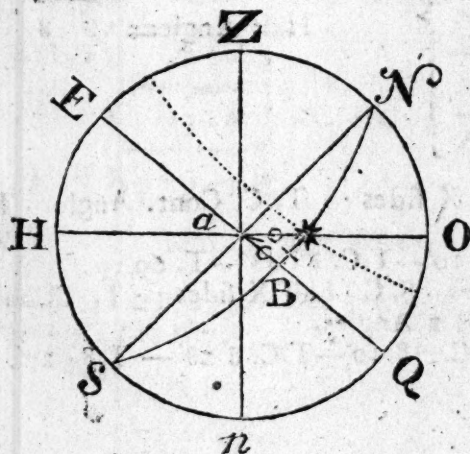
These may suffice for Problems relating to the *Sun* :
Next follows the Doctrine of the *Primum Mobile*, or Di-
urnal Motion of the Fixed Stars.

S E C T. II.

To find the Time of the Rising and Setting of the fixed Stars,
their Amplitudes, Azimuths, Hour of the Night, &c.

P R O B. I.

THE Latitude of a Place (suppose $51^{\circ} 30'$ North)
and a Star's Declination (admit the Star *Arcturus's*,
whose Declination is $20 \ 53'$ North) given; to find its
Amplitude, and Time of its rising and setting (*April 1*,
1732 ?)



This is projected
as *Prob. 8* of *Right
angled Astronomy* of
the *Sun* and its So-
lution the same as
there, viz.

1. As S.C. Lat. ..
S. Star's Declin. : :
R. .. S. Star's Ampl.
— S $38 \ 30'$ — S. 20
 $53'$ — S. 00 — S. 34°
 $56'$ E Nly.
2. As R. .. T. Lat.
: : T. Star's Decl. ..
S. Star's Asc. Diff.
— S.

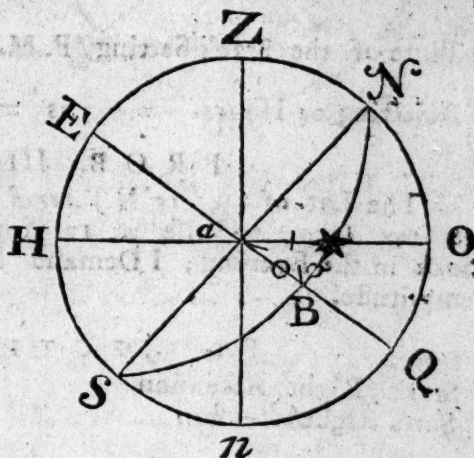
— S 90° — T. $51^\circ 30'$ T. $20^\circ 53'$ — S. $28^\circ 38'$ or in Time 1h $54' 3''$; which Added to 6h. is 7h. $54' 32''$.

P R O B. II.

The Latitude of a Place ($51^\circ 30' N^\circ$) and a Star's Amplitude (suppose the Star *Andromeda's Head*, on June 15, rose East $42^\circ N$.) given, to find his Declination and Ascensional Difference.

P R O J E C T I O N.

1. P.C.Q. Lat. set off, Axis and Equinoctial drawn. Make $a * = 42^\circ$ from the Half Tangents and thro' the points N, * and S, draw the Meridian Circle. N * S, and it's done. In the Triangle $a, B *$ are given the Hypo. $a * = 42^\circ =$ Star's Amplitude and the $\angle B a * = 38^\circ 30'$ the Comp Lat. to find the Perpend. $B *$ the Star's Declination and $a B$ the Base, its Ascensional Difference.



Therefore by Case 1, and 10, of *Right-angled Spherick Triangles*,

1. As $R \propto S. * \text{ Amp.} :: S.C. \text{ Lat.} .. S. * \text{ Declin. North.}$

— S 90° — S 42° — S. $38^\circ 30'$ — $24^\circ 37'$.

2. As T C. * Amp. $R \propto S.C. \text{ Comp. Lat.} .. T. * \text{ Ascensional Difference}$

— T.

206 *The Doctrine of the Sphere : Or,*

— T. C. 42° — S. 90° — S. $51^{\circ} 30'$ — T.
 $35^{\circ} 47'$ 2h. 23' 28"
 The Star's Right Ascension is 23h 52'
 The Sun's Right Ascension is 6 17

The Time the said Star comes to the S° . 17 35 00
 Star's Ascensional Diff. added to 6 h. subtr. 8 23 28
 Remainder is the Star's Rising in the Even. 9 11 32

Again,

To the Time of the Star's com. on the Merid. 17 35 00
 Add the Star's Ascensional Diff. $+ 6$ h. 8 23 28

Time of the Star's Setting P. M. 25 58 28
 24 00 00

Rejecting 24 Hours. is = 1 58 28

P R O B. III.

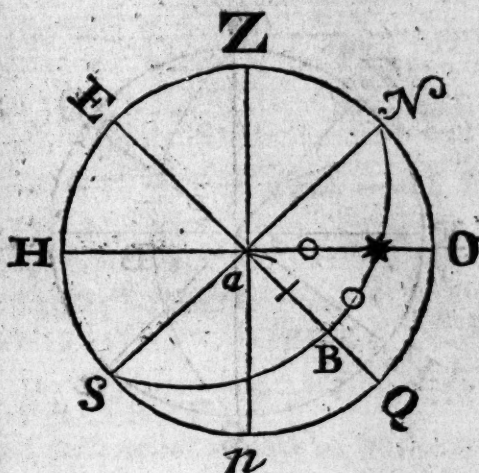
In The Lat. of ($44^{\circ} 15' N.$) August 14th, 1732. I found
 the Star *Lyra*; to rise at 17 Minutes after 4 of the
 Clock in the Evening; I Demand his Declination and
 Amplitude.

P R O J E C T I O N.

Star's Right Ascension 18h 18'
 Sun's Right Ascension 10 14
 Star's Coming to the Meridian 8 12 P.M.
 Star's Rising subtra& 4 17 P.M.
 Star's Ascensional Difference 3 55
 Or $58^{\circ} 45'$ = Therefore,

R. P. C. Q.

2. P.C.Q. Lat. c off, Axis and Equinoctial drawn, Make $aB = 58^{\circ} 45'$ from the Half Tangents and thro' the Points N, B, and S, describe the Meridian NBS. and it's done. Then in the Triangle $aB*$ are given the Base $aB =$ Ascensional $X \angle 58^{\circ} 45'$ the $\angle B a *$ at the Base $=$ Comp. Lat. $45^{\circ} 45'$ to find the Hypoth. $a *$ $=$ to the Amp. and Perpend. $B *$ equal to the Star's Declination, by Case 4, and 9, of *Right-angled Spherick Triangles*.



As T.C. Comp. Lat. .. $R :: S. * \text{ Ascen. } X .. T. * \text{ Declination } N.$

— $T. 44^{\circ} 45'$ — $S. 90^{\circ}$ — $S. 58^{\circ} 45'$ — $T. 41^{\circ} 10'.$

As $T. * \text{ Ascen. } X .. R :: S.C. \text{ Comp. Lat. } .. T.C. * \text{ Amp. } N.$

— $T. 58^{\circ} 45'$ — $S. 90^{\circ} - 44^{\circ} 15' - T. 22^{\circ} 56'$
Comp. to 90° is $67^{\circ} 04'$. That is,

The Star $\left\{ \begin{array}{l} \text{rises — E. } 67^{\circ} 04' \text{ N. } ? \\ \text{sets — W. } 67^{\circ} 04' \text{ N. } ? \end{array} \right\}$ in Latitude.

$44^{\circ} 15'$ North; when his Ascensional Difference is 3h. 55m. and his Declination is 4° deg. 10 m. North, found as above,

P R O B. IV.

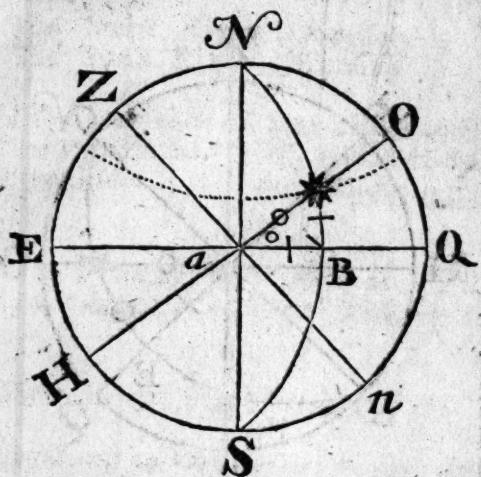
A Fixed Star's Declination: (as suppose the Star *Castor*, whose Declination is 32° deg. $33'$ N.) and his Ascensional Difference given (as admit, I observ'd the said Star *Castor* to rise 1h. 48' 44" on *February* 20, 1732.) to find thereby the Latitude of the Place, and the said Star's Amplitude.

CON-

CONSTRUCTION. With the Ascensional Difference

found as below, and
Declination of the
Star, 32 deg. 33' m:
you may project
and solve this Tri-
angle as in *Prob. 17*,
of *Right-angled A-*
stronomy, i. e.

By making $\angle B =$
Ascen. X. 52°
34 m. and drawing
the Meridian-Circle
thro' the points N,
B, and S, to cut the
Parallel of the Star's
Declination in *.



Lastly, drawing thro' a and $*$ the Horizon $H a O$, and at Right Angles thereto the Prime Vertical $Z a n$, in the Triangle $a B *$ are given AB the Ascen. $X 52$ deg. 34 m. and $B *$ the Perpendicular $= *$ Declination 32 deg. 23 m. to find the $\angle B a *$ $=$ Comp. Lat. and Hypoth. $a *$ $=$ Amplitude, by Case 6, and 8 of *Right-angled Spherick Triangles*.

Star's Right Ascension		7h 14'
Add		24 00
Sum		31 14
Subt. $\odot$ Right Ascension		22 55
		8 19
Add		12 00
Sum		20 19 00
Subt. Star's Rising		10 48 44
Star's Ascension X + 6h.		09 30 16
Abate		06 00 00
Star's Ascension X's =		03 30 16
in Time		X. 5
		17 31 20
		X.
Ascension X's =		52 34 00

1. *As* T. * Decl. .. *R* :: S. * Ascen. X .. T.C.
Comp. Lat.

— T. 32 deg. 33 m. — S. 90 deg. — S. 52 deg.
34 m. — T. 51 deg. 12 m. which is the Latitude of
the Place Northerly.

2. *As* *R* .. S. C. * Decl. :: S. C. * Ascen. X. ..
S.C. * Amplitude.

— S. 90 deg. — S. C. 32 deg. 33 m. — S. C.
52 deg. 34 m. — S.C. 30 deg. 47 m.

Whose Compliment to 90 deg. is $59^{\circ} 13'$.

That is, the Star's { E. $59^{\circ} 13' N.$ } at his rising { *W.W.R.*
Amplitude is { W $59^{\circ} 13' N.$ } at his setting {

PROB. V.

A Star's Ascensional Difference (suppose as in the last
Star *Castor's*, which is $52^{\circ} 34'$) on *Feb. 20, 1732*,) and Am-
plitude East $59^{\circ} 13'$ North, given; to find its Declinati-
on and Latitude of the Place of Observation.

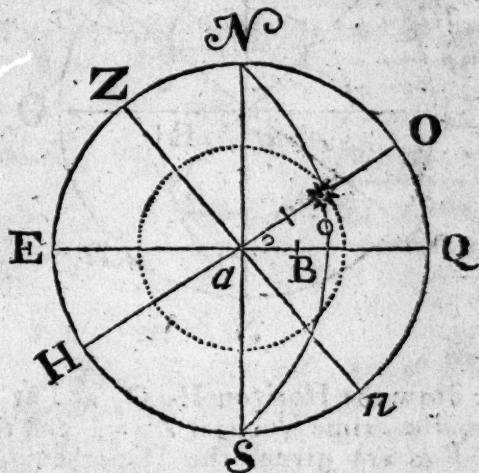
PROJECTION.

1. P.C.Q. Make *N* the North, *S* the South Pole, and
E & *Q* the Equino-
tial.

2. Make *a* B the
Base = $52^{\circ} 34'$, the
Sun's Ampl. from
the half Tangents;
and thro' the Pts.
N, *B*, and *s* draw
the Meridian *NBS*.

3. With the half
Tangent of $59^{\circ} 13'$
= Star's Ampli-
tude, and one Foot
in *a* describe a Pa-
rallel to the Pri-
mitive, to cut *NBS*
in *.

Lastly, Thro' *a* and * draw the Horizon *H* & *O*,
and at Right Angles to it, the Prime Vertical *Z* & *s*;
then in the Right-angled Triangle *a* B * are given the
Base *a* B $52^{\circ} 34'$, and Hypoth. use *a* * $59^{\circ} 13'$, to find
the Perpendicular B * = Decl. and Angle B * * at the
Base = Comp. Latitude, by Case 7, and 13, of Right-
angled Spherick Triangles, viz.



Practical Astronomy.

$\angle$ S * Amp. .. R : : S. Declin. .. S. Comp. Lat.
 $\text{---} 59^{\circ} 13' \text{---} 59^{\circ} \text{---} 32^{\circ} 23' \text{---} 38^{\circ} 47'.$
 Comp. to $90^{\circ} = 51^{\circ} 13'$ Latitude.

As s. c. * Amp. R : : s. * Declin. .. S. c. * Ascen.
 Difference.

$\text{---} \text{S. C. } 59^{\circ} 13' \text{---} \text{S. } 90^{\circ} \text{---} \text{S. C. } 32^{\circ} 23' \text{---} \text{S. C. } 3^{\circ} 25'.$
 Comp. = $52^{\circ} 35'$, Star's Ascensional difference required. Then,

To Star's Right Ascension

Add

h. 7 14
 24 00

Z

©'s Right Ascension subtract

31 14
 21 15

Feb. 20, the Star on the Meridian

Star's Ascen. X in time

3h. 30' 20"

Add

6 00 00

Star's Asc. diff. + 6h = 9 30 20

To the Star's coming on the Meridian

8h. 19' 00"

Add

12 00 00

Z

Star's Ascen. difference + 6h, subtract

20 19 00
 9 30 20

Star rises P. M. at

10 48 40

And added, is = Z

29 49 20

Subtract

24 00 00

Star sets A. M. at

5 49 20

PROB. VII.

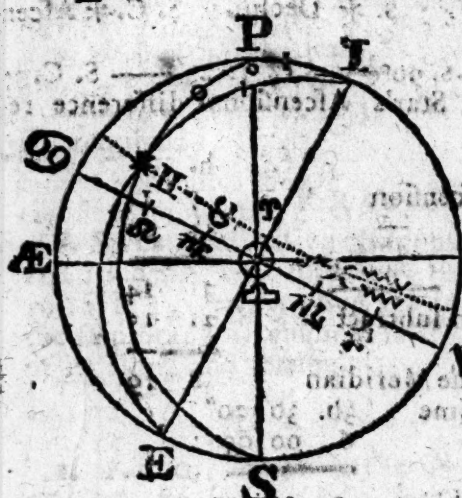
The Longitude, and Latitude of a fixed Star given, to find its Right Ascension and Declination.

Note, 1. The Longitude of a Star is an Arch of the Ecliptick, contained between the Beginning of Aries, and that Circle of Longitude, which passeth by the Star's Center; Or an Angle between the * and ☉ or ☿.

2. The Latitude of a Star is an Arch of a Circle of Longitude contained between the Center of the Star and the Ecliptick.

The Doctrine of the Sphere : Or,

EXAMPLE.



Anno 1732, I find
 the Longitude of
 the Star Apollo, or
 Castor, in the Head
 of the Twins Gemini,
 II, to be 15°
 32' in Cancer, G,
 whose Latitude is
 $16^{\circ} 40'$ N. I de-
 mand his Right As-
 cension and Decli-
 nation.

Explanation of the Scheme.

- P S Q The General Meridian or Solstitial Colure;
- E Q Q the Equinoctial.
- G V the Signifier, or Ecliptick, divided into the 12 Signs, and I, E, its Poles; P, and S, the Poles of the World.
- * the Star's place in the Ecliptick, being the Comp of its Longitude from G, and G its Longitude from S.
- P I = E S. the Dist. betwixt the Poles of the World and the Poles of the Ecliptick, or Signifier, = $23^{\circ} 30'$.
- I * the Comp. Lat. Apollo or Castor. M. A. and 111.
- The $\angle$ * I P its Longitude from G.
- The $\angle$ * P I Comp, of its Right Ascension.
- * the Comp. of its Declination.

PROJECTION.

1. P. C. Q. with the Equinoctial, E Q Q and Axis P S.
2. Set off $23^{\circ} 30'$ from E, to G and from P to I, and draw the Ecliptick G V (which you may divide into the 12 Signs by setting from the Center Q the Half Tangent of 30° and 60° both ways on the Ecliptick, and characterize it with the 12 Signs, as you see in the Scheme.) Draw also I Q E and Parallel of the *'s

Latit

Latitude $79^{\circ} 58'$ distant from I, and make the $\angle * I P = 14^{\circ} 28'$ the $*$ Longitude from $\odot$, (i. e. from γ to $\odot$ $30^{\circ} + 113^{\circ} + \odot 15^{\circ} 32' = 75^{\circ} 32'$ the Star's Longitude from γ .) to cut the Parallel of Latitude in $*$. Lastly, thro' the Points $P*$, and S , draw the Meridian $P*S$, and 'tis done. Then in the Oblique Triangle, $P*I$ are given, the side $I* = \text{Comp. its Lat. } 79^{\circ} 58'$ the Obliquity of the Ecliptick $P I = 23^{\circ} 30'$ and the Contained $\angle P I * 14^{\circ} 28'$ the Star's Longitude from $\odot$ to find the $\angle * P I$ its Right Ascension, and side $P* = \text{Compl. Star's Declination}$; by Prob. 3, of *Oblique Astronomy* foregoing.

PREPARATION:

side $I* = 79^{\circ} 58'$	$\angle P I * = 14^{\circ} 28'$
side $I P = 23^{\circ} 30'$	
Z <u>103 28</u>	Half $\angle$ is <u>07 14</u>
	<u>90 00</u>
Half Z <u>51 44</u>	Comp. is <u>88 46</u>
X <u>56 28</u>	

Half X 28 14
As S Half Z sides :: S Half X sides :: T.C. Half
Cont. $\angle$:: T Half X $\angle$'s.

— S . 51 deg. 44 m. — S . 28 deg. 14 m. — T. 82 deg. 46 m. — T 78 deg. 06 m.

As $S.C.$ Half Z sides :: $S.C.$ Half X sides T.C. Half
Cont. $\angle$:: T Half Z $\angle$'s.

— $S.C.$ 51 deg. 44 m. — $S.C.$ 28 deg. 14 m. — T. 82 deg. 46 m. — T. 84 deg. 54 m.

Then Half Z 84 54
Subt. Half X 78 06

Rem. is 6 48 the $\angle$ of $*$'s Position

Add Sum 163 00 the $*$ Right Asc. from γ

Subr. γ vs 90 00 $*$ Rt. Asc. from γ reqd.

Rem. 73 00

As S . $\angle$ Posit. :: Ob. Eclip. :: S $*$ Long. :: $S.C.$ $*$
Declination.

— S . 6 deg. 48 m. — S . 23 deg. 30 m. — S . 14 deg. 28 m. — S . 57 deg. 16 m.

PART II.

Q 2

Whole

Whose Comp. to 90° is $32^\circ 44'$ m. the Star's Declination North.

P R O B. VIII.

The Right Ascension and Declination of a fixed Star given, to find the Longitude and Latitude thereof.

E X A M P L E.

The Right Ascension of * *Caster* is $73^\circ 6'$ from γ and his Declination $32^\circ 44'$ N. I demand its Longitude and Latitude?

See the last Scheme.

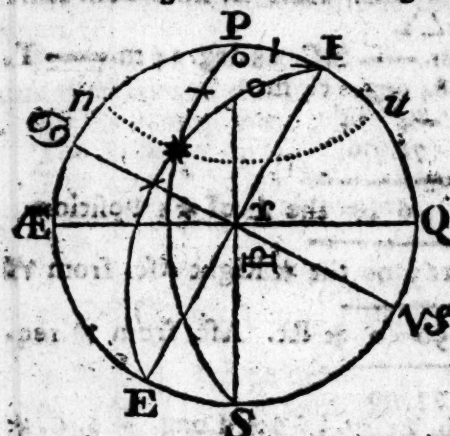
Here are given the side $P* = 57^\circ 16'$, side $PI = 23^\circ 30'$ with the Contain'd Angle $*PI (73^\circ + 90^\circ =) 163^\circ$, the *'s Right Ascension from VS . to find the $\angle$, $*IP$ its Longitude from $\mathfrak{S}$ and $I*$, its Latitude, after the same manner as in the last, or as in *Prob. 3*, of *Astronomy Oblique* foregoing. The Longitude is $14^\circ 28'$ from $\mathfrak{S}$, or from γ $75^\circ 32'$ m. and Latitude $10^\circ 10'$ m. N $^\circ$.

P R O B. IX.

The Longitude and Declination of a fixed Star given; to find its Latitude and Right Ascension.

E X A M P L E.

The Longitude of the Bright * in *Perseus*'s right side



is $27^\circ 08'$ m. of $\mathfrak{S}$ and its Declination is $48^\circ 36'$ m. N. What's its Latitude and Right Ascension?

Here, are given the Obliquity of the Ecliptick $PI = 23^\circ 30'$ the $\angle *IP = 32^\circ 52'$ (i.e. $\gamma 30^\circ + \delta 27^\circ 08'$ Comp. to $90^\circ = 32^\circ 52'$) *'s Long. from $\mathfrak{S}$, and $P* = 41^\circ 24'$ Comp. *'s Declin. viz.

two sides and an Angle Opposite, to find the $\angle *PI =$ its Right Ascen: and $I* =$ Comp. Lat. for the Projection and Solution of which see *Prob. 2*, of *Oblique*

Astr.

Practical Astronomy

Astronomy. Lat. is $30^{\circ} 08'$ Right Ascension is $134^{\circ} 48'$ from $\gamma 3$, or $44^{\circ} 48'$ from γ required.

P R O B. X.

The Latitude and Right Ascension of a fixed Star given; to find its Longitude and Declination.

E X A M P L E.

The Latitude of the Bright Star in *Perseus's* side is $30^{\circ} 08' N.$ and its Right Ascension from γ is $44^{\circ} 48'$ (or $2h. 59' 12''$) Its Longitude and Declination is required?

See the last Scheme.

Here are given the Obliquity of the Ecliptick $P I = 23^{\circ} 30'$ the Comp. Lat. $I * = 59^{\circ} 52'$ and Opposite $\angle * P I = 134^{\circ} 48'$ * Right Ascension from $\gamma 3$ (for $90^{\circ} + 44^{\circ} 08' = 134^{\circ} 48'$) to find $\angle * I P$ the Star's Longitude from $\gamma 3$, and $P *$ the Comp. of its Declination; for which see *Prob. 4. of Ob. Astronomy.*

Star's $\left\{ \begin{array}{l} \text{Long. from } \gamma 3 \text{ is } 33^{\circ} 42' \\ \text{Declination } 47^{\circ} 59' N^{\circ}. \end{array} \right\}$ required.

P R O B. XI.

A fixed Star's Latitude (suppose $30 \text{ deg. } 05' m. N$) his Declination ($48^{\circ} \text{ deg. } 36 m. N.$) and Right Ascension ($2h 59'$ or $44 \text{ deg. } 45 m.$ from γ , or $134 \text{ deg. } 45 m.$ from $\gamma 3$) Given; to find its Longitude, Obliquity of the Ecliptick, and Angle of Position

3. Describe a Parallel to an Oblique Circle 19 deg. 05 m. (= Posit) Distant from T. the last Pole found; so cut the Equinoctical in the point p , by Prob. 9, Case 3, of Stereog. which is the Pole of the requir'd Oblique Circle or Meridian.

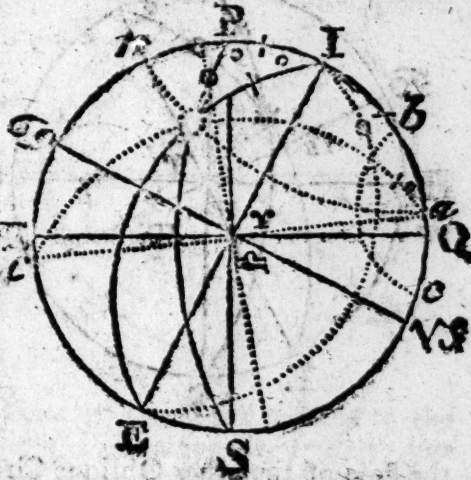
4. Measure $\overset{*}{P} \overset{*}{I}$ on the Half Tangents) which let be 44 deg. 45 m.) and set of its Comp. (viz. 45 deg. 15 m.) from $\overset{*}{P}$ to n , and thro' the points Pn and S draw the Oblique Circle or Meridian PnS , and it's done: Then in the Triangle $P \overset{*}{I}$, are given The two Angles $\overset{*}{I} P \overset{*}{I} = 32$ deg. 50 m. = $\overset{*}{I}$ Longitude $P \overset{*}{I} = 19$ deg. 05 m. = $\angle$ Posit. and Opposite side $P \overset{*}{I} = 23$ deg. 30 m. the Oblique Ecliptick, to find the side $\overset{*}{I} \overset{*}{S}$ Comp. Star's Latitude side, $P \overset{*}{S}$ Comp. of its Declination and $\angle \overset{*}{S} P \overset{*}{I} =$ Stars Right Ascension from Cancer by Prob. 6. of Oblique Astronomy.

The Star's { Declination is $48^{\circ} 36'$
Right Ascension $45^{\circ} 15'$ from γ } Req.
Latitude — $30^{\circ} 06' N^{\circ}$.

PROB. XIII.

The Latitude of a fixed Star ($30^{\circ} 05' N.$) And Angle of Position ($19^{\circ} 05'$) also the Distance of the Poles of the World, and that of the signifier, or Ecliptick $23^{\circ} \frac{1}{2}$ Given; to find its Longitude, Declination and Right Ascension.

Here are given the two sides $\overset{*}{I} \overset{*}{S} =$ Comp. Lat. of $\overset{*}{S}$ 59 deg. 55 m. $P \overset{*}{I}$ equal 23 deg. 05 m. $\angle$ Posit and the Oppos. $\angle \overset{*}{I} \overset{*}{S} P$ 19 deg. 05 m. to find the other two Angles $P \overset{*}{I} \overset{*}{S}$ equal Star's Longitude from γ , & $P \overset{*}{I}$ the $\overset{*}{S}$'s Right Ascension from γ , and side $P \overset{*}{S}$ the Comp. of its Declination; for the Projection and Solution of which see Prob. 15, Case 4, of Spher. Geom.



The Doctrine of the Sphere : Or,

Stars { Right Ascension from γ $44^{\circ} 48'$
Declination — $48^{\circ} 35'$ } Req.
Longitude from $\odot$ $32^{\circ} 50'$

P R O B. XIV.

The Obliquity of the Ecliptick $23^{\circ} \frac{1}{2}$ or $30'$ and a fixed Stars Declination (48 deg. 36 m. N.) and its Angle of Position (19 deg. 05 m. Given ; to find its Longitude, Latitude, and Right Ascension ?

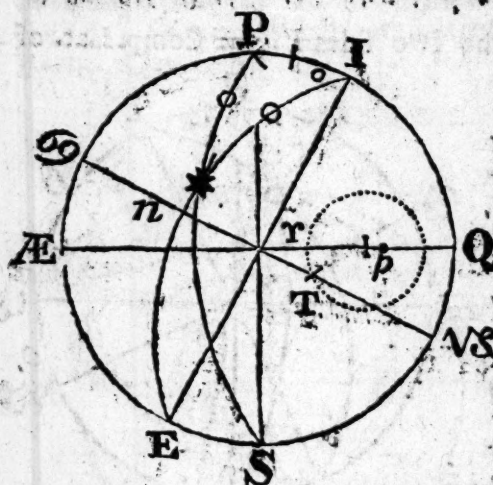
In this Prob. also are given, The side P^* equal 41 deg. 24 m. Comp. Declin. side PI equal 23 deg. 30 m. and its Opposite Angle P^*I equal 19 deg. 05 m. to find the $\angle^*$ equal Stars Longitude from $\odot$. $\angle^*PI$ its Right Ascension from γ . and I^* the Comp. of its Latitude, as in the last Problem.

Stars { Longitude from $\odot$ is $32^{\circ} 50'$
Latitude — $30^{\circ} 07'$ } Req.
Right Ascension from γ $44^{\circ} 49'$

P R O B. XV.

The Right Ascension of a fixed Star, (suppose the bright Star in *Perseus's* Right Side $134^{\circ} 45'$ from γ) and its Angle of Position ($19^{\circ} 05'$.) Given ; to find its Latitude, Longitude and Declination ?

Here are Given, the two Angles $IP^* = 134^{\circ} 45'$, $I^*P = 19^{\circ} 05'$, and



opposite side $PI = 23$ deg. 30 m. Obliq. Eclip. to find the side I^* the Comp. * s Lat. side P^* the Comp. of its Lat. and $\angle^*PI^*$ its Longitude from $\odot$, in Prob. 12th foregoing ; only in Projection of this you must describe the Parallel to an Oblique Circle 19 deg. 05 min. distant from the Pole P , to cut

the Pole of the other Oblique Circle, or Circle of Longitude in T .

The $\star$'s { Latitude is $30^{\circ} 00'$
Longitude is $32^{\circ} 56'$ from $\odot$ } Req.
Declination $48^{\circ} 27'$ North.

P R O B. XVI.

In the Latitude 51° deg. 32 m. N. April 1. 1732. the brightest Star in the Northern Crown comes to the Meridian 14 Hours after the Sun (or Afternoon) his Declination 27° deg. 51 m. N. I demand its Height and the Time when it will be due East or West?

P R O J E C T I O N.

1. P.C.Q. Lat. set off, Axis, Equinoctial, and Parallel of the Star's Declination, drawn as before taught. Then,

2. Thro' the Intersection of the Star's Parallel of Declination, with the prime Vertical at $\star$ and P and S, describe, the Hour-Circle or Meridian P $\star$ S.

Then in the Right-angled Spherical Triangle Υ B $\star$, are given the $\angle$ at $\Upsilon =$ Lat. 51° deg. 32 m. N. and B $\star$ the Star's Declination 27° deg. 51 m. N. to find $\Upsilon \star$ its Altitude, and Υ B the Time when it will be due East or West? by Prob. 8. of Astronomy.

As S Lat. .. S $\star$ Decl. :: Υ .. S $\star$ Alt. E. or W.

— S. 51° deg. 32 m.

— S. 27° deg. 51 m. S.

90° deg. — S. 36° deg.

39 m.

As Υ .. T. C. Lat.

:: T $\star$ Decl. S. of an

Arch.

T. 45° deg. — T. H

38° deg. 28 m. — T.

37° deg. 51 m. S. 24° deg.

51 m.

Whose Comp. to

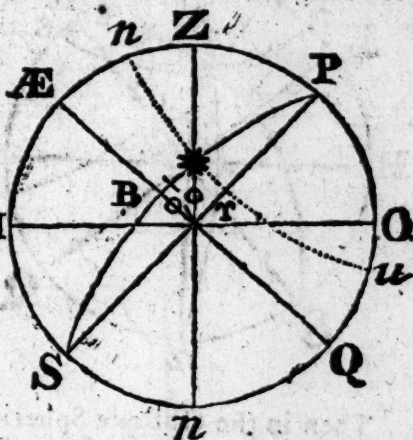
90° deg. 65° deg. 09 m.

converted into Time,

is $4^h 20' 28''$. This

subtracted from the Time of the Star's coming to the Meridian, gives the Time it will appear due East; but the said Arch of Time, added to the Time of its passing over the Meridian, shews the Time it will be due West. For Instance;

From



From the Time the Star is on the Meridian April 1. is $\frac{14}{00}{00}$
 Subtr the Arch last found $\frac{4}{20}{28}$

Time when the * is due East P.M. $\frac{9}{39}{32}$

Again,
 The Star on the Meridian $\frac{14}{00}{00}$
 Add the said Arch $\frac{4}{20}{28}$
 it gives $\frac{18}{20}{28}$
 Reject $\frac{12}{00}{00}$

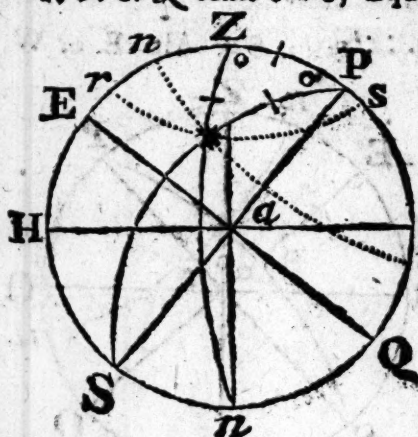
The Time the * is due West A.M. $\frac{6}{20}{28}$

P R O B. XVII.

The Altitude of a fixed Star (suppose the brightest in the Crown 45 deg. high) its Declination 27 deg. 51 m. N. and the Latitude of the Place (51 deg. $\frac{1}{2}$ North) given; to find its Azimuth, and the Hour of the Night April 1. 1732.

P R O J E C T I O N.

1. P. C. Q Axis P S, Equinoctial E ; Q and Parallel of Declination (62 09' from P the North Pole) drawn as 22.



2. Describe a Parallel of the Star's Altitude 45 deg. dist. from Z the Zenith; as $r s$, to cut the parallel of Declination in *.

3. Thro' the Points P, *, and S, draw a Meridian P-S; and thro' the Points Z, * draw an Azimuth-Circle Z-n.

Then in the Oblique Spherical Triangle ZP* are given, ZP equal to Comp. Lat. 38 30', Z* the Comp. of the Star's Altitude 45 deg. and P*, the Complement of its Declination 62 09'; to find the Angle *ZP, the Star's Azimuth from the North, and the Angle ZP*, the hour from Noon, by Prob. 10, of Oblique Astronomy. Or otherwise thus:

Then

Then say,

PREPARATION.

Comp. { Lat. $38^{\circ} 30'$ } At Ec. S.C. Lat. S.C. Alt. 4th Sn.
 { Alt. $45^{\circ} 00'$ } S. 90° — S. $38^{\circ} 30'$ — S. 45° — S. 26
 { Dec. $62^{\circ} 09'$ } 07'.

Sum $145^{\circ} 39'$

Half Z $72^{\circ} 49'$

Difference $10^{\circ} 40'$

At 4th S. S. half Z sides; $\therefore$ S. X. 7th Sino.

$26^{\circ} 07'$ — S. $72^{\circ} 49'$ — S. $10^{\circ} 40'$ — S. $23^{\circ} 41'$

Whose Logarithm is

9.603913

Radius added, is

19.603913

S.C. $50^{\circ} 40'$

9.801956

Doubl. is $101^{\circ} 20'$, the Star's Azimuth from the North at that Altitude.

2. For the Hour of the Night, 'twill be,

At S.C. Decl. S. Azim. S.C. Alt. S. Arch.

$52^{\circ} 09'$ — S. $101^{\circ} 20'$ — S. 45° — S. $52^{\circ} 57'$, the

Angle ZP*; which converted into Time, is 3h 33'.

This subtracted from the Star's Southing (14 h. (before-

found) gives the Hour of the Night 10h. 27', if the Star

were on the East side of the Meridian: But added to

the time of the Southing, gives 17 h. 33' (or 5h. 33') af-

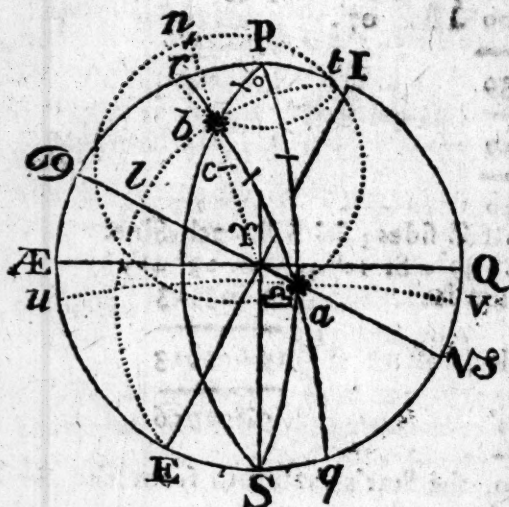
ter Midnight, the Hour of the Night when the said

Star is on the West side of, or is past the Meridian.

PROB.

P R O B. XVIII.

The Meridian Altitude, or Declination of an unknown Star or Comet, and its distance from a known fixed Star, given, to find the Latitude and Longitude of the said unknown Star.



Example.

In the Latitude of $51^{\circ} 32'$ North, the Meridian Altitude of an unknown Star is $30^{\circ} 36'$ (or its Declination $7^{\circ} 52'$) South

(for the Meridian Altitude $30^{\circ} 36'$ subtracted from the Comp. Lat. $38^{\circ} 28'$, remains the Declination $1^{\circ} 52'$;) Also his distance from the Star in *Cepheus's Girdle*, is $84^{\circ} 32'$ given, to find his Latitude and Longitude.

* *Cepheus's Girdle*, Longitude $1^{\circ} 36'$ of $\odot$, or $31^{\circ} 36'$ from Υ , its Declination $69^{\circ} 17'$ North, Comp. to $90^{\circ} = 20^{\circ} 43'$; *Cepheus's* distance from the North Pole, and its Right Ascension $21^{\text{h.}} 25'$, or $321^{\circ} 15'$.

P R O J E C T I O N .

1. P.C.Q. The Ecliptick $\odot \Upsilon \vee \Omega$, drawn; also the Obliquity thereof $P I$ set off, and $I \Upsilon E$ drawn; then,

2. Set off the known Star's Longitude, $31^{\circ} 36'$ from Υ to l , and describe the obscure Oblique Circle $I l E$.

3. Draw the Parallel of the Star's Declination, $r t 20^{\circ} 43'$ from P , the North Pole, to cut the last Circle in b ; and through P, b , and S , draw the Meridian Circle $P b S$.

4. Describe the Parallel of the unknown Star's Declination, as $u v 82^{\circ} 01'$ distant from S , the South Pole.

5. On b , as a Pole of an Oblique Circle describe a Parallel $n r 84^{\circ} 32'$ distant therefrom (to cut the last Parallel of Declin. $u v$ in a , the unknown Star) by Prob.

9. Case 3. of *Stereog.*

6. Thro'

6. Thro' P, α , and S, describe the Meridian of the unknown Star P α S. And lastly, thro' the two Points b and a describe an Oblique Circle $ra q$, by *Prob. 3*, of *Stereog.* and it's done. Then in the Oblique Spherick Triangle P $b a$, there are given three sides, viz. P $b = 20^{\circ} 43'$ (the Complement of the known Star's Declination) P $a = 97^{\circ} 52'$ (the unknown Star's distance from P the North Pole) and $b a = 84^{\circ} 32'$ (the distance between the two Stars, to find the Angle α P b (the difference of Right Ascension between *Cepheus's* Girdle and the unknown Star. Therefore *Prob. 10*, of *Oblique Astronomy*,

$$\begin{array}{l} \text{Side } \left\{ \begin{array}{l} P a = 97^{\circ} 52' \\ P b = 20^{\circ} 43' \\ b a = 84^{\circ} 32' \end{array} \right. \quad \begin{array}{l} \text{Or } 82^{\circ} 08' \text{ Sine Co. Ar. } 0.001105 \\ \text{Sine Co. Ar. } 0.451306 \end{array} \\ \hline Z \quad 203 \quad 27 \end{array}$$

$$\text{Half } Z \quad 101 \quad 43 \quad \text{Or } 78 \quad 17 \quad \text{Sine} \quad 9.990855$$

$$\text{Diff.} \quad 17 \quad 11 \quad \text{Sine} \quad 9.470455$$

S. Comp.

$$\begin{array}{r} Z \quad 19.916721 \\ \hline 24^{\circ} 42' \quad 9.958366 \end{array}$$

Which doubled, is

$$49 \quad 24 \quad \text{the } \angle \alpha P b.$$

And added to *Cepheus's* Rt. Asc. }
because the unknown Star is to the } 321 15
Eastward of the known Star

Reject a Circle, or

$$\begin{array}{r} 370 \quad 39 \\ \hline 360 \quad 00 \end{array}$$

Remains the Right Ascension }
of the unknown Star } 10 39

And now, having the unknown Star's Declination $82^{\circ} 08'$ South, and its Right Ascension $10^{\circ} 39'$, its Latitude and Longitude may be found by *Prob. 8*. foregoing.

P R O B. XIX.

The Latitudes and Longitudes of two known Stars, with the distance of a Planet, Comet, or New Star, from each of them being given; to find the unknown Star's Latitude and Longitude.

EX.

Then, In the Triangle OIM, are given the two sides
 $IO = 40^{\circ} 58'$ Comp. Lat. of the Swan's Bill $IM = 59^{\circ} 55'$ the Comp. Lat. of Perseus's side, and the Contained
 Angle OIM $= 120^{\circ} 33'$ the Difference of Longitude be-
 twixt the two Stars, to find the Angle MOI, and side OM,
 by Prob. 3. of Astronomy.

As S. Half Z. sides .. S. Half X sides :: T. C. Half
 Cont. $\angle$.. T. Half $\angle$ X

— S. $50^{\circ} 26'$ — S. $9^{\circ} 38'$ — T. — $39^{\circ} 44'$ —

As S.C. Half Z. sides .. S.C. Half X sides :: T.C.
 Half Cont. $\angle$.. T. Half $\angle$ X

— S.C. $50^{\circ} 26'$ — S.C. $9^{\circ} 26'$ — T. $39^{\circ} 44'$ —

Then $41^{\circ} \text{ deg. } 29 \text{ m.} \div 6^{\circ} \text{ deg. } 66 \text{ m.} = \angle \text{ MOI} =$
 $48^{\circ} \text{ deg. } 25 \text{ m.}$

As $\angle \text{ MOI} .. \text{S. side NM} :: \text{S. } \angle \text{ OIM} .. \text{S.}$
 side OM

— S. $48^{\circ} \text{ deg. } 25 \text{ m.}$ — S. — $59^{\circ} \text{ deg. } 55 \text{ m.}$ —
 S. $120^{\circ} \text{ deg. } 33 \text{ m.}$ — S. $85^{\circ} \text{ deg. } 02 \text{ m.}$

a. In the Ob. Triangle MNO are now given the three
 sides $MO = 85^{\circ} \text{ deg. } 02 \text{ m.}$ the Dist. between the two
 known Stars, ON the unknown Star's Dist. from the
 Swan's Bill, $= 49^{\circ} \text{ deg. } 05 \text{ m.}$ and MN equal $88^{\circ} \text{ deg. } 57 \text{ m.}$
 the Dist. from Perseus's sides, given; to find S. $\angle$
 MON, by Prob. 17.

Side { OM $85^{\circ} 02'$ S. Co. — Ar. 6.0016337
 ON $49^{\circ} 05'$ S. Co. — Ar. 0.0416719
 MN $88^{\circ} 57'$

Z $223^{\circ} 04'$

Half X $111^{\circ} 32'$ S. 95685783

Diff. $22^{\circ} 35'$ 9.5843615

Z 19.676454

S.C. $46^{\circ} 10'$ Half Z 9.8381227

Doubled, is $92^{\circ} 56'$ the $\angle \text{ MON}$ reqd.

To which add $\angle \text{ MOI } 48^{\circ} 25'$

Sum is the $\angle \text{ NOI} = 141^{\circ} 21'$

S. There

3. There are now given in the Triangle NOI the two sides IO equal 48 deg. 58 m. ON 49 05' and the Contained Angle NOI = 141 21', to find the Angle OIN, the Difference of Longitude the *Swan's Bill* and the unknown Star, and the side NI the Comp. of the unknown Star's Latitude, by *Prob.*

1. S. Half Z. sides .. S Half X. sides :: T.C. Half Cont. $\angle$.. T Half X. $\angle$ s.

S. 45 deg. 01 m. — S. 04 deg. 02 m. — T.C. 70 deg. 40 m. — T 82. 88 m.

2. S.C. Half Z. side .. S.C. Half X. sides T.C. Half Cont. $\angle$ T Half Z. $\angle$ s,

— S.C. 45 deg. 01 m. — S.C. 04 deg. 03 m. — T.C. 70. deg. 40 m. — T 160 20

Add the Half X above found 03 88

Sum is the Angle OIN = 18 20

To which add Long. of * *Swan's Bill* 27 20 V5

Subtract one Sine = 55 27

30 00

Unknown Star's Longitude is 25 27 of 20

As S. $\angle$ OIN .. S. side ON :: S. $\angle$ NOI .. S. side NI.

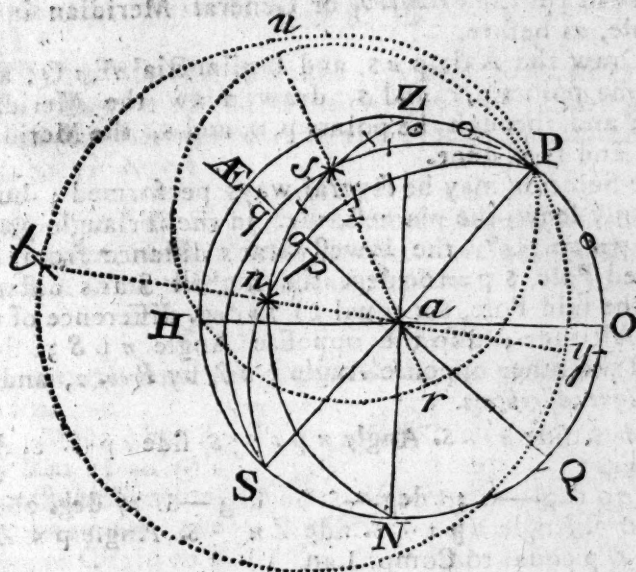
— S. 28 deg. 20 m. — S. 49 deg. 05 m. — S. 141 deg. 21 m. — S. 83 deg. 57 m. Which subtracted from 90 deg. remains 6 deg. 03 m. the Unknown Star's Latitude Northerly. *W.W.R.*

PROB. XX.

The Altitudes of two known fixed Stars when they are both in the same Azimuth, being Given; to find the Latitude of the Place of the Observation.

EXAMPLE.

In the Northern Hemisphere admit I had observed two known Stars in one Azimuth-Circle (*viz.* a Plumb-line being held up, cuts both the said Stars) and suppose the Altitude of the highest Star was 75 deg. this Star's Declination being 30 deg. North and Right Ascension from γ 80 deg. the other Star's Altitude 50 his Declination 25° South and Right Ascension from γ 60 deg. I demand the Latitude of the Place? **PROB.**



PROJECTION.

First, from the Right Ascension of one Star $a q = 80^\circ$ subtract the Right Ascension of the other $a p = 60^\circ$; remains their difference of Ascension $p q =$ Angle $n P S$; that is, 30° . Also from 90 , equal to $a E$, take Star's Rt. Ascension $a q$, equal to 80 , remains $q E 10^\circ$, equal to $\angle SPZ$, and $\angle n P S 30^\circ +$ the $\angle SPZ 10^\circ$ equal to $\angle n P Z$ equal to 40° , and the Comp. of the highest Star's Altitude $Z s$ equal to 15° . Therefore in the superior Triangle $Z P S$ are given the two sides $P s$ equal to 60° , the Comp. of the highest Star's Declination, $Z s$ Comp. of its Altitude 15° , and opposite Angle $Z P s$ equal to 10° , to find the Star's Azimuth from the North.

As s . side $Z s$.. $S. \angle s P Z :: S. s P$.. $S. \angle s Z P$ Obtuse Star's Azimuth.

— $S. 15$ deg. — $S. 10$ deg. — $S. 60$ deg. — $S. 35$ deg. 31 m. Comp. to 180 deg. equal to 144 deg. 29 m.

With which describe the Angle $s Z P$, or Azimuth-Circle $Z s n$.

2. Set off the highest Star's Altitude, as at s , the other at n on the said Azimuth-Circle, and on the points n , s , and n , as Poles of Oblique Circles, describe the Parallels $n p n$, and $H r p u$, to intersect each other in P , the North pole (or you need only describe the last Parallel distant from $S 60$ deg. Comp. of the highest Star's Declin.

clin. 'twill cut the *Primitive* or General Meridian in p , the Pole, as before.

3. Draw the Axis $p \alpha s$, and Equinoctial $\alpha n Q$, and thro' the points p, s , and s , draw draw the Meridian $p q s$; and through the points p, n , and s , the Meridian $p n s$, and it is done.

The Solution may be several ways performed; but I shall only shew the plainest, *viz.* in the Triangle $p n s$, given $p n = 115^\circ$, the lowest Star's distance from the elevated Pole, $s p = 60$ deg. the highest Star's distance from the said Pole, $s n$ equal to 70 deg. difference of the Star's Altitude; also the opposite Angle $n p s$ 30 deg. to find the other opposite Angle $p n s$, by *Prob. 2*, and 6, of *Oblique Astronomy*.

1. *As* S. side ns .. S. Angle $n p s$:: S. side sp .. S. Angle $p n s$.

—S. 70 deg.—S. 30 deg.—S. 60 deg.—S. 27 deg. 26 m.

2. *As* S. Angle $n p z$.. S. side $Z n$:: S. Angle $p n Z$.. S. side $Z p$ equal to Comp. Lat.

—S. 30 deg.—S. 85 deg.—S. 27 deg. 26 m.—S. 45 deg. 34 m. Comp. is 44 deg. 26 m. the Latitude required.

Having now laid down great Variety of Problems in Astronomy, both in relation to the Sun and fixed Stars, in a very plain Method, both as to their Construction and Calculation, as projected on the Plain of the General Meridian, and solstitial Colure, I proceed now to give some Projections on the Plains of the other Great Circles, *viz.* the Equinoctial, Horizon, Ecliptick, and prime Vertical; but shall omit the Calculations, to avoid prolixity, they being performed in the same manner as before is taught. *Vide* the *Projection of the Sphere* on each Circle.

PROB

P R O B. I.

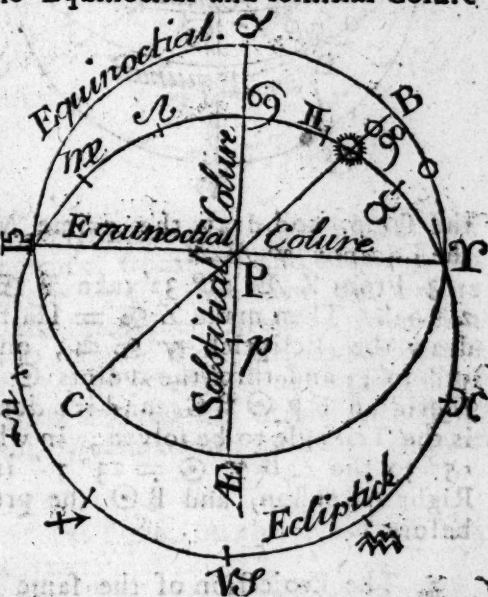
THE Sun's Longitude (25 deg. 30 m. of $\odot$) at his greatest Declination (23 deg. 30 m.) given to find his Right Ascension and present Declination, Lat. 51 32' North.

The Learner is desired to look back to the Projection of the Sphere on the abovesaid Circles, and also to the Astronomical Definitions.

P R O J E C T I O N.

1. P.C.Q. And the Equinoctial and solstitial Colure drawn.

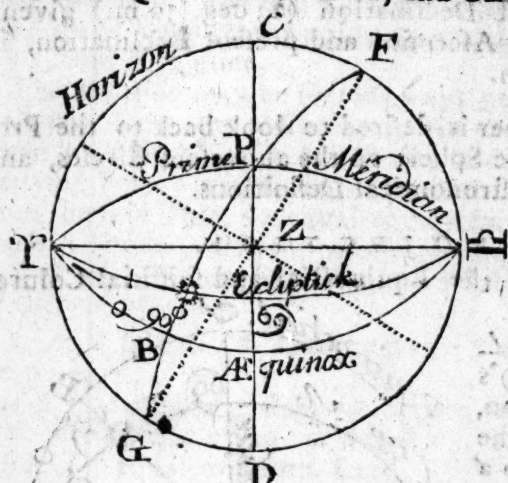
2. Make the $\angle \odot \gamma B = 23\ 30'$ $\odot$'s greatest Declination, and continue the Oblique Circle to a perfect Circle, as $\gamma \odot \vee S$; which you may divide into the 12 Signs, by dividing the Oblique Circle or Ecliptick by the Pole p , into every 30 Deg. from the point γ to $\odot$, II, &c. all round, by Prob. 6, Case 3. of Stereog.



3. After the same manner, by the said Prob. make the Hypothense $\gamma \odot$ equal to the Sun's Longitude from γ , viz. 55 deg. 30 m. and through $\odot$ and the Center p ; or the Pole of the World, draw the Meridian $B \odot p C$, and it's done. Then shall the primitive Circle $\simeq Q \gamma A$ represent the Equinoctial, and P , its Center, the North pole, $\simeq P \gamma$ the Equinoctial Colure, $Q P A$ the solstitial Colure, $B \odot p C$ the Sun's Meridian; and in the Triangle $\gamma \odot B$ are given $\gamma \odot$ Hypoth. Sun's Long. 55 30' $\angle$ adjacent $\odot \gamma B$ 23 30' Sun's greatest Declination, to find γB , the Base or Right Ascension, and $B \odot$ the Perpendicular or present Declin. by Case 1. of Right-angled spherick Triangles, or Prob. 1. of Astronomy foregoing.

2. To Project the same on the Plain of the Horizon, it's thus ;

1. P.C.Q. with $\gamma Z \perp$, and CZD. Here the Primitive $\gamma C \perp D$ is the Horizon, and Z the Center the Zenith.



2. The Distance from the Zenith to the Pole is the Comp. Lat. and from the Zenith to the Equinoctial the Lat. Ergo, Set off, or make $ZP = 38^{\circ} 28'$ Comp. Lat. from the Tangents and $Z \bar{A} = 51^{\circ} 52'$ from

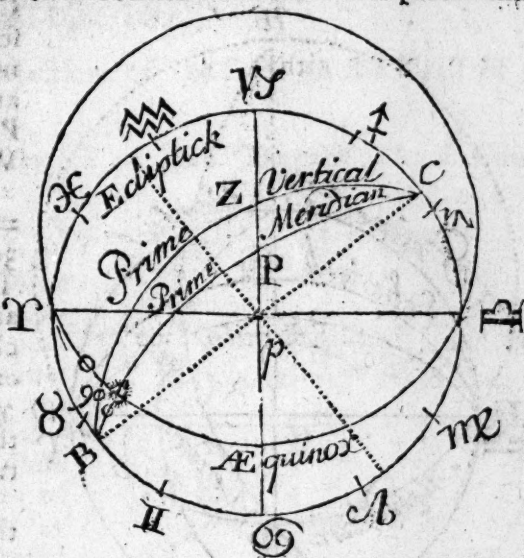
the same, and draw the prime Meridian $\gamma P \perp$, also the Equinox $\gamma \bar{A} \perp$.

3. From Z $\bar{A}$, $51^{\circ} 32'$ take $\bar{A} \bar{G} = 23^{\circ} 30'$; Remains $28^{\circ} 02'$. Then make $Z \bar{G} =$ Half Tangents $28^{\circ} 02'$ and draw the Ecliptick $\gamma \bar{G} \perp$; on which make $\gamma \odot = 55^{\circ} 30'$; and thro' the Points $\odot$ and P describe the Meridian $F P \odot B G$, and it's done; and then $\gamma B \odot$ is the Triangle to be solved; in which $\gamma \odot =$ Hypoth. $55^{\circ} 30'$ the $\angle B \gamma \odot = 23^{\circ} 30'$ is given; to find γB Right Ascension, and $B \odot$ the present Declination, as before.

3. The Projection of the same on the Plain of the Ecliptick.

1. P.C.Q.

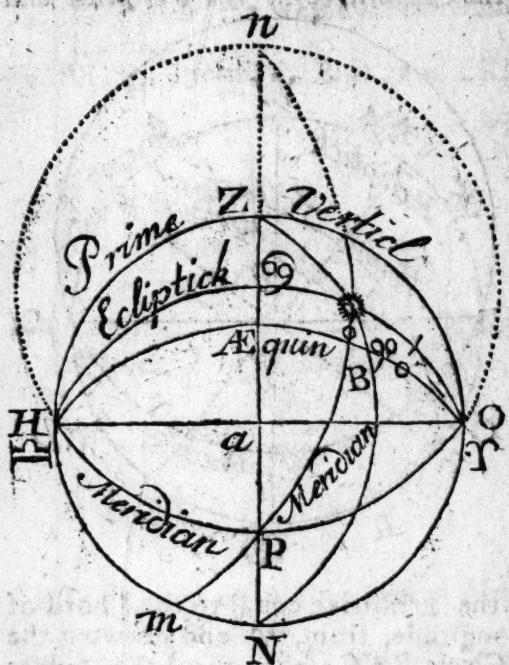
2. Make also
 $P \text{ } \Lambda \text{ } = \text{Half Tangent } 66^{\circ} 30'$
 the Distance from the
 Poles of the E-
 cliptick to the
 Equinoctial, and
 and draw the E-
 quinoctial Circle,
 as $\gamma \text{ } \Lambda \text{ } \omega$.



3. Make γB on the Primitive equal to the Chord of $55^{\circ} 30'$ the Sun's Longitude, from, γ and drawing the Diameter or Right Circle BPC ; then thro' the Points B and C , describe the Oblique Circle or Meridian BPC ; so shall $\gamma B \odot$ be the Triangle, in which are given $\gamma B = \text{Long. } 55^{\circ}$; $\angle B \gamma \odot$ equal 23° ; Sun's greatest Declination to find $\gamma \odot$ his Right Ascension, and $B \odot$ his present Declination by the former Problems.

4. The Projection of the same, on the plain of the Prime-Vertical Circle.

1. P.C.Q. Make $Z\bar{A}$ equal to the Lat. or $\bar{A} =$ to its



Comp. and describe the Equinoctial $\bar{A}\bar{E}\gamma$, and find its Pole P , the Pole of the World.

2. Make a $\odot = 38^{\circ} 28' + 23^{\circ} 30' = 61^{\circ} 58'$ (the Half Tangent) and draw the Ecliptick $\bar{A}\bar{E}\gamma$, on which make $\gamma\odot = 55^{\circ} 30'$ the Sun's Longitude from γ .

3. Then continuing the Meridian HPO to a perfect Circle, describe the Sun's Meridian thro'

the Points n , $\odot$, and P ; also the Azimuth Circle $Z\odot N$. and it's done, $O\odot =$ Hypoth $55^{\circ} 30'$ $\angle B O \odot$ the Sun's greatest Declination $23^{\circ} 30'$ $\angle B$ Radius given, to find OB the Base equal Sun's Longitude, and the Perp. $B\odot$ equal Sun's present Declination.

P R O B. II.

The Latitude ($51^{\circ} 30'$ N.) and the Sun's Declination ($23^{\circ} 15'$ N.) Given, to find his Amplitude, and his Rising and Setting, and his Ascensional Difference, consequently, his Rising, and Setting, with the Length of the Day and Night.

Projection on the Plain of the Equinoctial.

1. P.C.Q. And PZ made equal Half Comp. Lat. $38^{\circ} 30'$ and PH equal $51^{\circ} 30'$ from the Half Tangents. Describe the Circle AZQ , for the prime Vertical, and AHQ for the Horizon.

2. With the Half Tangent $66^{\circ} 45'$ Comp. Declination describe a parallel to the Primitive, or Equinoctial as $\odot BCD$. and it cuts the Horizon in $\odot$; then thro' the Center P , and $\odot$ draw the Right Circle or Meridian $P\odot M$, and it's done, so shall $A\odot M$ be the Triangle, in which $M\odot$ equal Declin. $23^{\circ} 15'$ $\angle MA\odot$ equal Comp.

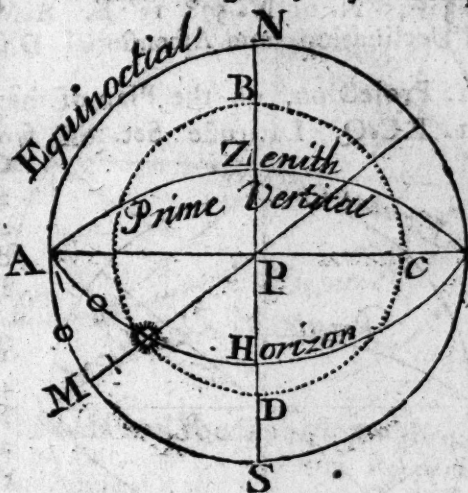
Lat.

Lat. $38^{\circ} 30'$ be given; to find $A \odot$ equal Amp: and AM equal Ascensional Difference.

2. The Projection of the same on the Plain of the Horizon.

1. P.C.Q. Make $ZP =$ Half Tangent $38^{\circ} 30'$ Comp. Lat. and $Z\bar{E} = 51^{\circ} 30'$ from the same, as before; and draw the prime Meridian APQ and Equinoctial $A\bar{E}Q$, producing it without the Primitive, to M.

2. On P. as a Pole with the Comp. of the Declin. $66^{\circ} 45'$ describe a Parallel to an Oblique Circle to cut the Primitive or Horizon, in $\odot$.



Lastly, Thro' the point $\odot$ and Center Z, draw the Right Circle, or Azim. $\odot ZS$; and then thro' the Points S, P, and $\odot$, describe the Sun's Meridian $SP\odot$, and produce it till it meet with the Equinoctial continued in M, so shall $AM\odot$ be the Triangle to be solved; in which $\odot M$ Perpend. $=$ Declin. $23^{\circ} 15'$ $\angle MA\odot$ the Comp. Lat. $= 38^{\circ} 30'$ are given; to find the Hypoth $A\odot =$ Amplitude, and AM the Base $=$ Ascensional Difference,

PART II.

P 4

11

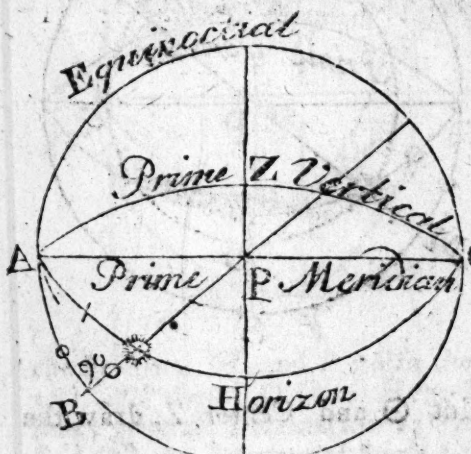
 *Note*, Any Arch, (Side) or Angle without the Primitive, are Measured by the same Rules as those within the same.

P R O B. III.

The Lat. of a Place ($50^{\circ} 30'$) and Sun's Altitude (E. $32^{\circ} 45'$ N. or N. $57^{\circ} 15'$ E. A.M.) Given; to find his Declination, and Ascensional Difference.

1. Projection, on the Plan of the Equinoctial.

1. P.C.Q. Latitude Set. off from P to H. and its Comp. from P to Z. and the Horizon, and prime Vertical, drawn, as in the last Prob.



2. Make A $\odot$ on the Horizon = $32^{\circ} 45'$, and draw the Sun's Meridian P.B. and 'tis done. In the Triangle AB $\odot$ are given A $\odot$ Hypoth. equal $32^{\circ} 45'$ Amp. $\angle$ BA $\odot$ equal Comp. Lat. $30^{\circ} 30'$

to find the Perpen. $\odot$ B the Declin. and Base AB. the Ascensional Difference.

2. The

2. The same projected on the Plain of the Horizon.

I. P.C.Q. The prime Meridian and Equinoctial

drawn, as in the last Problem. Then

make A ⊙ equal

to the Amp. 32°

45 and produce

the Equinoctial $\mathcal{A}$

A, and draw the
Sun's Meridian C

Sun's Meridian C
PQ continued till

© continued the
it coincide or in-

perfect the Equi-

no official without the

Primitive at B, the

(Right Angle,) and

its done ; then in

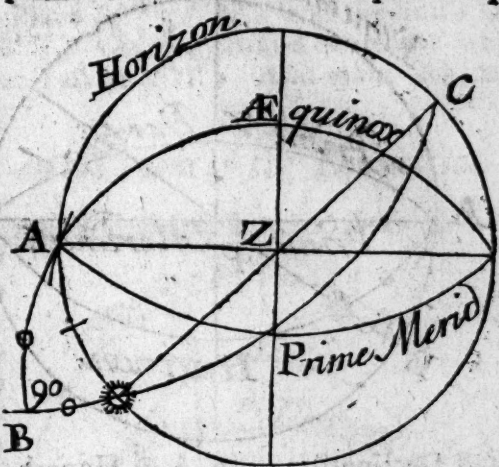
the Triangle ABC

are given $A \odot$ the

$\angle B A \odot$ equal $\angle$
 $\odot B$ the Destination

©B. the Declination
ence

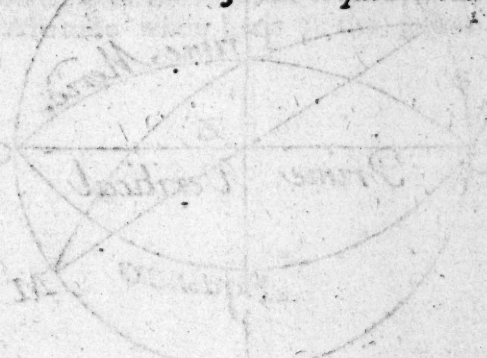
Fence.



P R O B. IV.

The Lat. ($51^{\circ} 31' \text{ N.}$) Ascensional Difference (30°)
Given; to find the Sun's Declination and Amplitude.

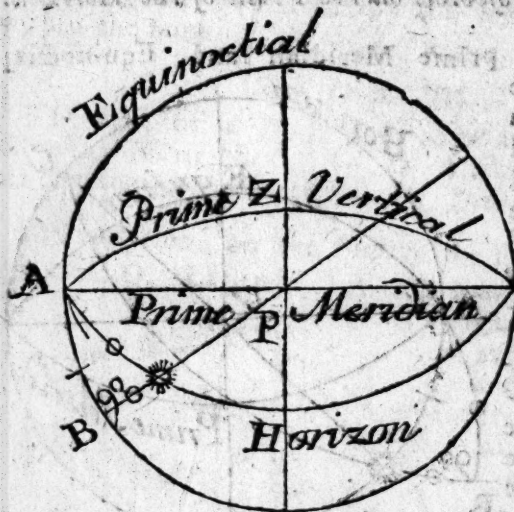
1. Projection on the Plan of the Equinoctial.



1. P.C.Q.

236 *The Doctrine of the Sphere : Or,*

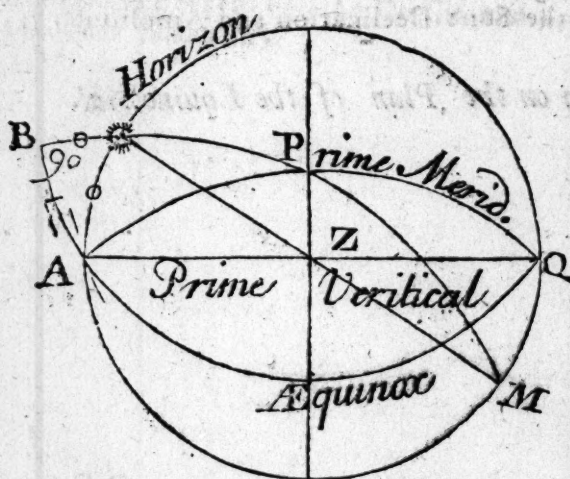
1. P.C.Q. Prime Vertical, and Horizon drawn as in Prob. 2.



Sun's Declination; also A $\odot$ Hypoth. equal Sun's Amplitude required.

2. To Project the same on the Plain of the Horizon.

1. P.C.Q. Prime Meridian, and Equinoctial drawn, as in Prob. 2.



2. Produce the Equinoctial towards B, and make AB equal 30° and thro' the two Points B and P. draw an Obl. Circle, or the Sun's Meridian BPM to cut the Primitive or Horizon in $\odot$; so shall AB $\odot$ be the Triangle without the

Primitive: In which are given, AB equal 30° , the Base equal Ascensional Difference $\angle$ BA $\odot$ equal $38^\circ 28'$ Comp. Lat. to find $\odot$ B the Perpendicular, equal Declination

nation and Hypoth. $A\odot$, equal Sun's Amplitude, as before.

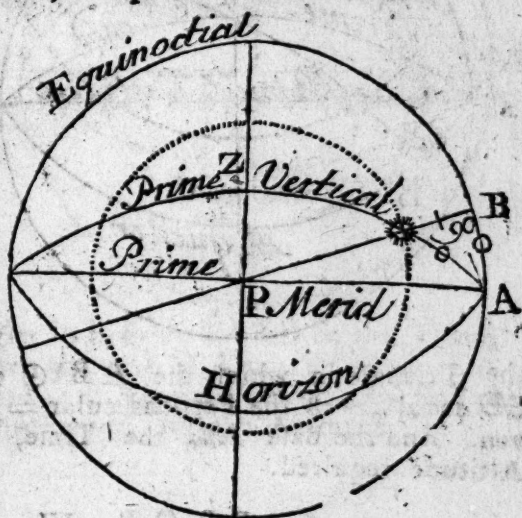
P R O B. V.

The Lat. of a Place ($50^{\circ} 31' N^{\circ}$) and the Sun's Declination : ($22^{\circ} 58' N.$) Given ; to find the Time when the Sun will be due East or West : And what Altitude he then will have ?

I. Projection on the Plain of the Equinoctial.

1. P.C.Q. Horizon, and prime Vertical drawn.

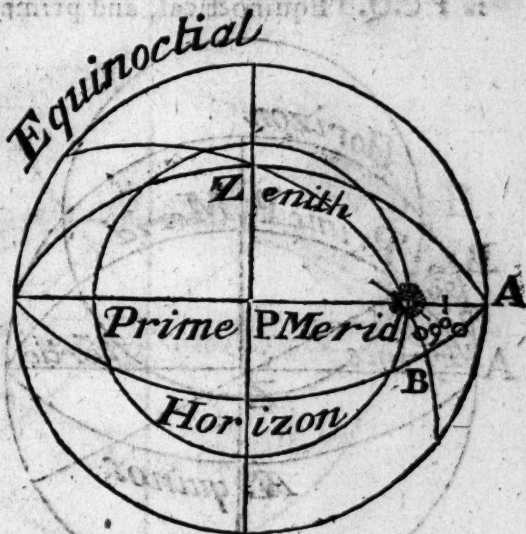
2. Describe a Circle parallel to primitive $67^{\circ} 02'$ (Comp. Declination) dist. from P. the Pole, to cut the prime Vertical Circle in $\odot$. Then through $\odot$ and the Center P draw the Sun's Meridian $P\odot B$, and 'twill form the Triangle $AB\odot$, in which are given, the Perpendicular $B\odot =$



Sun's Declination $22^{\circ} 58'$ $\angle BA\odot$ equal $50^{\circ} 31'$ the Lat. to find AB the Base equal to the Time from 6, and Hypo $A\odot$ the Sun's Altitude when he is on the prime Vertical.

1. Projection on the Plain of the Equinoctial.

1. P.C.Q. Horizon, prime Vertical, and Parallel of Declination $66^{\circ} 31'$ Dist. from P the Pole of the Equinox or Primitive, being described, will cut the prime Meridian in the Point $\odot$.



2. Thro' the Point $\odot$ and Z the Zenith, draw the Sun's Azimuth-Circle Z $\odot$ B to cut the Horizon H BA at Right Angles in B. So shall AB $\odot$ be the Triangle, in which are given the $\angle B \odot A = 52^{\circ} 30'$ the Comp. Lat. Hypoth. $\odot A = 23^{\circ} 29'$ Declination, to find $\odot B$ Perpend. the Altitude and AB the Azimuth.

1. Projection on the Plain of the Equinoctial.

1. P.C.Q. Prime Vertical, and Horizon drawn and produc'd to B.

2. Make

A $\odot$ Hypo-

path $= 33^\circ$

45' (the

Time from

Noon) and

draw the

Sun's Meri-

dian $\odot$ P.C.

3. Thro'

the Points

CZ and $\odot$

describe an

Azimuth C

Z $\odot$ and

produce it

to B; so shall

AB $\odot$ be the

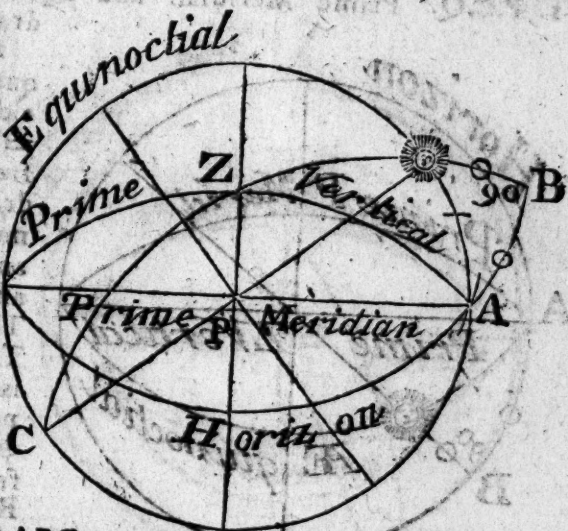
Triangle, in

which are given

$\angle$ BAC $= 39^\circ 10'$ Comp. Lat. Hypoth. AC $= 33^\circ$

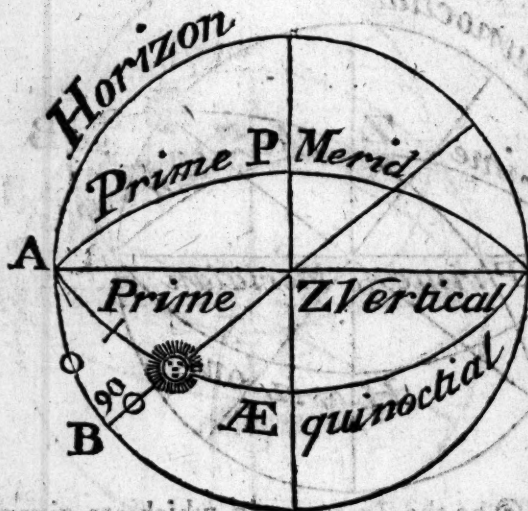
45' Hour from Noon to find AB the Base or Az-

imuth and BC, Perpendicular or Altitude.



2. To Project the same on the Plain of the Horizon.

1. P.C.Q. Prime Meridian and Equinoctial being drawn ;



2. Make A $\odot$ on the Obl. Circle or Equinox $= 33^{\circ} 45'$ (hour from Noon and draw the Azimuth thro' $\odot$ and Z. as B $\odot$ Z, and 'tis done. Then in the Triangle AB $\odot$ is given the Hypoth. A $\odot$ $= 33^{\circ} 45'$ the hour from Noon $\angle$ BA $\odot$ equal to the Comp. Lat.

to find AB Base = Azimuth and B $\odot$ the Perpendicular or Altitude. Thus much may suffice for Practical Problems in Right-angled Astronomy. I shall insert but two more in Oblique Astronomy projected on the Plain of the Equinoctial, and Horizon; and leave the rest to the Learners Industry.

P R O B. VIII.

The Lat. of a Place ($51^{\circ} 30' N$) Sun's Declination ($18^{\circ} 27' N.$) and Hour of the Day ($\frac{1}{2}$ past 1. P.M. or $\frac{1}{2}$ past 10. A.M. each of which are 1h. and half from Noon) Given; to find his Altitude and Azimuth, at that Time?

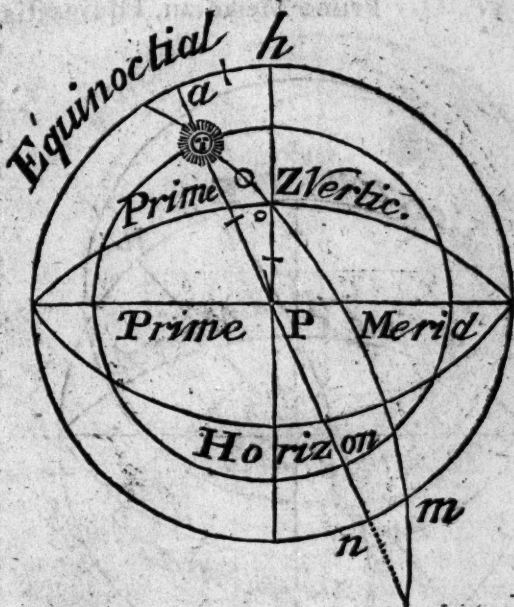
Projection on the Plain of the Equinoctial.

1. P. C. Q. Prime Vertical, Horizon, and Parallel of Declination $71^{\circ} 33'$ Dist. from the Pole P or Center; being drawn.

2. Make $ba = 21^{\circ} 30'$ the Hour from Noon, and draw the Sun's Meridian aPn to cut the Parallel of Declination in $\odot$.

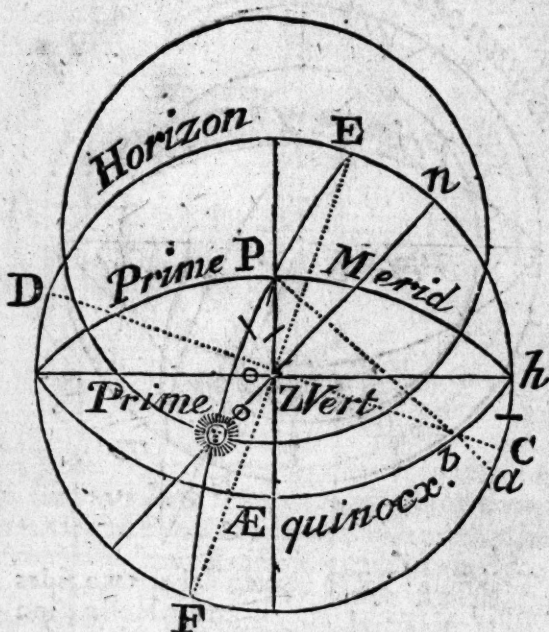
3. Thro' the Points $\odot$ and Z, draw an Azimuth, as $\odot Z m$, and it's done. Then

in the Oblique Triangle $PZ\odot$ given the two sides $PZ = 38^{\circ} 30'$ Comp. Lat. $= 71^{\circ} 33'$ Comp. Declination and contain'd $\angle Z P \odot 21^{\circ} 30'$ Hour from Noon, to find $Z \odot =$ Comp. Altitude and $\angle P Z \odot$, the Azimuth.



2. To Project the same on the Plain of the Horizon.

1. P.C.Q. Prime Meridian, Equinoctial, and Parallel of Declination $71^{\circ} 33'$ Distance from P being drawn as before.



2. Make b equal $22^{\circ} 30'$, and laying a Ruler over a and P , 'twill cut the Equinoctial in b ; then thro' b , and the Center Z draw the Right Circle CZD , and at Right Angles to it the Right Circle FZE .

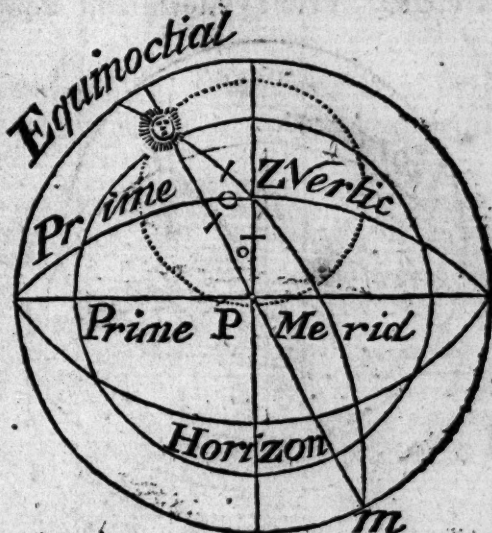
3. Thro' the Points EP and F draw the Oblique Circle, or Sun's Meridian EPE , which cuts the Parallel of Declination in $\odot$. Lastly, thro' the Center Z and $\odot$ draw the Azimuth-Circle $\odot Z n$, and 'tis done. Then in the Oblique Triangle $Z\odot P$ are given $ZP = 38^{\circ} 30'$ $P\odot = 71^{\circ} 33'$ and Cont. Angle $\odot PZ = 22^{\circ} 30'$ the hour, to find $Z\odot$ Comp. Altitude and $\angle \odot ZP$, the Azimuth as before.

P R O B. IX.

The Latitude of the Place ($51^{\circ} 30' N.$) the Sun's Declination ($16^{\circ} 27' N.$) and Altitude ($52^{\circ} 30'$) given; to find the Azimuth, and Hour of the Day.

1. Projection on the Plain of the Equinoctial.

1. P.C.Q. Prime Vertical, Horizon and Parallel of Declination $70^{\circ} 33'$, Distance from the Pole or Center P being drawn. Then 2dly, describe a Parallel of Altitude $= 37^{\circ} 30'$ (Comp. Alt.) Distance from Z the Zenith to cut the Parallel of Declination in $\odot$.



3. Thro' the Points $\odot$ and Z describe an Oblique-Circle or Azimuth $\odot Z m$, and it's done. Then in the Triangle $PZ\odot$ are given, the Three sides $PZ = 38^{\circ} 30'$ Comp. Lat. $P\odot = 41^{\circ} 33'$ Comp. Declination $Z\odot = 37^{\circ} 30'$ to find the Angles $\odot Z P$ the Sun's Azimuth and $Z P \odot$ the Hour of the Day.

2. To Project the same on the Plain of the Horizon.

1. P.C.Q. Prime Meridian, Equinoctial and Parallel of Declination $71^{\circ} 33'$ Dist. from P. being drawn.



2. Describe a Parallel of Altitude $37^{\circ} 30'$ distance from Z the Zenith, the Center to cut the Parallel of Declination in O.

3. Thro' the Points O and P. describe the Oblique Triangle O P n, and it's done. Then in

the Oblique Triangle O Z P, are given the Three sides $PZ = 38^{\circ} 30'$ $PO = 71^{\circ} 33'$ Comp. Declination and $ZO = 37^{\circ} 30'$ Comp. Altitude to find the $\angle O Z P$, the Azimuth and $\angle OPZ$ the Hour of the Day.

Thus much for *Practical Astronomy by Construction and Calculation*. Next, to solve the practical Problems by *Inspection*.



S E C T. II.

The Practical Questions in Astronomy solved by Inspection in our Tables following by a general Example, viz.

November 14th 1732, In the Latitude of 43° N. I Demand the Sun's Declination, Amplitude, Rising and Setting, Length of the Day and Night, the Sun's Right Ascension, &c.

1. In the Table of the Sun's Declination, look for the Year, and Month November at Top, and the Day of the Month 14, in the side Column, and in the common Angle of Meeting you have the Sun's Declination $20^{\circ} 52'$ South.

2. After the same manner, in the Table of the Sun's Right Ascension, you'll find the Sun's Right Ascension November 14, to be 16h. 03m.

3. To find the Sun's Amplitude, which is the Distance of the Rising or Setting of the Sun (or Star,) from the East or West Points of the Horizon.

E X A M P L E. I.

Let it be required to find the Sun's Amplitude on the 20th of October 1732. In the Lat 43° N. the Sun's Declination for that Time is 14° S. Therefore look for the Declination 14° in the first Column, and the Lat. 43° at the Head of the Table, and in the Angle of meeting is the Sun's Amplitude $19^{\circ} 18' S.$ that is, the Sun rises E. $19^{\circ} 18' S.$ and sets W. $19^{\circ} 18' S.$ for the Sun's Amplitude is always of the same Name with his Declination.

E X A M P L E. II.

The Lat. 43° N. and Sun's Declination (suppose $20^{\circ} 25'$ North) what is his Amplitude?

Here because the Declination is not an even Degree, we must take the Proportional part for the odd Minutes of the Sun's Declination. Thus;

Sun's Amplitude } for { 20° } Declination is { $27^{\circ} 53'$
 tude in Lat. 43° } { 21 } { $29 27$

Subt. and their Difference is $1 34 = 94'$

Then say, by Proportion,

As $1^{\circ} = 60'$.. $94' :: 25' .. 39'$ Then,

Lat. 43° Declination 20° @ Amplitude is $27^{\circ} 53'$
 To it Add the proport. Part above $0 39$

Gives Amplitude for Declination $23^{\circ} 25' = 28 32$ N.

That is, the @ { rises } E. $28^{\circ} 32'$ } North.
 { sets } W. $28 32$ }

EXAMPLE. III.

Let the Lat. be $43^{\circ} 30'$ N. and Declination, as in the last Example, $20^{\circ} 25'$ N. What is the Sun's Amplitude?

For Lat. 43° and Decl. $20^{\circ} 00'$ { $27^{\circ} 53'$ } { $60' .. 94' :: 25$
 the Ampl. is { $29 27$ } { $.. 39'$ Facit.

For Lat. 43° and Decl. $21^{\circ} 00'$ $29 27$
 And $27^{\circ} 53' + 35' = 28^{\circ} 32'$ the @'s ———
 Amp. for $20^{\circ} 25'$ N. Decl. in Lat. 43° N. : $34 = 94'$

Again,

For Lat. 43° and Declination { 20 } { $28^{\circ} 23'$ } { Amp.
 { 21 } { $29 46$ }

Their Difference is = $1 23$

As $60' .. 83' :: 25' .. 34'$ prop. Part.

And $28^{\circ} 23' + 34' = 28^{\circ} 57'$ the Amplitude in Lat. 44° with $20^{\circ} 25'$ Declination.

Now, because the Given Lat. $43^{\circ} 30'$ is in the middle, between 43° and 44° , therefore the Medium of the Amplitudes before-found is the Amplitude required, Thus;



Lat.

Lat. $\left\{ \begin{array}{l} 43^{\circ} \\ 44 \end{array} \right\}$ Declination $20^{\circ} 25'$ $\left\{ \begin{array}{l} 28^{\circ} 32' \\ 28 57 \end{array} \right\}$
the Amplitude is

$$\text{Sum} = \underline{57 \ 29}$$

Half the Sum is the Amp, requir'd $28 \ 44$

Thus may the Amplitude be found for any Odd Minutes of Lat. or Declination, tho' the Table is calculated for whole Degrees only.

EXAMPLE IV.

In the Lat. of 43° N. the Sun's Declination being $23^{\circ} 30'$ N. I demand the Time of the Sun's Rising and Setting, with the Length of the Day and Night? Find the Lat. 43° at the Head of the Table, (of the Sun's Rising and Setting) and the Degrees of the Sun's Declination in the Left-Hand-Column; and in the Common Angle of meeting, you have the Sun's Setting 7h. 36m. because the Declination is North; otherwise it would be the Sun's Rising, if the Declination had been South.

Sun setting	—	—	7h 36'
Subt. from	—	—	12 00
Sun-rising is	—	—	4 24
Sun-setting doubled	}		15 12
is the Length of the Day	}		
Sun-rising doubled	}		8 48
is the Length of the Night	}		

 If there be Odd Minutes of the Latitude, or Declination, or both, you may find the Proportional part, after the same manner as was exhibited in Example 2, and 3, of the Sun's Amplitude.

EXAMPLE V.

The Sun's Declination being 14° N. and his Amplitude at Rising E. 20° N. I demand the Latitude of the Place of the Observer?

In the Table of Amplitudes look the Declination 14° in the left hand Column; and running along the same Line till you meet with 20° Amplitude, then at the Head of the said Column you have the Lat. 45° N.

PART II.

Q 4

Ex.

EXAMPLE VI.

The Sun's Declination $23^{\circ} 29'$, I found the Sun to set at 7h 36'. I demand the Latitude of the Place? In the Table of the Sun's Rising, and Setting, look for the Declination $23^{\circ} 29'$ in the left hand Column; and running along the same Line, till you meet with 7h, 36m, then at the Head of this Column you'll have (43°) the Lat. North.

EXAMPLE VII.

To find when any known fixed Star comes on the Meridian, do thus; by this *Rule*.

Having look'd the Right Ascension of both the Star and the Sun in their proper Tables, then subtr. the Right Ascension of the Sun from the Right Ascension of the Star: But if the Star's Right Ascension be less than the Sun's, (so that you cannot subtract, because the Sun and the Star are on contrary sides of the first Point of *Aries*) add thereto 24 Hours, and then subtract, and the Remainder, if under 12 Hour, is the Time of that Stars coming on the Meridian, Afternoon; but if the Remainder be more than 12 Hours, subtract 12 Hours from it, and the Remainder is the Time after Mid-Night.

Or Thus;

To the Comp. of the Sun's Right Ascension, add the Right Ascension of the Star; the Sum is the Time of that Stars Southing. If the Sum exceed 12, or 24 Hours, take 12 or 24 Hours from it, and the Remainder is the Time of the said Stars Southing.

See the Operation.

On *December 1. 1732* I demand at what Time the Bull's South Eye or *Aldebaran* comes to the (South or on the) Meridian.

	h	m
Right Ascension of the Bull's Eye.	4	17
Add	24	00
Sum =	28	17
Subt. Sun's Right Ascension <i>December 1.</i>	17	15
Bull's Eye South at Night	11	02
	Or,	

Or thus;

	h. m.
To the Comp. © Right Ascension	6 45
Add the Right Ascension of the *	4 17
It gives as before	11 02

EXAMPLE VIII.

To find at any Time, what Star is then upon the Meridian.

To the Sun's Right Ascension, add the Time from Noon given, and the Sum is the Right Ascension of that Point of Heaven, that will come to the Meridian at the proposed Time; with which enter the Table of the Star's Right Ascension, &c. and see what Star hath such Right Ascension, or nearest thereto, and that is the Star sought for.

So on November 6, 1732. let it be required to know what Star will be upon the Meridian at 12 a-Clock at Night?

	h
Sun's Right Ascension	15 27
The Time from Noon	12 00
Sum =	27 27
Reject.	24 00
Star's Right Ascension =	3 27

The Star that is nearest in Right Ascension is the brightest of the seven Stars or *Pleiades*, whose Right Ascension is 3h, 28', *vere*.

EXAMPLE IX.

December 4, 1732. I demand the Time of the Rising and Setting of the *Bull's Eye* or *Aldebaran*, whose Declination is $15^{\circ} 48'$ North, in the Latitude of 43° North.

Look in the left side of the Table (of *Sun's Rising and Setting*) (nearest to $15^{\circ} 48'$) of Declination, and the Latitude 43° in the Head thereof; and in the Common Angle of Meeting you have the Stars Semi-diurnal Arch, (or Half the Time) that Star doth continue above the Horizon in that Latitude, or the Distance of Time that Star is in going from the Horizon to the Meridian on the East side; likewise from the Meridian

252 Of Practical Astronomy by Inspection.

Meridian to the Horizon on the West side of the Meridian; in this Case 7h. 02m.

Now if you substract these Hours and Minutes from the Time of the Star's coming to the Meridian (which will be found by *Example 7*, to be at) 10h 48' the remainder will be the Time of the Star's Rising; and if you add, the Sum will be the Time of the Star's Setting,

OPERATION.

	h	'
The Time of the Star's Southing	10	48
Its Semi-diurnal Arch substract	7	02
Time of the Stars Rising in the Evening	3	46
Added is	17	50
Reject	12	00
Time of the Stars Setting in the Morning	5	50

Note, 1. If the Sum of the Addition exceed 12 Hours, cast away the 12 Hours.

2. And when you can't substract, add 12 Hours to the Stars Southing, and then substract; what remains, is its Setting.

3. If you are in North Latitude, and the Star hath South Declination, the Tables give the Star's Semi-nocturnal Arch; which substract from 12 Hours, the Remainder is the Stars Semi-diurnal Arch, as before.

4. If You are in South Latitude and the Star hath North Declination, do the same as when you are in North Latitude and the Star hath South Declination.

EXAMPLE X.

In the Latitude 52° N^o. the Brightest of the Seven Stars or *Pleiades* Declination is $22^{\circ} 55'$ N. I demand its Amplitude?

Look in the Left side Column (of the Table of the Sun's Amplitude) for 23° (being nearest to $22^{\circ} 55'$) the Stars Declination, and in the Head thereof for 52° of Latitude, and the Angle of meeting, you'll find the said Stars Amplitude to be $39^{\circ} 24'$ North, because the Stars Declination is North.

Thus may you find by these Solar Tables the Rising, Setting, and Amplitude, of any fixed Star whose Declination exceeds not that of the Sun's, viz. $23^{\circ} 29'$. To which Tables you have here annexed an Astral Table of the principal Stars in the Northern, Southern, and Zodiacal Constellations, their Longitudes, Latitudes, and Magnitudes, the use of which needs no Explanation, but is Obvious to the meanest Capacity of the Young Student.

The Poets have a Three-fold Rising and Setting of the Stars.

Viz. $\left\{ \begin{array}{l} \text{Cosmical} \\ \text{Acronical} \\ \text{Heliacal} \end{array} \right\}$ Rising and Setting of the Stars.

See *Virgil* in his first Book of *Georgicks*, and *Ovid* in his first Book *De Ponto* and in his Second Book of *Fasts*.

1. Cosmical (from *Cosmos* the World) A Star is said to rise Cosmically when it rises with the Sun. So *Virg. Geor. Lib. I.*

Candidus auratis aperit cum cornibus annum,
Taurus, &c.

i. e. When the Sun enters *Taurus*, and they both rise at once.

2. And a Star sets Cosmically when it sets at Sun-Rising or rises at Sun-setting. So *Virg. Geor. Lib. I.*

Antè tibi Eoa Atlantides abscondantur,
Debita quam sulcis committas Semina, quamq;
Invita properes anni spem credere terræ.

Harvest-Time is meant when the Sun being in *Scorpio* the *Pleiades* set cosmically.

A Star is said to rise Achronically, when it rises at Sun-setting.

Et careo vobis Scythicus detrusus in oras,
Quatuor autumnos Pleias Orta facit. Ovid.

The

The Sun being in *Scorpio*, the *Pleiades* rise achronically, and set cosmically.

A Star is said to set achronically, when it sets with the Sun. So *Ovid. Fast. L. 2.*

*Quem modo calatum Stellis Delphina videbas
Is fugiet visus nocte sequente tuos.*

Feb. 3, When the Sun being in *Aquarius*, the *Dolphin* sets achronically.

Hence the Sign in which the Sun is, rises Cosmically and sets Achronically, and the Opposite Sign, at Night rises Achronically and in the Morning sets Cosmically.

4. The Heliac Rising of a Star is when the Star appears in View, after having been for some time lost in the Sun's greatest Light. So *Ovid. Fast. L. 2.*

*Jam levis obliqua, subsedit Aquarius urna
Proximus athereos excipe Piscis equos.*

About the Latter end of February.

*Ante tibi Eos Atlantides abscondantur,
Gnosque ardentis decedat Stella corona
Debita quam sulcis committas Semina, &c. Virg. G. 1.*

The Sun being in *Scorpio*, the *Pleiades* set cosmically, and the *Corona Septentrionalis* (i. e.) the Northern Crown) arises heliacally.

5. The Heliac Setting of a Star is when the Sun's greater Light obscures that Star.

*Candidus auratis aperit cum cornibus annum
Taurus & adverso cedens canis occidit astro.*

Of the Poetical Rising and Setting of the Stars, more plainly defined,

Viz. { Cosmical
Achronical } Rising and Setting of the Stars.
Heliacal

A Star said to Rise Cosmically (*viz.* from *Cosmos*, the World) is when he riseth with the Sun, or with that Degree of the Ecliptick in which the Sun is; And the Cosmical Setting is, when a Star setteth in the

the Morning; or goeth down under the West Horizon in the Morning, at the same time when the Sun rises in the East.

Achronical (from the Greek *Acronichos*, belonging to the Evening) rising, is, when a Star riseth in the East Horizon at that Moment that the Sun sets; and *Achronical* Setting, is when a Star goeth down under the West Horizon with the Sun.

Heliacal (from *Helios*, the Sun) Rising of a Star is when having been some time Combust or hid by the Sun-beams, now begins to appear, that same Star being at greater distance from the Sun; and a Star is said to set *Heliacally*; which hath some small time before been seen, but now by its Near Approach to the Sun becomes Inconspicuous, Combust of, or hidden under his Beams: See *Virgil* in his first Book of *Georgicks*, and *Ovid* in his first Book *de Ponto*, and in his Second Book of *Fasts*.

Note, that what Stars rise *Cosmick* will set *Achronick*; and what Stars do rise *Achronick*, will set *Cosmick*, &c. And to know when a Star begins to be Combust, and to be freed from the Sun-beams, no Certain Rule can be given: For the Magnitude of a Star, the Difference of the Climate, the Cloudiness, or Serenity of the Air, may much Alter; but the Opinion of the Antient Astronomers was, that,

A Star of the	{	1	Magnitude may be	{	12	Degrees below;
		2			13	
		3			14	
		4			15	
		5			16	
		6			17	
		seen when the Sun is		the Horizon.		

And those which are Nebulous, or Dark, cannot be seen till the Sun is 18 Degrees below the Horizon.



Of the Images, or Constellations, upon the
Celestial Globe.

THe Causes why the Antient Astronomers assumed these kinds of Images, are likely these three :

1. The Likeness which those Stars have in dispositions with those Images.

2. The Efficacy, which the same Astronomers superstitiously ascribed to some Constelations.

3. Immortality ; to which some were Consecrated to great Heroes, which would have their Names (as it were) Inscribed in Heaven viz.

4. For the more easy finding them on the Globe, or in the Heavens.

Northern Constellation.

The Army of the Starry Sky,
Declares the Glory of God most high.
Seen and perceiv'd of every Nation,
In Eighty and Forty Constellations :
First, near unto the Northern Pole
The *Dragon* and two *Bears* do rowl,
Whose Hinder Parts and Tail contain
The Greater and the Lesser *Wain* ;
The *Hare*, the *Bear*, *Ward* and the *Crown*,
And thence comes *Hercules* kneeling down.
And next below a place doth take
Great *Serpentarius* with the *Snake*.
Under the Harp of *Orpheus*,
The *Eagle* and *Ationius*.
The *Silver Swan* her Wings doth spread
Above the *Dart* and *Dolphins* Head.
Then *Pegasus* comes on amain ;
Andromeda follows in her Chain.
The *Triangles*, below her stands,
And at her Feet in *Perseus* hands.
The *Georgians* Head, above are seen
Her Parents *Cephus* with his Queen
Cassiope ; not far below,
Heniochus his Goat doth Show ;
On his Left Shoulder, in his hand
He doth the Stormy *Kids* command.

Zodiacal

Zodiacal Constellations; or, the 12 Celestial Signs.

Here in the Zodiack begins,
 The Ram (♈) the Bull (♉) and ♊ the Loving Twins.
 The Crab (♊) the Lyon (♌) and Virgo (♍) Virgin tender
 The Ballance (♎) Scorpio (♏) and (♐) Sagitta Bow-bender.
 the Goats (♑) Waterman (♒) then (♓;) Fishes Twain
 Shall bring you Round to the Ram ♈ again:

Southern Constellations.

Fifteen Images appear
 In the Southern Hemisphere.
 The Monstrous Whale above the rest
 Eridanius wets his Breast.
 Over the Hare, Orion bright
 Sparkles in a Cold Winter's Night.
 Then comes the great Dog, at whose Tail
 The famous Argo spreads her Sail.
 Above the Little Dog, doth flame
 For whom the Latines had no Name.
 Long Hydra in her Tail allow,
 Carries the Pithers and the Crow
 The Centaur holds the Wolf by the heell
 The Alter and Ixion's Wheell
 Are never seen of us; but here
 The Southern Fish brings up the Rear.

Of the Planets.

Under those fixed Stars above
 Seven Planets in their Orbs do move.
 The Highest is Saturn; thirty Year
 He spends encompassing his Sphere.
 Twelve, Jupiter; Mars in Twain
 Sets forward, and comes round again.
 Then in One Year the Sun displays
 Three hundred sixty and five Days,
 And near a Quarter, which in four
 Encompassings makes one Day more.
 Between the Sun and us there fly
 Fair Venus and swift Mercury.
 Those always with the Sun we find
 Not far below, nor far behind.
 The Moon is lowest, who in seven
 And twenty Days goes round the Heaven
 And about two Days more that she must run
 Before she overtakes the Sun;

So Twenty Nine and half in all
 To make a Month Synodicall
 The Planets make their Course to the East,
 Though they be farther hurled West;
 And Nine Degrees the rest may stray
 Besides the Sun's Ecliptick Way.
 Should I pretend to mount the Skies as some,
 Or might I touch the Planets as they run,
 And *Icarus* Wings for my assistance get,
 (And keep them free from *Titan's* scorching Heat;)
 Then might I tell you what the Stars portend,
 Which Nation, they will plague, and which befriend.

There are Twelve Constellations, posited about
 the South Pole, which were added by *Frederica*
Houtmaino; inhabiting in the Island *Samatra*; who
 being accommodated with *Tycho Brahe's* Instruments,
 hath observed the Latitude and Longitude of the
 following Constellations, *viz.*

Phenix, Grus, Indus, Xiphias, Pavo, Anser, and Hydrus;
Passer, Apus; Triquetrum, Musca, Chamaeque, Leon.

The Circles of the Sphere described.

SIX greater Circles mark you shall,
 Which equally divide the Ball;
 Just in the Midst between the Poles,
 From East to West the Equator rowls
 The Ecliptick cuts it, and doth slide
 Twenty three Degrees and half beside.
 Th' Horizon even with the Ground
 From Stars below our sight do bound.
 Meridian streight up doth Rise;
 Parting the East and Western Skies.
 Two Colures through the Pole do run,
 Quartering the Circle of the Sun.
 One where the Spring and Fall begin,
 The other where Longest Days come in.

The smaller Circles described.

Four Lesser Circles mark you well;
 Are to the Equator Parallel.

Two Tropicks bound the Sun's high Way,
Shewing the longest and the shortest Day.
The Artick Circle cuts the Bears;
The Antartick Opposite appears

And other Circles imagined on the Globe, are,
Meridians half Twenty four
For Hours; and Degrees, Nine score,
Through both the Poles directly pass,
And Equinoctial downright cross;
And Ninety Parallels have that Line,
By which Stars North and South decline.
The Ecliptick hath its Longitude
And Parallels of Latitude
For Stars; but in Geography
The Towns, besides the Equator lye
Over our Heads, and under our Feet
The Ninety Azimuths do meet,
And there are as many Parallels
Of Altitude, Horizontals,
Longitudes, and Meridians, all,
And Azimuth great Circle call;
But all the Parallels of Heaven
Being Lesser; cut the Globe uneven.
Degrees three hundred and Threescore
Hath every Circle, and no more.
God with an Upright Eye did Man endue,
And gave him Power the Starry Sky to view.

Vide Sturmy's Mathem. Magazine, First and best Edit.

A *TABLE* of the Principal Fixed Stars,
Rectify'd to the Year 1733. For Years to
come, Add 50'' (*the Præcession of the Æ-*
quinox, per Annum,) to the Star's Longitude
in this *Table*, and their Longitudes will be
correct, their Latitudes being always the same.

S T A R S Names.	Longit.	Latit.	Magn.
	S. ' ''	' ''	
<i>A</i> RIES, first * in the former Horn,	γ 29 27	7 8 N	4
Brightest * in the Head,	δ 3 56	9 57 N	3
End North Horn δ, or Foot of <i>Auriga</i> ,	π 18 49	5 20 N	2
Bright Foot of <i>Gemini</i> ,	σ 15 21	6 48 S	2
Head of <i>Hercules</i> , <i>Pollux</i> ,	σ 15 33	6 38 N	2
Middle and Brightest in the <i>Lion's</i> Neck,	ε 25 49	8 47 N	2
<i>Cor Caroli</i> ,	κ 22 7	40 3 N	2
Head of <i>Hercules</i> ,	χ 12 23	37 23 N	3
In the <i>Swan's</i> Bill,	ν 3 7	34 49 N	3
<i>Fomahant</i> , or the last in the Effu- sion of <i>Aquarius</i> ,	≈ 30 3	21 0 S	1
Head of <i>Andromeda</i> ,	γ 10 37	25 42 N	2
Girdle of <i>Andromeda</i> ,	γ 26 32	25 59 N	2
Brightest * in the Chair of <i>Cassio-</i> <i>peia</i> ,	δ 1 25	51 14 N	3
Breast of <i>Cassiopeia</i> , <i>Scheder</i> ,	δ 4 7	46 35 N	3
Left or South Foot of <i>Andromeda</i> ,	δ 10 26	27 46 N	2
Head of <i>Algol</i> , <i>Medusa</i> ,	δ 22 27	22 22 N	1
Left Foot of <i>Orion</i> , <i>Rigel</i> ,	π 13 07	31 11 S	1
Belly of the <i>Hare</i> ,	π 15 56	43 57 S	3
<i>Hircus</i> the He-Goat, <i>Capella</i> ,	π 18 02	22 50 N	1
First * in <i>Orion's</i> Belt,	π 19 40	23 38 S	2
Middle * in Ditto,	π 19 44	24 33 S	2
Hinder * in Ditto,	π 20 56	25 21 S	2
Pole *, <i>Arctic</i> ,	π 24 52	66 2 N	2
Right Shoulder of <i>Orion</i> ,	π 25 02	16 7 S	2
Right Shoulder of <i>Auriga</i> ,	π 27 42	21 27 N	2

STARS

261

STARS	Names.	Longit. S. ° ' "	Latit. ° ' "	Magn.
Great Dog,	Syrus,	♄ 10 25	29 30 S	1
Higher Head of	Gemini,			
Cassor,	Apollo,	♄ 16 31	10 02 N	2
Lesser Dog,	Procyon,	♄ 22 8	15 57 S	2
Shoulder of the	Great Bear, or 4th			
Wheel Pointer,		♂ 11 24	49 40 N	2
Flank of the	Bear, 3d Wheel or			
Pointer,		♂ 15 33	45 03 N	2
Hydra's Heart,		♂ 23 35	12 24 S	1
Northern * in the	Lion's Neck,	♂ 23 47	11 50 N	3
Left Thigh of the	Bear, 1st Wheel,	♂ 27 15	51 37 N	2
Brightest * in	Leo's Back,	♃ 7 31	14 20 N	2
Ursa Major, 2d	Horse, or Middle *			
in the Tail of	Helice,	♃ 11 46	56 22 N	2
Third * in the	Dragon's Tail,	♃ 4 01	66 36 N	2
Tail of Leo,		♃ 17 53	12 18 N	2
Great Bear's Tail,	or first Horse of			
the Chariot,		♃ 3 02	54 25 N	2
Virgo, Vindematrix,		♈ 6 13	16 15 N	3
Bootes, left Shoulder,		♈ 14 55	49 33 N	3
Arcturus in the	Skirts of Bootes,	♈ 20 29	31 2 N	1
Brightest * in the	Northern Crown,	♊ 18 28	44 23 N	2
Brightest * in the	North Ballance,	♊ 15 38	8 35 N	2
Brightest * in the	Serpent's Neck,	♊ 18 20	25 35 N	2
Right Shoulder of	Hercules,	♊ 27 17	42 48 N	3
Left Shoulder of	Hercules,	♊ 11 00	47 47 N	3
Head of Hercules,		♊ 12 21	37 23 N	3
Head of the Dragon,		♊ 24 14	75 03 N	3
Shining * in the	Harp,	♊ 11 33	61 47 N	1
Tail of the Vulture,		♊ 16 5	36 16 N	3
Mouth of the Swan,		♊ 27 34	49 02 N	3
North Horn of the	Goat,	♋ 0 08	7 02 N	3
Left Shoulder of	Aquarius,	♋ 19 41	8 42 N	3
Breast of the Swan,		♋ 21 15	57 9 N	3
Lower Wing of the	Swan,	♋ 23 59	49 26 N	3
Mouth of Pegasus,		♋ 28 12	22 27 N	3
Tail of the Swan,		♋ 1 42	59 56 N	2
Wing of Pegasus,		♋ 19 46	19 26 N	2
Southern * in the	Whale's Tail,	♋ 28 46	20 47 N	2
In the Girdle of K.	Cephas,	♌ 2 37	7 N	3
Last in the Dragon's	Tail,	♌ 6 28	57 7 N	3

S T A R S Names.	Longit.		Latit.		Magn.
	S.	°	°	'	
The Northern in the <i>Harp</i> ,	V318	01	56	06 N	3
The <i>Swan's</i> Bill,	V327	34	49	02 N	3
<i>Ophiuchus</i> , in the Head,	218	40	35	57 N	3
In the Temples of the <i>Serpent</i> of <i>Ophiucus</i> ,	M17	58	35	25 N	3
In the Breast of <i>Antoni</i> us,	V326	40	21	38 N	3
In the Head of the <i>Dolphin</i> ,	15	32	32	47 N	3
In the Head of <i>Andromeda</i> , <i>Coma</i>	V10	37	35	42 N	3
<i>Berenices</i> ,	M20	07	28	25 N	3
The <i>Whale's</i> Mouth,	5	44	12	02 S	3
<i>Argo Navis</i> , in the top of her Stern.	7	47	43	18 S	3
* Brightest of the <i>Pleiades</i> , or 7 Stars,	26	15	04	00 N	3
<i>Taurus</i> his Nostril,	102	02	05	46 S	3
North Eye of <i>Taurus</i> ,	104	42	02	36 S	3
South Eye of the Bull, <i>Taurus</i> , <i>Alde-</i> <i>baran</i> , <i>Palitium</i> ,	106	02	05	31 S	1
In the South Horn of <i>Taurus</i> ,	121	05	02	14 S	3
Higher Foot of <i>Gemini</i> ,	127	12	00	53 S	3
In the upper Knee of the <i>Twins</i> ,	306	14	02	11 N	3
In the Left Knee of the <i>Twine</i> ,	311	19	02	06 S	3
In the Belly of the S. <i>Twin</i> ,	314	48	00	13 S	3
<i>Cancer</i> , his Southern Claw,	9	56	05	08 S	3
The S. Star in the <i>Lion's</i> Neck,	24	13	04	52 N	3
The <i>Lion's</i> Heart, <i>Regulus</i> , <i>Basilica</i> ,	26	07	08	26 N	1
<i>Virgo</i> , the middle * in her Foot,	17	47	28	33 S	3
Second in the Left Wing of <i>Virgo</i> ,	206	25	02	50 N	3
The <i>Virgin's</i> Spike, <i>Arista</i> ,	20	06	01	59 S	1
South Ballance,	11	21	00	26 N	2
In the Forehead of <i>Scorpio</i> ,	29	26	01	05 N	2
The <i>Scorpion's</i> Heart, <i>Antares</i> ,	206	03	04	27 S	1
Southern in the North part of the Bow of <i>Sagittary</i> ,	V302	37	02	00 S	3
South Horn of the <i>Goat</i> ,	300	21	04	41 N	3
Brightest * in the Tail of the <i>Goat</i> ,	18	04	02	26 S	3
Following * in the <i>Goat's</i> Back,	19	50	00	29 S	3

The S T A R S compleat one Revolution thro' the
Zodiack in 25920 Years.

* These Stars lie in the Moon's Way, Lat. Lond. 51°
32' North.

7. The

1. The Starry Heaven, or that Glorious Celestial Canopy embroidered with those Sparkling Diamonds, which hang upon the Dusky Cheeks of the Night, as a Rich Jewel in an *Emperor's* Ear: This is the most Noble Part of the Universe, incircling our Terraqueous Globe: At an almost unmeasurable Distance, sparkle the innumerable Lights or Stars in the immense expansion of the Firmament.

These Celestial Lights appear to us no more than so many Lucid Points, tho' some thousand times more in Magnitude than our Globe of Earth and Sea: They are said to be fixed, because they always keep the same invariable Distance from one another, and from the Ecliptick; whereas the Planets, or Erratick Stars, sometimes are in the Ecliptick, at other times have North Latitude or South Latitude; at one time moving direct, or according to the Order of the Signs; at another time Retrograde, or backward, contrary to the Order of the Signs, as in appearance to us at the Earth.

2. As for the substance of the fixed Stars, Modern Astronomers suppose them to be compounded of Elementary Matter, formed into fiery Globes partly Liquid, and partly Solid; the Liquid being fluid and moveable, like Flame; their Light being, innate; that is, they shine with their own Light, as the Sun does. And as to their Scintillation or Twinkling, 'tis imputed to a constant Evibration of Lucid Matter, or a continual *Effluvia*, or expiration of Fiery Vapours; in the same manner as the Fulgurations and Ebullitions in the Sun.

3. As to the Number of the Stars, they may be said to be innumerable; for looking with larger and better Telescopes, still more Stars appear; yet we consider them only which are most Notable and Visible, as they are reduced to six Degrees, or Sizes of Magnitude; not minding the *Nebulosa* or Obscure, we shall find them according to *Ptolemy* 1026, to *Pliny* 1600, to *Copernicus*, 1022, to *Tycho Brahe* 773 to *Kepler* 1147; to *Ricciolus*, 1468, to *Bayers*, 1725, to *Hévelius*, and our *Dr. Halley*, 1888; to the Late Royal Astronomer *Mr. Flamsteed*, 3000; all rectified to the Year 1726, as may be seen in his third Volume of his *Historia Cælestis*. *Galileus* informs us, that by the Telescope he discovered in the small Asterism of the *Pleiades* no less than 72

Stars; whereas the bare Eye can see but six. *Tycho Brahe* rectified 780 of the Fixed Stars; the Landgrave of *Hesse*, 386; *Hervelius*, 1534, in their Times.

4. The fixed Stars, have no Diurnal, nor Menstrual Parallax, (as the Planets have,) but according to *Dr. Hook*, the Earth's Annual Orb, had a sensible Parallax, to the Star in the Head of the *Dragon*, viz. about 27" or 30" in the Winter-solstice, nigher to the Vertical point of *Gresham-College*, than in the Summer; so that according to his Computation, the distance of that Star from the Earth was 13742 Semidiameters of the Earth's Orbit. But the great *Mr. Flamsteed* considering, that the Polar Star, seeing it never sets (here) but is seen all Night, (if clear Weather) and has that Altitude from the Horizon as to free it from Refraction, as also its great Latitude or distance from the Ecliptick, he thought it the most proper Star (for these Reasons) to discover if there was any Annual Parallax; and by comparing 15 Sets of Observations made for 7 Years together, he found that the Pole-Star had a Lesser Latitude or distance from the Ecliptick about the Summer-solstice than about the Winter Solstice; the difference amounting to 42" and that of the great *Dog Sirius* 30". Therefore,

As S. Lat. * *Syrus* $39^{\circ} 30'$.. *R.* S. $90^{\circ} ::$ S. $30'$
To S. $47''$.

The Parallax being $47''$ that the fixed Stars make with the Earth's Annual Orbit.

Therefore the Diameter of the Earth's Orb, bears the same proportion to the Distance of the fixed Stars which the Sine of $47''$ doth to the Sine of 90° . Hence it will appear that the Distance of the fixed Stars are 9000 Semidiameters of the Earth's Annual Orbit, or 70000000000 *English* Miles. Whereas by *Dr. Dee*, no more than 69036540 Miles, and *Tycho Brahe* 48184000 miles.

5. Hence we may Observe, That the Earth's Annual Orbit is but a meer Point in comparison of the Distance of the Earth and Fixed Stars.

6. That could we advance 99 parts of the aforesaid Distance and only $\frac{1}{100}$ Part remaining, the Stars would seem no bigger to us there than they do here; for they would shew no otherwise than they do thro' a Telescope, which magnifies an Hundred-fold.

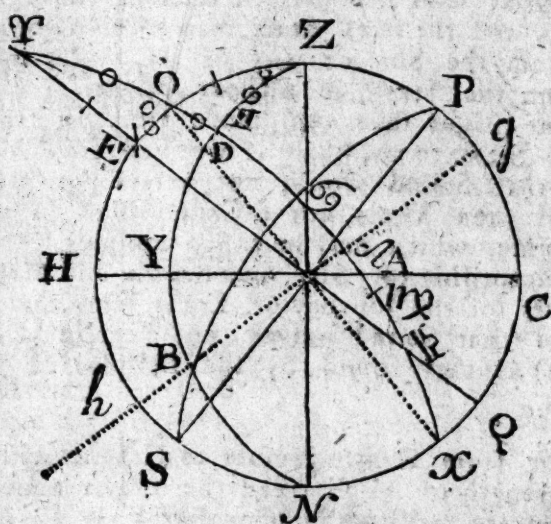
7. That

7. That at least nine parts in Ten of the Space between us and the fixed Stars, can receive no greater Light from the Sun or any of the Stars than we have from the Stars in a Clear Night.

8. That Light takes up more time in travelling from the Stars to us, than if we had travelled 3000 Miles; that Sound would not arrive to us from thence in 50000 Years, nor a Cannon-Ball in a much longer time, which may be easily computed, by allowing according to Sir. *Isaac Newton*, and Mr. *Pound*, 7 Minutes for the Journey of Light from the Sun to us: And (that Sound moves 968 Feet in a Second of Time) as they found by the Eclipses of *Jupiter's* Satellites.

We tho' from Heav'n remote to Heav'n will move,
With Strength of Mind tread the Abyss above,
And penetrate, with an interior Light
Those upper Depths, which Nature hides from sight.
Pleas'd we will be to walk along the Sphere
Of shining Stars, and travel with the Year;
To leave this heavy Earth, and Scale the Height
Of *Atlas*, who supports the Heavenly Weight;
To look from upper Light, and thence survey
Mistaken Mortals wandring from the Way.

Ovid's Metamor. Lib. 15.



CONSTRUCTION

1. To the time given find the Sun's place; which in this Example is $\Pi 7^{\circ} 30'$ and his Right Ascension is $65^{\circ} 41'$ by *Prob. 2. Astron.*

2. Because 15° of the Equinoctial pass the Meridian every Hour; therefore if the Time $\left\{ \begin{array}{l} \text{before} \\ \text{after} \end{array} \right\}$ Noon be converted into Degrees and Minutes and $\left\{ \begin{array}{l} \text{taken from} \\ \text{Added to} \end{array} \right\}$ the Sun's Right Ascension, it shall give the Right Ascension of the *Medium Celi* or Point of the Ecliptick upon the Meridian *viz.* $65^{\circ} 41' - 37^{\circ} 30' = 28^{\circ} 11'$.

3. Having thus obtain'd the Right Ascension of the M.C. *viz.* $28^{\circ} 11'$ draw the Primitive Circle, the Horizon H O, and Equinox E Q, and Axis P S.

4. Set off the Right Ascension $28^{\circ} 11'$ from Q, to n, and lay a Ruler from S n, to give the Point m, which is the Point of the Intersection of the Equinoctial and Ecliptick; thro' which draw the Ecliptick Y X making an Angle of $23^{\circ} 29'$ with the Equinoctial E Q.

5. Find the Pole of the Ecliptick B, and set off 6 Signs from γ to ϖ , containing 30° each by *Prob. 6. Ster.*

6. Thro' the Pole B strike the Vertical Circle Z DBN; so shall the Triangles $\gamma E \delta$ and $\delta D Z$ exhibit what's Requir'd in the Problem following.

PROB.

PROB. I.

The Latitude of the Place, the Day, and Hour of the Day given; to find what point of the Ecliptick Culminates, and its Altitude; also what Angle it makes with the Meridian?

Example. In Latitude 51° Half N: On May 18, at 9h: 30' in the Morning I demand what point of the Ecliptick is upon the Meridian and its Altitude, &c.

Vide the former Page.

In the $\triangle PED$ $\left\{ \begin{array}{l} PE = 28^{\circ} 11' \\ \angle P = 23^{\circ} 29' \end{array} \right\}$ to find $\left\{ \begin{array}{l} PD \\ ED \\ \angle D \end{array} \right\}$ by R.S.

1. As T: $PE = 28^{\circ} 11'$.. R: $\therefore$ Co. S. $\angle P = 23^{\circ} 29'$.. Co. T: $PD = 30^{\circ} 18'$.. $\therefore$ $DO = 18'$ Culminates.

2. As Co. T. $\angle P = 23^{\circ} 29'$.. R: $\therefore$ S. $PE = 28^{\circ} 11'$.. m. $ED = 11^{\circ} 35'$ = Declination *Med. Cali.*

3. As R: $\therefore$ S $\angle P = 23^{\circ} 29'$.. Co. S. $PE = 28^{\circ} 11'$.. Co. S $\angle D = 69^{\circ} 26'$ = the Meridian $\angle$.

To the Height of the Equator $HE = 38^{\circ} 30'$,
Add (in this *Example*) the Decl. M.C. $ED = 11^{\circ} 35'$

Gives the Altitude of *Medium Cali* $50^{\circ} 05'$

But if the M.C. be in S. Signs subtract, &c. And its place is $08^{\circ} 18'$ of $\varnothing$.

PROB. II.

To find the Place, Altitude, and Azimuth of the Nonagesime Degree of the Ecliptick, and consequently the points Ascending and Descending, and the Angle Orient, or Angle of the Ecliptick and Horizon.

Example and Projection (as in the last Prob.)

For having obtained the Requisites in the last Prob. there will be given

In the $\triangle DZ$ $\left\{ \begin{array}{l} \angle D = 69^{\circ} 26' \\ \angle Z = 39^{\circ} 55' \end{array} \right\}$ To find $\left\{ \begin{array}{l} DZ \\ DZ \\ \angle Z \end{array} \right\}$ by R. A.

1. As Co. T. $\angle Z = 39^{\circ} 55'$.. R: $\therefore$ Cos. $\angle D = 69^{\circ} 26'$.. T. $DZ = 16^{\circ} 23'$ &c.

2. As R: $\therefore$ S $\angle Z = 39^{\circ} 55'$.. S. $\angle D = 69^{\circ} 26'$ S. $DZ = 36^{\circ} 55'$ Half = Comp. DY.

3. *As* Co. T. $\angle \delta = 69^\circ 26'$.. *R* :: Co. S. $\delta Z = 39^\circ 55'$.. Co. T. $\angle Z = 26^\circ 04'$ the Azimuth *&c.*

To the Place of the Medium Cali

Add the Arch of the Ecliptick

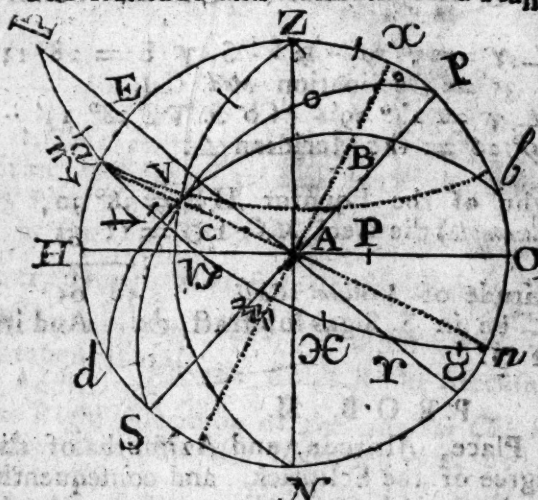
$\delta, 0^\circ 18'$
 $\delta D 16 23$

Sum is the Place of the Nonagesime Deg. at D, viz. $\delta 16 41$

To which Add 3 Sgns, and it gives the Point Ascending A. viz. $\delta 16^\circ 41'$, and the Opposite Point of the Ecliptick viz. $\approx 16^\circ 41'$ Descending, the West; Also $DZ = 36^\circ 55'$ is the Comp. of $DY = 53^\circ 4'$ the Altitude of the Nonagesime at D, and the Measure of the Oriental Angle at A.

PROB. III.

The Altitude and Azimuth of a Planet, Comet, or New Star, together with the Time of Night Given; To find its Declination, Right Ascension, Latitude and Longitude?



Ex. Anno
 1732, Jan. the
 2, at 6h. 41'
Mane (suppose)
 I observed a
 new Star or
 Comet and

found its Altitude $= 29^\circ 30'$ and Azimuth $31^\circ 30'$ from the South, I demand *&c.*

CONSTRUCTION.

1. At the given Time I find the Sun's Right Ascension $= 19h. 39'$ to which add the time Afternoon viz. $18h. 41'$, gives the Right Ascension of the M. C. $14h. 20' = 35^\circ$ from $\approx$

2. Project the Sphere according to the Directions of the last two Problems, and set off the Signs from $\approx$, in the Ecliptick.

3. Draw the Parallel of the new Stars Altitude a, b , and it's observed Azimuth ZCN which Intersects the Parallel in the Point C, the true Place of the Star in the Heaven.

4. Thro'

4. Thro' the Pole of the Ecliptick B and the Point C
Strike the Circle of the Longitude B C d

Then in $\triangle CZP$ $\left\{ \begin{array}{l} CZ = 60^{\circ} 30' \\ ZP = 38^{\circ} 30' \\ \angle Z = 148^{\circ} 30' \end{array} \right\}$ to find $\left\{ \begin{array}{l} \angle P \\ CP \end{array} \right\}$ $\left\{ \begin{array}{l} \text{equal} \\ \text{to} \end{array} \right\}$ $\left\{ \begin{array}{l} EV \\ CV + 20' \end{array} \right\}$

OPERATION.

1. As S half Z two sides CZ & ZP .. S half their
X :: Co. T half cone. $\angle Z$.. T half X. Opposite $\angle$'s.
— S — $49^{\circ} 30'$ — S $110^{\circ} 00'$ — T $74^{\circ} 15'$.. T $4^{\circ} 30'$
2. As C.S. half Z two sides CZ & Z B C.S. half their
X :: Co. T. half cont. $\angle Z$.. T half Z Opposite $\angle$'s.
— $49^{\circ} 30'$ — $110^{\circ} 00'$ — $74^{\circ} 15'$ — $23^{\circ} 35'$, then

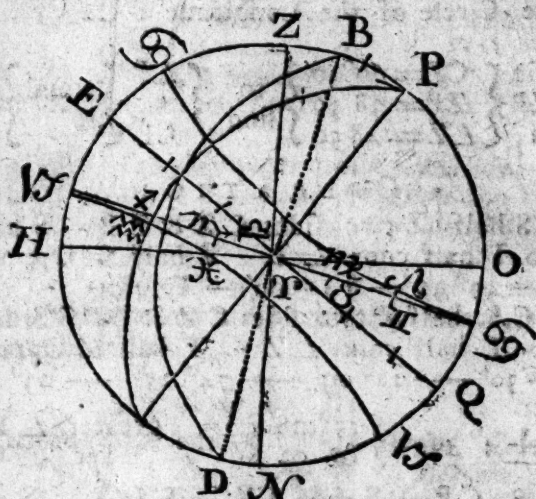
$23^{\circ} 05' + 4^{\circ} 30' = \left\{ \begin{array}{l} 27^{\circ} 08' \\ 19^{\circ} 02' \end{array} \right\}$ the greater $\angle P = EV$
lesser $\angle C = V$

3. As S. $\angle P$.. S. side CZ :: S $\angle Z$.. S. side CP.
— $27^{\circ} 08'$ — $60^{\circ} 30'$ — $148^{\circ} 30'$ — $85^{\circ} 40'$ Comp. to
 $180^{\circ} = 94^{\circ} 20'$.

So that the New Stars $\left\{ \begin{array}{l} \text{Declination, is} = 4^{\circ} 20' \text{ S.} \\ \text{Right Ascension} = 62^{\circ} 48' \end{array} \right\}$
from α or $242^{\circ} 8'$ from γ .

Now having found the Right Ascension and Declina-
tion, 'tis best to project another Scheme as here under,
to find the Longitude and Latitude, to avoid Confu-
sion of Lines, &c. Where

In



In the ΔCBP $\left\{ \begin{array}{l} CB = 94^\circ 20' \\ BP = 23^\circ 29' \\ \angle P = 27^\circ 52' \end{array} \right\}$ to find $\left\{ \begin{array}{l} CB \\ \angle B \end{array} \right\}$ etc.

OPERATION.

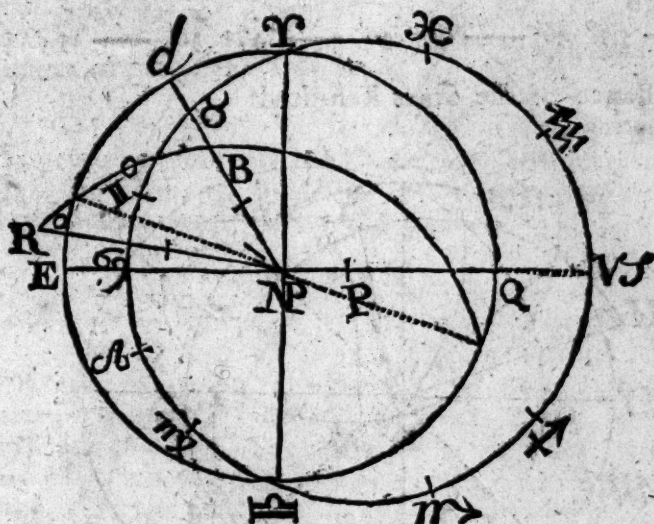
1. *As* S half Z 2 sides CP and BP .. S half their X
 $\therefore$ C.T. half cont. $\angle P$.. T half X's $\angle$'s.
 $\rightarrow 58^\circ 54'$ half $\rightarrow 35^\circ 25'$ half $\rightarrow 13^\circ 56'$ $\rightarrow 69^\circ 52'$.
2. *As* Co. S. half Z two sides CP and BP .. C.S. half
 their X $\therefore$ C.T. half cont. $\angle P$.. Z half Z $\angle$'s.
 $\rightarrow 58^\circ 54' \rightarrow 35^\circ 25'$ half $\rightarrow 13^\circ 56' \rightarrow 81^\circ 04'$.

Sum gives the $\angle B = 150^\circ 56'$

3. *As* S. $\angle B$.. S. sides CP. $\therefore$ S $\angle P$.. S. side
 CB.
 $\rightarrow 150^\circ 56' \rightarrow 94^\circ 20' \rightarrow 27^\circ 52' \rightarrow 73^\circ 36'$.

By which there $\left\{ \begin{array}{l} CB = 73^\circ 31' \text{ the Comp. Lat.} \\ \angle B = 150^\circ 56' \text{ the Long. from } \end{array} \right\}$ and
 is found

Consequently $\left\{ \begin{array}{l} \text{Longitude is } 73^\circ 31' \\ \text{Latitude } 16^\circ 29' \text{ North} \end{array} \right\}$
 the Stars



PROB. IV.

The Right Ascension and Declination of two Stars given ; to find their Distance, asunder.

EXAMPLE.

The { Bright Star of γ Rt. Af. $27^{\circ} 45'$ Dec. $32^{\circ} 04' N.$ }
 { Foot, of Orion Rigel Rt. Af. $75^{\circ} 15'$ Dec. $08^{\circ} 34' S.$ }
 I demand their Distance ?

In the $\triangle RBN$ { $\angle NPB = 67^{\circ} 56'$ } and the $\angle NP =$
 is given { $\angle NPR = 98^{\circ} 34'$ } $47^{\circ} 30'$ to find BR.

1. As S half Z sides NPB and NPR .. S half their X
 $\therefore$ Comp. T half the cont. $\angle NP$.. T half opposite $\angle$'s,
 $\text{--- } 83^{\circ} 15' \text{ --- } 15^{\circ} 19' \text{ --- } 23^{\circ} 45' \text{ --- } 31^{\circ} 09'.$

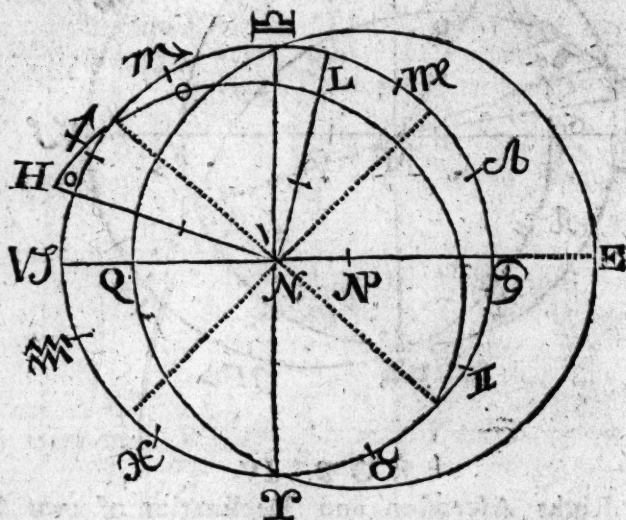
2. As C.S half Z sides NPB and NPR .. C. S. half X
 $\therefore$ Comp. T half $\angle NP$.. cont. $\angle NP$.. T half Z
 opposite $\angle$'s
 $\text{--- } 83^{\circ} 15' \text{ --- } 15^{\circ} 19' \text{ --- } 23^{\circ} 45' \text{ --- } 86^{\circ} 55'$
 Subtr. the half X = $31^{\circ} 09'$

Remainder is the lesser $\angle R = 55^{\circ} 46'$

3. As $S \angle R$.. S sides BNP :: $S. \angle NP$.. $S.$
side RB .

— $55^{\circ} 46'$ — $67^{\circ} 56'$ — $47^{\circ} 30'$ — $55^{\circ} 44'$

the Distance of the Stars Required.



The Latitudes and Longitudes of two Stars Given to
find their Distance.

P R O B. V.

The $\left\{ \begin{array}{l} \text{Lyons Tail Long. } \mathcal{M}. 17^{\circ} 37' - \text{Lat. } 12^{\circ} 16' \text{ N} \\ \text{Scorpions Heart Long. } \mathcal{X} 5^{\circ} 50' - \text{Lat. } 4^{\circ} 26' \text{ S} \end{array} \right\}$
I demand their Distance.

In the $\left\{ \begin{array}{l} \text{HN} = 94^{\circ} 26' \\ \text{LN} = 77^{\circ} 44' \\ \angle \text{HNL} = 78^{\circ} 13' \end{array} \right\}$ To find the Arch LH.
is given

O P E R A T I O N.

1. As S half Z sides HN and LN .. S half X their
sides :: Comp. T half cont. $\angle \text{HNE}$.. T half $X \angle$'s.
— $86^{\circ} 05'$ — $8^{\circ} 21'$ — $39^{\circ} 6'$ — $10^{\circ} 09'$.

2. As $C.S.$ half Z sides HN and LN .. $C.S.$ half X
their sides :: Comp. T half cont. $\angle \text{HNL}$.. T half $Z \angle$'s.
 $86^{\circ} 05'$ — $8^{\circ} 21'$ — $39^{\circ} 6'$ — $86^{\circ} 47'$

From which take the half X 10 09

Remains the lesser $\angle H = 76.38$

3. As

3. As $S \angle H$.. S. sides $\angle N :: S. \angle N$ " S. side.
or Arch HL.

$76^{\circ} 38' - 77^{\circ} 44' - 78^{\circ} 13' - 79^{\circ} 29'$, The
Distance of the Stars required.

To know the Stars by the Help of a Planisphere.

You must observe in the Chart the Constellation proposed, how many Principal Stars are therein, and compare them with those in the Heaven.

EXAMPLE.

*How to know the Great Bear vulgarly called Charles's Wain,
and by the French, the Great Chariot.*

Observe first upon the Chart, that that Constellation is not far from the North Pole of the World, and that there are therein seven principal large Stars, whereof four of them are in the Body of the great Bear, which make a sort of a Long Square; the three others are in his Tail. Thus having the Idea of the Figure of which these seven Stars are composed, it is impossible to mistake them in the Heavens; if you look to the North Pole, in respect that that Constellation is alone, and separate from all others.

One may also know this and all other Constellations by the Hour of their passing the Meridian.

How to know the other Constellations by the Help of this former one.

You may observe, that *Cassiopeia*, is on the other side of the Pole opposite to the Stars that are in the Tail of the Great Bear; that when the last Star of the great Bear shall be upon the Meridian above the Pole, *Cassiopeia* shall be upon the Meridian under the Pole; and when the Bear shall be East, *Cassiopeia* shall be West, &c.

These two Constellations are much the same Distance alike from the Pole.

Cassiopeia contains five principal Stars, whereof that which is farthest from the Pole, is called, the Breast of *Cassiopeia*.

Of the North Star.

The North Star, which is at the End of the Tail of the little Bear, is pretty near the middle of a right Line, which one might conceive to be drawn from the Stars of the Tail of the great Bear to those of *Cassiopeia*.

Note, That the North Star is but two Degrees from the Pole of the World.

The brightest of the Guards of the North Star, is in the Shoulder of the *Little Bear*, and is almost between the North Star, and that of the End of the Tail of the *Great Bear*; but it is a little higher the North Star, and hath a little more Right Ascension, or is more to the East than the Tail of the *Great Bear*.

Now suppose one draw a Line from the brightest of the Guards thro' the North Star, it would pass thro' the bright Star, in the Side of *Perseus*, and so thro' the bright Star in the *Whale's Jaw*, which is in the South.

The Constellation of *Orion* is easy to be known, in as much as it contains three bright Stars in a strait Line in the Belt of *Orion*, and by the *French* is commonly called the *three Kings*. There are two other bright Stars in his Shoulders more to the Northward, and two bright Stars in his Feet which are to the Southward, with abundance of small Stars in his Buckler towards the West, somewhat more than those in his Shoulders.

The Belt and Feet of *Orion* are in the Southern; but his Shoulders are in the Northern Hemisphere.

The Constellation of the *Bull* is a little more to the West than that of *Orion*; it contains in his Neck the *Pleiades* or seven Stars; but that bright and red Star in the South Eye of the *Bull* is between the *Pleiades* and the Belt of *Orion*, called *Aldebaran*.

That bright and clear Star call'd *Capella*, or the *Goat*, in the Shoulder of *Auriga*, is between the North Star and *Orion*.

Suppose a Line was to pass from the North Star, between *Capella* and the *Great Bear*; would go between the two Heads of *Gemini*, or the *Twins*) and then thro' that clear and bright Star in the *little Dog* called *Procyon*.

A Line drawn from the brightest of the Guards, thro' the middle of the Square of the *Great Bear*, passes thro' that very clear and bright Star, the *Lion's Heart*, called *Regulus*.

A Line drawn from the North Star, and thro' the same Square, a little nearer the hinder part, passes thro' that bright Star in the *Lion's Tail*. Besides, the *Lion's Tail* and *Heart*, there are six other bright Stars, but of a lesser magnitude, viz. three in his Shoulder and three in his Thigh.

A Line drawn from the North Star, passing between the brightest of the Guards, and that of the End of the Tail of the *Great Bear*, passes thro' the *Dragon* near his last Knot, and so thro' that very clear and bright Star called *Arcturus*, in the skirts of *Bootes*.

A Line drawn from the North Star thro' the least of the two Guards, passes thro' that small Star in the last Knot of the *Dragon*, and so thro' the Northern Crown, and thro' the Head and Heart of the *Serpent*.

A Line drawn from *Capella* thro' the North Star, passes to the other side of the Pole, thro' the third Knot of the *Dragon*, and thro' *Hercules*, and thro' *Serpentarius*.

A Line drawn from the Star in the Square of the *Great Bear* nearest his Head, thro' the brightest of the Guards, passes through the Constellation of the *Swan* and the *Dolphin*.

The Stars in the Wings of the *Swan*, that bright Star in her Tail, and the little Star in her Beak, make a compleat Cross, and the four Stars in the *Dolphin* make a Figure like a Diamond.

A little nearer the West than the *Dolphin*, are three bright remarkable Stars, whereof the middlemost is the largest; in a streight Line they are in the *Eagle's Neck*.

There is a bright Star, called the Brightest in the *Harp*, near the *Swan*, but a little more to the West.

A Line drawn from the Star at the Rump of the *Great Bear*, passing between the North Star and the brightest of the Guards, passes through *Cepheus*, a Constellation near the Pole, and through those in the fore part of *Pegasus*, or the *Flying Horse*.

A Line drawn from the North Star, and through the Westernmost of *Cassiopeia* passes through that in the Head of *Andromeda*, and through that at the End of the Wing of *Pegasus*.

These Four, viz. that in the Head of *Andromeda*, that at the End of *Pegasus* Wing, that at the Bending of the same Wing, and that in his Shoulder, make together a large Square very easy to be observed.

A Line drawn from the North Star thro' *Cassiopeia* and *Perseus*, passes thro' the three small ones that form the Triangle, and to that bright one in the Head of *Aries*, or the *Ram*.

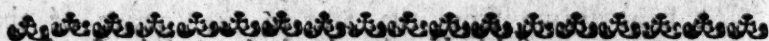
A Line drawn from the *Pleiades* thro' the three Stars in *Orion's Belt*, passes thro' that very bright and splendid

did Star in the Mouth of the *Great Dog*, called *Sirius*, which is in the South.

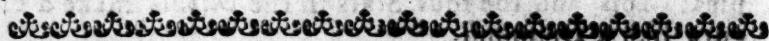
It is very easy to draw in the Heavens strait Lines from the *North Star* through all the others which one knows; by holding a long Stick between your Eye and the Stars; and as one observes in the Chart, the Stars that are in a strait Line with the *North Star*, it will be as easy to know them all in a little time without the Chart.

The Declination and Right Ascension of the Fixed Stars is another Means to know them by.

Vide my Treatise of Navigation.



The Projection of the Sphere plainly laid down and Demonstrated by the Celestial Globe.



SECT. V.

Of the Projection of the Sphere.

1. **T**HE Sphere (or Globe) may be projected on a Plain several ways, *viz.*

1. Stereographical.
2. Orthographical, and
3. Gnomonical.

The Stereographick Projection of the Sphere being the most common, and indeed the most intelligible (in Trigonometrical or Astronomical Operations) which is projected upon the Plain of that Great Circle, that passes through (on the Globe) the Summer and Winter Solstitial Points of *Cancer* and *Capricorn*, called the *Solstitial Colure*, and by some, the *General Meridian*; we shall begin first with that.

DEFINITIONS.

1. Stereographick Projection is a Branch of Perspective, which shews us how to describe or draw the Appearances of the Circles of the Globe, upon a Plane passing through the Center thereof; supposing the Eye

in the Surface of the Globe, in the Pole of the Circle on which the Projection is to be made, and on that side of the Sphere opposite to the side to be projected. This Projection thus made, shall truly represent all the Appearance of that half of the Sphere opposite to that in whose Pole the Eye is placed, as (says a late * Author) may be illustrated by the following Similitude.

Suppose (says he) a Sphere, or Globe were made of Glass, or any transparent Metal, on which were described all the Points and Circles of the Sphere (as Zenith, Nadir, Equator, Ecliptick, &c.) in black Lines, that might be discerned through the Globe; and admit there were a Glass-Plain passing through the Center of the Globe, cutting it in two equal Parts in that Great Circle, upon whose Plain the Projection is to be made. As for instance, suppose I were to project the Western Hemisphere upon the Plain of the Meridian, having a transparent Globe prepared as before described, and the transparent Plain to pass thro' the Center of the Sphere, and cut it in two parts in the Meridian (the Great Circle upon which the Projection is to be made) if the Eye be supposed to be placed in the East Point of the Horizon, upon the surface of the Sphere; and observe, where the Meridians, Azimuths, &c. on the West side of the Sphere cut the Plain passing through the middle of the Sphere, these Circles upon that Plain shall truly represent the Projection of the Western Hemisphere upon the Plain of the Meridian. But more Obvious,

Thus, on the Celestial Globe, bring the solstitial Point *Cancer* to the Brazen Meridian; then shall the Equinoctial Point *Libra* be in the East Point of the Horizon; elevate the Globe, suppose to the Latitude of London, 51° and half North; screw the Quadrant of Altitude on the Zenith-Point on the Degrees 51° and half, and bring it to the East Point of the Horizon; and imagine a Line drawn by the Edge thereof, to represent the Prime Vertical to 90° , and the Parallels of Altitude to pass thro' every Degree of the Quadrant of Altitude, parallel to the Horizon; also the Parallels of Latitude to pass thro' every Degree of the Brazen Meridian; and the Meridian to pass thro' every Degree of the Equinoctial.

* *H. Wilson's Trigonometry improv'd.*

In this Position of the Globe, the prime Vertical, prime Meridian, Equator, and Ecliptick will all coincide in the East Point of the Horizon. Now supposing the Eye in the East Point of the Horizon, and the Globe made of a flexible Matter, and flatted down; that is, all the Eastern Hemisphere, into the Brazen or General Meridian, or solstitial Colure; I say, then the Equinoctial, prime Vertical, prime Meridian, Ecliptick, and Horizon, will become Right Lines, Diameters, or Right Circles, the Parallels of Altitude Small Circles parallel to a Right; that is, the the Horizon, the parallels of Latitude, Lesser Circles parallel to a Right, *viz.* the Equinoctial, the Meridians, and Azimuth-Circles, being Oblique on the Globe will become Oblique Circles or Arches in the Projection; so likewise the Tropicks and Polar Circles being parallel to the Equinoctial, will become Lesser Circles parallel to a Right in the same manner as the Parallels of Latitude.

These two Similitudes may suffice to shew the Nature of the *Stereographick Projection of the Sphere*. My Design here is to render it plain and easy to the Practical Learner; and for the more curious and speculative ones, I refer them to *Heynes's Trigonometry*, from Page 83, to 93; *Mr. Hodgson's Theory of Navigation*, from Page 197, to 204; *Mr. Whity's Doctrine, of the Sphere*, and others, that have compiled a whole Treatise of the same.

I proceed now to the Projection of the Sphere upon a Plain, on any Great Circle; and here I would advise the Learner to consider again what was said before, relating to the Points and Circles of the Sphere, in Astronomy, in the Definitions of Stereographick Problems, Page 33 and in the Astronomical Definition hereafter, where they be more fully explained.

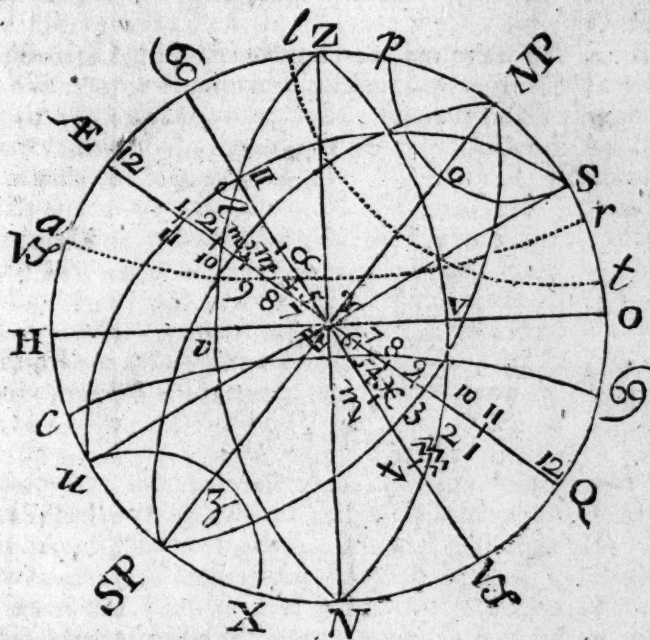
1. To project half the Surface of the Sphere upon the Plain of the General Meridian; or solstitial Colure, Latitude $51^{\circ} 30'$ North.

C O N S T R U C T I O N.

1. With a Chord of 60° describe the primitive Circle, or solstitial Colure, HZON, and cross it at Right Angles with the Diameters HO, and ZN, then shall HO represent the Horizon, H the South part, O the North, and the Center of the Primitive Υ the East, or West part of the Horizon, and Z the Zenith, N the Nadir, and the Diameter or Circle, ZN the prime Vertical or East and West Azimuth.

2. Set

2. Set off the Latitude $51^{\circ} 30'$ (which is the Pole's Elevation above the North part of the Horizon O) taken from the Chords from O, to NP, and draw the



Axis-Line thro' the Center, as NPVSP; and at Right Angles to this, the Equinoctial Line ÆVQ; upon which Time and Longitude is accounted; to divide which into Time or Hours of the Day and Night, the Center V represents 6, a-Clock, Morning and Evening, the Point Æ 12 at Noon, and Q 12 at Midnight. For the other Hours set off the half Tangent of 15° 30° 45° 60° 75° , both ways from the Center V towards Æ and Q on the Equinoctial Line Æ, and you have the Hour-Points, to be numbred as in the Scheme above.

3. Set off the Chord of $23^{\circ} 30'$ the Sun's greatest Declination, N; or South from Æ and Q both ways, to G and VS, and draw the Ecliplick Line (in which the Sun always moves;) to divide which into the 12 Cœlestial Signs. V. O II. G. Q. M. N. M. J. VS. and H, set off the half Tangent of 30° , 60° , &c. from the Center V both ways towards G, and VS; and prefix the Characters of the Signs as in the Scheme.

4. By Prob. 9 Case 2. Stereog. describe Parallels to the Equinoctial ÆQ at $23^{\circ} 30'$ Distance and $66^{\circ} 30'$ Distance

PART II.

S 3

from

from the North and South Poles thereof NP and SP, as P, p is the Arctick Polar Circle and u, x the Antartick Polar Circle, and $\odot, \odot$, and $\vee\delta, \vee\delta$; the first is the Northern Tropick of *Cancer*; and the other the Southern Tropick of *Capicorn*,

5. After the same manner any Parallel of Latitude or Parallel of Declination of the Sun may be describ'd at any Degrees distant from the Equinoctial, $\mathcal{A}EQ$, or the Comp. of the Latitude or Declination's Distance from the North or South Poles, as the Parallel of Latitude, It , of $45^{\circ} 00'$ North.

5. To describe any Parallel of Altitude, or Height of the Sun or Star above the Horizon, is after the same manner; only these must be parallel to the Horizon HO, as ar , a Parallel of 20° Altitude, from the Horizon, or 70° distant from its Pole Z; and so likewise the Parallels of Depression, parallel to, but under the Horizon, whose Pole is N, as the Parallel, C, $\odot$ of 18° below the Horizon, or 72° from its Pole N.

6. To describe any Meridian Circle.

These Circles intersect the North and South Poles of the World cutting the Equinoctial at Right Angles, through any Degree thereof, whose Centers are always in the Equinoctial, produced if need be. Let it be required, to draw a Meridian to cut the Equinoctial in m , or M at 45° from γ ; describe an Oblique Circle NP n SP or NPMS P , through the three Points N P, m , S, P, or NP, M, SP, and it's done; but if the Meridian shall make an Angle of any Degrees at the Poles, as suppose here of 45° , then with the Secant of 45° and one Foot in the North or South Poles NP, or SP, with the other intersect the Equinoctial, (as here) at Q or $\mathcal{A}$, which is the Center of the Circle, or Meridian required.

7. And Lastly, to describe any Azimuth-Circle.

These Circles intersect the Zenith and Nadir; that is, in the Points Z, and N, cutting the Horizon HO at Right Angles, thro' any Degrees, whose Centers lie in the Horizon produc'd if need be. Suppose I would draw an Azimuth-Circle, through the Point v or V. to cut the Horizon in 45° ; draw a Great or Oblique Circle through the three Points Z v N or ZVN, and it's done; but if it must make an Angle of 45° (or its Comp. 135°) at Z, or N, the Zenith or Nadir; with the Secant of 45° and one Foot in Z or N, turn the other about to intersect the Horizon in any point as O, or H, which is the Center of the Azimuth-Circle required.

Circles

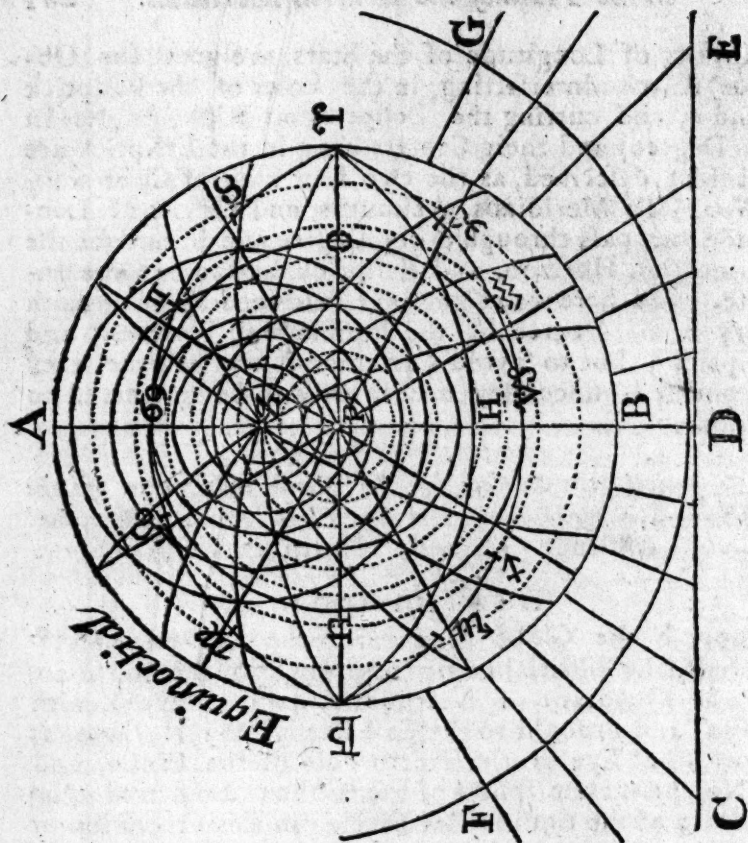
Circles of Longitude of the Stars, are great (or Oblique) Circles intersecting, in the Poles of the Ecliptick S and α , and cutting the Ecliptick at Right Angles in any Degree, and their Centers lying in the Ecliptick are therefore described, as the two Last kind of Circles.

Note, The Meridians, Azimuths, and Circles of Longitude may pass through every Degree and Minute of the Equinoctial, Horizon, and Ecliptick, and so become Infinite. And here we might have described them through every 20th, Degree of the Equinoctial, Horizon, and Ecliptick; but to avoid Confusion in the Scheme, they are omitted, since they are so easy to be conceived to be done.

II. To project the Surface of the North Hemisphere of the Sphere, Stereographically, on the Plain of the Equinoctial Circle, Latitude 51° half North.

DEFINITION.

Suppose the Globe Elevated to the Latitude of 51° and half, the solstitial Point $\odot$ brought to the Meridian, and the Quadrant of Altitude screw'd to the Zenith $51^{\circ} 30'$, and brought to the East side of the Horizon, as before, the Eye in the North Pole of the Globe, and the Northern Hemisphere of the Globe to be flatted upon the Plain of the Equinoctial Circle, in this Projection or Position, the Equinoctial is the Primitive Circle or Plain of Projection; the Center thereof of the North Pole; The Parallels of Latitude will become Parallels indeed, or whole Circles parallel to the Primitive, or Equinoctial. the Meridians are streighr Lines intersecting the pole or Center, as also the solstitial Colure. But the Horizon, prime Vertical and Ecliptick cutting the Equinoctial at Oblique Angles, on the Globe, will become Oblique Circles in the Plain of Projection; and so will also the Azimuth Circles; and the polar Circles and Tropicks being of the same nature with Parallels of Latitude, will be whole Circles parallel to the Equinoctial or primitive Circle.



CONSTRUCTION.

1 Draw the Primitive Circle $A \gamma B$ with a Chord of 60° and Quarter it with $\sphericalangle P \gamma$, and $A P B$, the first Representing, the prime Meridian, the solstitial Colure, 2. P . the Center being the North Pole; with the half Tangent of $10^\circ, 20^\circ, 30^\circ, 40^\circ$, &c. in the Compasses, and one Foot in the Center P , describe the prickt concentrick Circles; which being parallel to the Equinox, represent the severall Parallels of Latitudes of $80^\circ, 70^\circ, 60^\circ, 50^\circ$, &c.

2. Divide the Equinoctial or primitive Circle $\gamma A \sphericalangle B$ into 360° or equal parts by a Line of Chords, with $10^\circ, 20^\circ, 30^\circ, 40^\circ$, &c. from γ towards A , and so round, you may number it to 360° , to the point γ again; which may represent Longitude or Time, from the Center P to each 10th, Degree (or as here in the Scheme, to every 30th Degree) to avoid too much Confusion in the Lines) draw Right Lines or Semi-diameters, which are so many severall Meridians.

3. Then

3. Then consider, that the Distance between the Zenith and North Pole is equal to the Complements of the Latitude, in this Case $38^{\circ} 30'$; take therefore the half Tangent of $38^{\circ} 30'$, and set it off from P to Z, and through the Points Δ , Z, Υ , draw the Oblique Circle Δ Z Υ , representing the prime Vertical.

4. Now the Horizon being distant from P the Pole, equal to the Latitude $51^{\circ} 30'$, set off the half Tangent of $51^{\circ} 30'$ from P to H, and draw through the Points Δ , H, Υ , the Oblique Circle Δ , H, Υ , which represents the Horizon.

5. Now the Tropick of *Cancer*, and Ecliptick being each $66^{\circ} 30'$ distant from the North Pole P, set off the half Tangent of $66^{\circ} 30'$ from P to Θ Υ , and draw thro' the points Δ Θ Υ the Oblique Circle Δ Θ Υ , for the Northern Semicircle of the Ecliptick; to divide which into Signs, (by *Prob. 6. Case 3. Stereog.*) lay 30° and 60 on the Oblique Circle from Υ and Δ towards Θ , and place thereon the six Northern Signs from Υ towards the left hand, as Υ δ Π Θ α Υ . (Δ) 4, the Tropick of Θ being also $66^{\circ} 30'$ from the North Pole, with the half Tang. of $66^{\circ} 30'$, and one Foot in P the Pole, sweep the black Line Circle n. δ . ONS, which is the Northern Tropick of *Cancer*.

6. After the same manner, with the half Tangent of $23^{\circ} 30'$, and one Foot in the Center P, sweep the Artick polar Circle represented in the Scheme by a small black Circle parallel to the Primitive or Equinoctial.

7. For the Almicanthers, or Parallels of Altitude,

On the Point Z, the Zenith and Pole of the Horizon, or Oblique Circle Δ H Υ , describe (by *Prob. 9. Case 3. of Stereog.*) Parallels to the Oblique Circle or Horizon at 10° , 20° , 30° , 40° , 50° , &c. from its Pole, and they will be the Parallels of Altitude of 80° , 70° , 60° , 50° , 40° , &c.

8. For the Azimuth-Circles.

Set off the Tangent of the Latitude ($50^{\circ} 30'$) from P, the Center on the Solstitial Colure, to D (P B being produced) or the Secant of the same from Z to D; and on Z, with the Radius Z D sweep the Semi-circle F D G; then opening your Sector, to the Radius Z D (having first

first drawn the Tangent-Line CE) set off the Chords of 10° , 20° , 30° , 40° , &c. from D towards F and G; and drawing Secant-Lines from Z through the Semi-circle FDG, to intersect the Tangent-Line CE, these Intersections shall be the Centers of the Azimuth-Circles.

Or thus, more briefly (the Sector being open'd to the foresaid Radius ZD) set off the Tangent of 10 , 20 , 30 , 40 , &c. on both sides D upon the Tangent-Line CE, and 'twill give the same Center-points.

Then extending your Compasses from each Center-point to Z, sweep the respective Oblique Circles, and they shall be the Azimuth-Circles in this Projection.

And Lastly, For the Circles of Longitude of the Stars, they must pass thro' the Point where the Arctick Circle cuts the Solstitial Colure, and cut the Ecliptick (Oblique) Semi-circle at Right Angles, by *Prob. 5. Case 4. of Stereog.*

III. To project half the Superficies of the Globe (Sphere) on the Plain of the Horizon, Stereographically, Latitude 51° and half North.

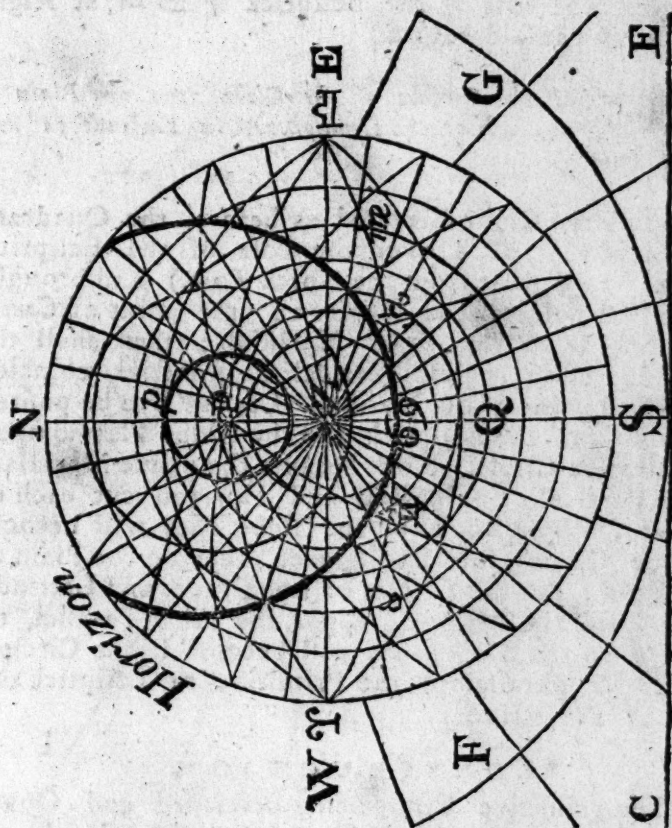
Suppose the Globe Elevated, &c. as before, and the Eye in the Zenith-point (or $51^{\circ} 30'$ on the Brass Meridian) the Globe flattened upon the Horizon, in this Position, the Horizon is the Plain of Projection, or primitive Circle, its Center the Zenith-point, the prime Vertical, or East and West Azimuth (the Solstitial Colure,) and other Azimuth-Circles become Right Circles, or Streight Lines; but the prime Meridian, Equinoctial, and other Meridians, and Ecliptick, making Oblique Angles on the Globe, make Oblique Circles in this Projection. The Parallels of Altitude are whole Circles, parallel to the Primitive, or Horizon; but the Parallels of Latitude, are Parallels to an Oblique Circle, viz. to the Equinoctial, the Circles of the Stars Longitude, as in the last.

C O N S T R U C T I O N.

The primitive Circle being describ'd with a Chord of 60° , and quarter'd, then shall WZE be the East or West Azimuth or prime Vertical, and NZS the Solstitial Colure: For the primitive Circle's Center is the Zenith-point, and the primitive Circle the Horizon; which divide into 360° beginning from E, towards N, and so round,

round, and drawing the Azimuth from Z, the Center, to every 10th Degree, as you did the Meridians in the former Projection; and also sweeping the Parallels to the Primitive at every 10° , distant from Z the Center, which in this are Parallels of Altitude, (as they were in the Last Parallels of Latitude.)

2. Make ZP equal to the half Tangent $38^{\circ} 28'$ the Complement of the Latitude, and draw the prime Meridian VPZ , as you did the prime Vertical in the last Scheme ; and because the Distance, between the Zenith and Equinoctial is equal to the Height of the Pole, or Latitude set off the half Tangent $51^{\circ} 30'$



from the Z to Q, (so shall ZQ+ZP be = 90° the distance between the North Pole and Equinoctial. Then draw an Oblique Circle $\gamma Q\alpha$, which is the Semi-Equinoctial.

Now 3:

3. Now the Distance between the Zenith and Ecliptick is equal to the Elevation of the Pole made less by the Sun's greatest Declination; that is, $51^{\circ} 50' - 23^{\circ} 30' = 28^{\circ} 00'$; therefore set off the half Tangent of 28° from Z, to $\odot$, and thro' γ , $\odot$, and Δ , draw the Oblique Circle, or Northern half of the Ecliptick, and divide it into Signs, as you did in the last. Also draw the Parallels of Latitude from P, and the Meridians through P; in all respects, as you did the Parallels of Altitude, and Azimuth-Circles in the former Equinoctial Projection, and it's done.

Circles of Longitude of the Stars in this Projection, are Oblique Circles passing through p, the Pole of the Ecliptick and cutting the Ecliptick $\gamma \odot \Delta$, at Right Angles, as was said before.

IV. *To project the Surface of the Globe upon the Plain of Ecliptick, on the Northern Hemisphere, in Latitude 51° and half North*

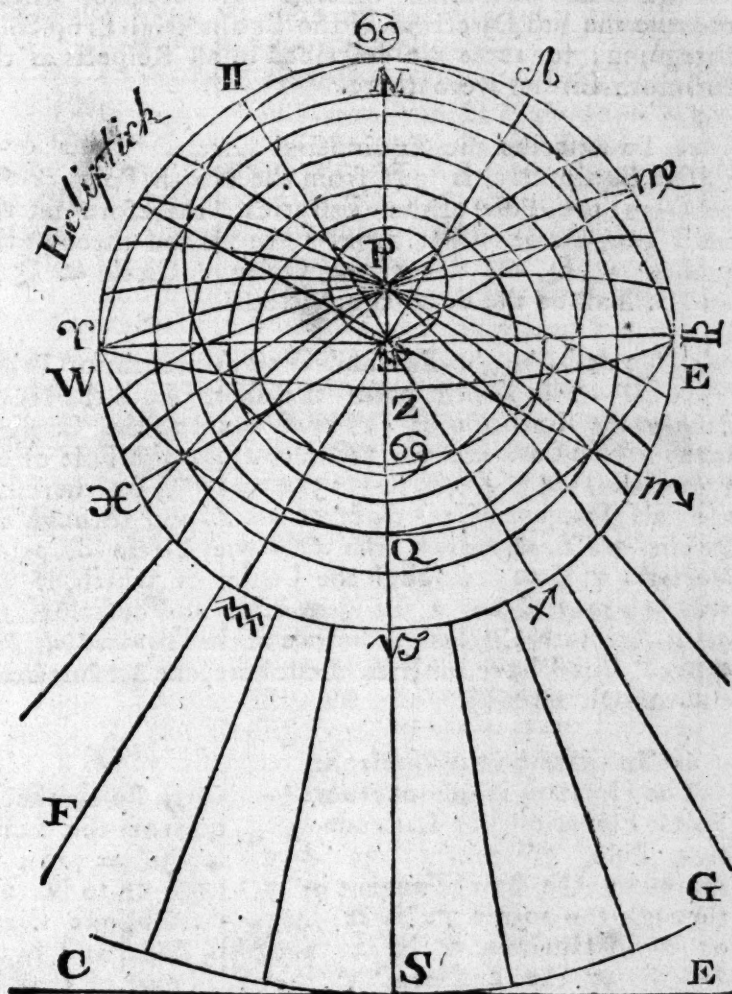
Suppose the Globe Elevated as before, the Quadrant of Altitude Screw'd over the Pole of the Ecliptick (*viz* $23^{\circ} 30'$ distant from the North Pole.) And brought to the East point of the Horizon; the point of Cancer $\odot$, brought to the Bra's Meridian; then shall the prime Meridian, prime Vertical, Equinoctial, coincide in the East point of the Horizon. Let the Eye be posited in the Pole of the Ecliptick, and the Globe Flatted upon the Ecliptick Circle. Now because the prime Meridian, prime Vertical, Equinoctial and Horizon cut each other at Oblique Angles on the Globe, they will become Oblique Circles intersecting each other on the Plain of Projection: But the Circles of Longitude, and Latitude of the Stars, the first being Right, the other Parallel, to the Eye, on the Globe, they will become Right Circles, and Circles parallel to the Primitive or Ecliptick the Plain of Projection.

CONSTRUCTION.

1. The primitive Circle being described and Quartered, as before then shall W γ EP Δ E be the Equinoctial Colure and $\odot$ N P EP Q VS. the Solstitial Colure; the first cutting the Primitive or Ecliptick, in the Equinoctial Points γ and Δ , the second in the Solstitial points $\odot$ and VS.

2. Divide

2. Divide the Ecliptick or Primitive into 12 Equal parts, (which may be done with the Chord of 30° :) Or (if you will, into 360 equal parts) then drawing Lines from the Center $\mathcal{H}$ P, to each point, of 33° in the primitive place thereon, the Characters of the 12 Signs, as you see in the Figure, and these Right Circles so drawn are the Circles of the Sun or Stars Longitude; and drawing Circles parallel to the Primitive, at any Degrees distant therefrom, shall be Circles of Latitude, of the (Sun or) Stars.



3. To

3. To describe the prime Meridian.

Consider the North Pole of the World is distant from the Pole of the Ecliptick 23° and half; set therefore the half Tangent of $33^{\circ} 30'$ from EP to P , and P is the North Pole; describe an Oblique Circle through the Points $\gamma P \infty$, and you have the prime Meridian, whose Center is S , (PS is = Secant and EPS to the Tangent of $90^{\circ} - 23^{\circ} : 3' = 66^{\circ} 30'$) With the Radius PS sweep the Arch ESG , and draw from the point S the Tangent Line CSE ; in which Line lie the Centers of the other Meridian Circles. To describe which, observe the 7th Direction of the Equinoctial Projection foregoing; for these are described in all Respects as the Azimuth-Circles were there.

4. To describe the Equinoctial Circle.

The Equinoctial is 90° from the North Pole, or $66^{\circ} 30'$ from the Pole of the Ecliptick; therefore set the half Tangent of $66^{\circ} 30'$ from EP to Q , and through the points γ, Q, ∞ , draw the Oblique Circle $\infty Q \gamma$, and it shall be the Semi-equinoctial.

5. To describe the East and West Azimuth, or Prime Vertical. The Zenith-point is distant from the North Pole in the Latitude of 51° half, equal to the Complement of the Latitude *viz.* $38^{\circ} \frac{1}{2}$ or from the Pole of the Ecliptick 15° (*i.e.* $38^{\circ} \frac{1}{2} - 23^{\circ} \frac{1}{2} = 15^{\circ}$;) set therefore the half Tangent of 15° from EP to Z , and through the Points γ, Z, ∞ , draw the Oblique Circle or prime Vertical $\gamma Z \infty$; through the Center of which, if you will you may draw a Tangent-Line and describe the other Azimuth-Circles as before in the *Equinoctial Projection*. But I have omitted them here, the Scheme being full enough already.

6. To describe the Horizon.

The Horizon is distant from the North Pole equal to Poles Elevation, or Latitude $51^{\circ} \frac{1}{2}$, or from the Ecliptick Pole, 75° *viz.* $51^{\circ} 39' + 23^{\circ} = 75^{\circ}$; set therefore the half Tangent of 75° from EP to N , and through the points γ, N, ∞ , draw the Oblique Circle or Semi-Horizon $\gamma N \infty$, and 'tis done, and N the North ∞ the East and γ the West part of the Horizon.

7. To

To describe the polar Circle.

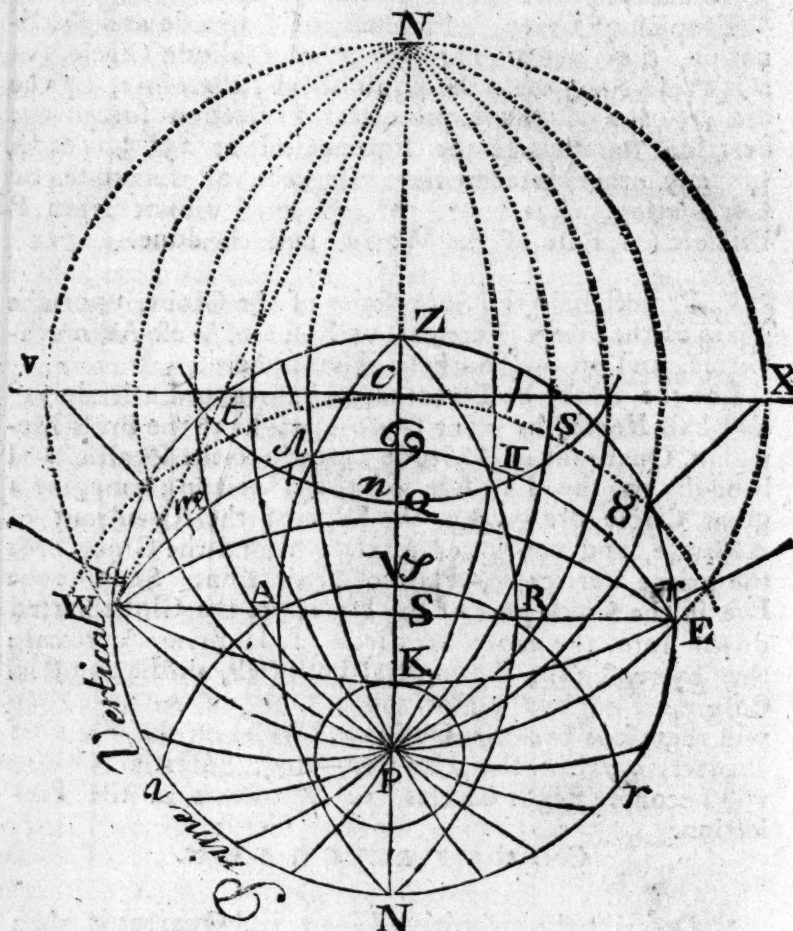
Tropick of *Cancer*, Parallels of Latitude and Declination, they are all Parallels to an Oblique Circle, (in this Projection) viz. the Equinoctial; therefore, by the 6th Direction of the Equinoctial Projection foregoing, describe Parallels to the Equinoctial at $23^{\circ} \frac{1}{2}$, $66^{\circ} \frac{1}{2}$, (or any other intermediate Degrees, of Latitude, or Declination, as 10° 20° 30° 40° , &c.) distant from P its Pole (or Pole of the World) and 'tis done.

8. Project half the Superficies of the Globe, upon the Plain of the prime Vertical, or East and West Azimuth-Circle, and on the Southern Hemisphere.

Let the Globe be Elevated, as before, to Latitude 51° and half North, the point *Cancer* brought to the Brass Meridian, Quadrant of Altitude Screw'd to the Zenith, and brought to the East side of the Horizon; imagine a great Circle drawn by the Edge of the Quadrant of Altitude, and continued directly round the Globe, for the prime Vertical or Plain of Projection. Suppose the Eye in the South part of the Horizon; the Globe flatted down into the aforesaid Circle of the prime Vertical; then by reason the Equinoctial Ecliptick, and Equinoctial Colure are cut by the Horizon at Oblique Angles; they will therefore become Oblique Circles on the Plain of Projection; but the Horizon, and Solstitial Colure will become Right Circles, or Diameters in the Projection.

CONSTRUCTION.

1. Describe the primitive Circle, and Quarter it, then shall the primitive Circle be the plain of the prime Vertical, the Right Circle W S E the Horizon, and the Right Circle Z S N, the Solstitial Colure the Point Z the Zenith, and N the Nadir, and the Center, S the South point of the Horizon, and W the West and E the East point thereof.



2. Set off the Latitude, 51° half from S to P, downward by the half Tangent; through the points W, P, E, draw the Oblique Circle WPE for the prime Meridian; through the Center of which, C describe the Line VX parallel to the Horizon WE; and making CP a Radius in your Sector, sweep an Arch WCE from P as a Center; and to this Radius, set off the Chords of 10, 20, 30, &c. on both sides C, upon the Arch WCE; or, if you will, as here, in the Figure, the Chord of 15, 30, 45, 60; and laying a Ruler from P to each Division, to Cut the Tangent-Line, VCX, in the the Centers of the other Meridians, Describe the Meridians, through

through P, (as you did the Azimuth Circles through Z in the 7th Direction of the Equinoctial Projection foregoing.

3. Set off the half Tangent of 38 Deg. and half (the Compr. of the Latitudes from S, upwards to Q, and through the Points W, Q, E, draw the Oblique Circle WQE for the Equinoctial.

4. The Ecliptick being 23 Deg. and half distant from the Equinoctial, set off the half Tangent of 38 Deg. and half + 23 Deg. and half = 62 Deg. from S to $\mathfrak{S}$, and through the Points W, $\mathfrak{S}$, E, describe the Oblique Circle W, $\mathfrak{S}$ E for the Ecliptick, and divide it into, and Character it with the Six Northern Signs, γ , δ , Π , δ , $\mathfrak{Q}$, $\mathfrak{M}$, and $\mathfrak{A}$, as before, in the 4th Direction, of the Equinoctial Projection foregoing.

5. The Azimuth-Circles are described, in all respects, as in the 7th Direction of the General Meridian Projection foregoing, as ZAN and ZRN.

The Parallels of Altitudes are also projected as in the 5th Direction of the said Meridional Projection foregoing; as t , n , s , &c.

7. On P, describe (by *Prob. 6. Case 3. of Stereog.*) a Parallel to an Oblique, *i. e.* the Equinoctial WQE, at 23 Deg. and half from the Pole P, for the Antartick, or South polar Circle.

After the same manner describe a parallel to an Oblique Circle (or the Equinoctial) at 66 Deg. and half distant from P, for the Tropick of Capricorn, as $\mathfrak{V}\mathfrak{S}\mathfrak{r}$.

8. The Pole of the Ecliptick, is distant from $\mathfrak{S}$, or the Ecliptick 90 Deg. or from S, the South part of the Horizon $90^\circ - \mathfrak{S}\mathfrak{S} 62^\circ = 28^\circ$, or from the Pole P 23 Deg. and half *i. e.* $\mathfrak{S}P 51^\circ$ Deg. and half $- 28^\circ = 23^\circ 30'$. Set therefore 28 Deg. from S to K (taken from the half Tangents) and through the Points $\mathfrak{A}$, K, γ , draw the Oblique Circle WKE, which is a Circle of Longitude, of the Stars that are in γ , and $\mathfrak{A}$; and for the other Circles of the Stars Longitude, describe them, to pass through the point K at every 30 Deg. 15 Deg. or 10

Degrees after the same manner as the Meridians are drawn thro' P.

VI. To project half the Sphere upon any Oblique Circle ; and to find in what Latitude and Difference of Longitude from the Place where the Oblique Circle deviates from the Meridian and Zenith (of a Known Latitude) shall be an Horizontal Plain.

In this Projection you have, (in some Measure ;) especially the difficult part of the whole Art of Dialing demonstrated.

CONSTRUCTION.

As great Circles of the Sphere. are Horizons in some place of the World or other ; so all Plains on which Sun-Dials are made are Horizontal Dials in some one part of the World, and such a Plain (or Great Circle) which I mean here, will best be represented by the Roof a House.

Now suppose in the Latitude of London 51 Deg. and half North, the side of a House should decline from the South, Westward 24 Deg. 20 Min. and the Roof thereof incline towards the North part of the Horizon 36 Degrees. I would know in what Latitude, and how much Differing in Longitude that part of the World is, in which this Oblique Plain to the Meridian and Zenith (or Horizon) of London will be an Horizon ?

1. Having

3



T 2

I. Having

1. Having drawn a primitive Circle $HXQD$ to represent this Oblique Plain (or Circle) and also drawn the Diameter HO , crossing it at Right Angles in Q with the Diameter CF ;
2. Take the half Tangent of the Plain's Inclination 36 Deg. and set it from Q to Z ; so shall Z be the Zenith of *London*, and Q of the place inquired.
3. Take the half Tangent of 54 Deg. (the Complement of the Plain's Inclination;) and set it from Q to B ; so shall B be one point through which the Horizon of *London* must pass.
4. Take the Tangent of 36 Deg. (the Plain's Inclination;) and set it from Q to C , or the Secant of 36 Deg. and set it from B to C ; either of which Distances shall give the point C , for the Center of the Horizon of *London* HBO .
5. Take the Tangent of 54 Deg. (the Comp. of the Inclination) and set it from Q to E .
6. Take 24 Deg. 20 Min. (the Declination of the Plain,) from the Chords, and set it from H to C , from q to d , and from O to e .
7. A Ruler laid from Z to c, d , and e , will give (in the Horizon HBO) the points W, S , and E , for the West South, and East points of the said Horizon of *London* HBO .
8. Draw FG perpendicular to QF , (or parallel to HO) and extend it as far towards G as you shall think requisite.
9. Draw a Line thro' the Point P and W (which if you have committed no former Error, will pass also thro' the Point Q the Center,) and extend it till it cut the Line FG last drawn; so shall the point G of the Intersection be the Center of the Meridian. Then with the Distance or Radius $GS = GZ$ describe the prime Meridian PZS , which is the Meridian of the Place where the Declining, Inclining Plain in *London* will be an Horizontal Plain.

10. A Ruler laid from W to Z, will cut the primitive Circle in *a*, from which point set the Chord of 38 Deg. and half, the Comp. Latitude London, to *b*; and a Ruler laid from W to *b*; will cut the Meridian in the Point *P*; for the North Pole of the World.

11. Set 90 Deg. of your Chords from *b* to *f*, and from *f* to *g*: A Ruler laid from W to *f*, will cut the Meridian in *Æ*, for the Point where the Equinoctial must cut the Meridian; and laid from W to *g* will cut the Meridian (Extended) in *M*, for the South Pole; and a Right Line drawn through *P*, *Q*, and *M*, shall be the Axis of the World.

12. The Equinoctial Circle must be drawn thro' the Points W, *Æ*, E; and to find the Center thereof, divide the Right Line W, E, into two equal parts in *R*; and upon *R* raise the Perpendicular RT, drawing it forth till it meet with the Axis of the World produced to the point *T*, which is the Center of the Equinoctial Circle; or if from *R* through *C*, the Center of the Horizon, a Right Line be drawn, and Extended, till it concur with the Axis of the World PQM; Extended, it would meet in *T*, the Center of the Equinoctial also.

13. Divide MP in two equal parts at Right Angles in *D*, and draw DG, extended infinitely.

14. Upon *P*, at the Distance PD or any other) describe the Semi-circle LDN, and Laying a Ruler from *P*, to *G*, the Center of the Meridian PZS, it will cut the Semi-circle LDN in *K*; at which point *K*, begin to divide the Semi-circle into 12 Equal parts at the Points $\odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot$. Then a Ruler Laid from *P*, to each of those 12 Equal parts $\odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot, \odot$ shall give the points of the Tangents of $15^\circ, 30^\circ, 45^\circ, 60^\circ, \&c.$ upon the Tangent-Line GD; in which shall be the Centers, of the several Meridians; *G* being the Center of the first Meridian of 12 a-Clock, or the Meridian of the Place. Or with your Sector, make PD equal to the Tangent of 45° , and so the Tangents of $15^\circ, 30^\circ, 45^\circ, 60^\circ, \&c.$ being set upon the Tangent-Line GD, both ways from *D*, shall give the Center of the other Meridians.

To find in what Latitude and Difference of Longitude from London, this will be an Horizontal Projection.

The Projection, thus finished as above if you take the Distance QP in your Compasses, and measure it upon your Scale of half Tangents you shall find it to contain $72^{\circ} 34'$, whose Complement to 90° is $PX = 17^{\circ} 26'$, and in that Latitude will this declining Inclining Plain be an Horizontal Plain.

And if you lay a Ruler from P to $\mathcal{A}E$, it will cut the primitive Circle in h and the Distance, n , (where the Axis PQn cuts the Primitive,) to h measured upon your Scale of Chords, shall be $14^{\circ} 41'$ which is the Plain's Difference of Longitude.

So that a Plain Declining $24^{\circ} 20'$, and Inclining to the Horizon 26° in Latitude 51 Deg. and half will be an Horizontal Plain in such part of the World which lies in the Latitude of $17^{\circ} 16'$, and differs in Longitude from London, $14^{\circ} 41'$.

Moreover, from this Projection may be reduced the several Requisites belonging to such a Declining Inclining Plain, in the Latitude of London: For,

1. The Elevation of the Meridian above the Horizon of London, is represented by the Arch HA ; which measured on the Chords, will be equal to $69^{\circ} 54'$.

2. The Difference of Meridians (which is the same with the Plain's Difference of Longitude) is represented by the Angle $QP\mathcal{A}E$, whose measure upon the Equinoctial is the Arch $V\mathcal{A}E$, or upon the primitive Circle nh , which will be found to be $14^{\circ} 41'$.

3. The Altitude of the Stile of the Dial above the Substile thereof, represented by the Arch of the Meridian PA whose Measure is the Complement of the Arch QP ; which measured upon the Scale of half Tangents, will be $72^{\circ} 34'$ whose Complement PA $17^{\circ} 26'$ is the Altitude of the Stile of the Dial above the Substile equal to the New Latitude.

4. The Distance of the Substile from the Meridian represented by the Arch of the primitive Circle AX , which is equal to the Chord of $4^{\circ} 30'$; and so far must the Substile be removed from the Meridian, upon the Dial's Plain.

5. And

PART

5. And for the Hour-distances upon the Plain, the Projection will give them also : For a Ruler laid from Q to the Intersection of the several Meridians upon the primitive Circle, Lines drawn hereby, will be True Hour - Lines of such an Inclining Declining Plain.



A Brief Synopsis of this Oblique Projection.

HXOD the primitive Circle ; or the Declining Inclining Plain.

QZ equal to the Half Tangent of 36° , the Plain's Inclination.

QB equal to the Tangent of 56° , or $BC = \text{Secant of } 36^\circ = \text{Plain's Inclination}$, and is the Center of the Horizon.

QF equal to the Tangent $54^\circ = \text{Comp. Plain's Inclination}$.

$Hc = gd = Oe = 24^\circ$ $2c' = \text{Plain's Declination}$. A Ruler laid from Z to c, d , and e , will give W, S, and E in the Horizon, GF to QF, or $\parallel$ HO.

EW extended, gives G for the Center of the Meridian.

A Ruler laid from W to Z, gives $ag = \text{Chord } 38^\circ$ and half $= \text{Comp Latitude}$; WB gives P for the Pole of the World, $bf = fg = \text{Chord of } 90^\circ$.

A Ruler laid from W to f , gives $\text{\AA}$ the Equinoctial ; Wg gives the South Pole M ; and PQM is the Axis of the World ; through W $\text{\AA}$ E draw the Equinoctial ; divide MP into two equal parts in D, and draw DO infinitely on both sides PDM ; upon P, at the Distance PD, describe the Semi-circle LDN.

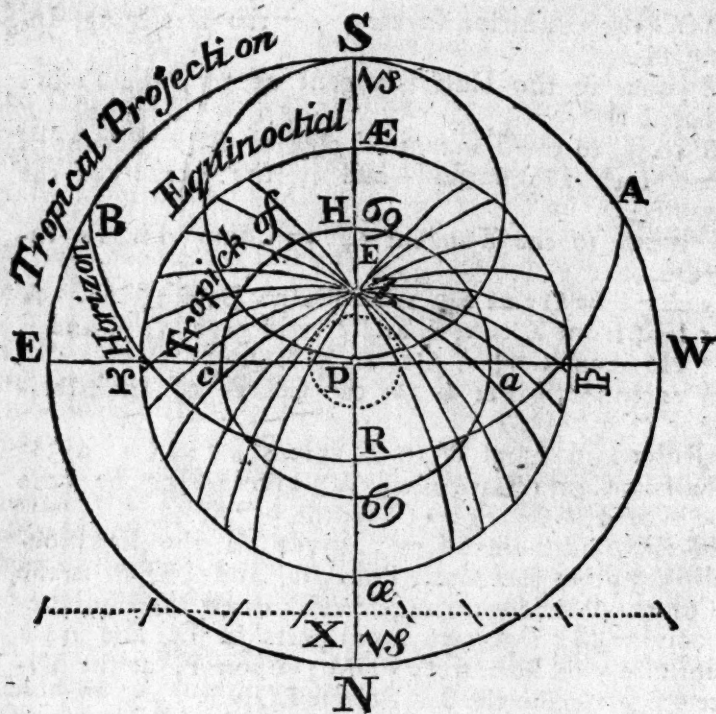
A Ruler laid from P to G, will cut the Semi circle in K ; at K begin to divide LDN into twelve equal Parts.

A Ruler laid from P to those equal parts, will give the Tangents of $15^\circ, 30^\circ, 45^\circ$, &c. upon GDO, and be the Centers of the Meridians, G being the Center of 12.

Lines drawn from Q through the Intersections of the Meridians, with the primitive Circle (representing the Declining Inclining Plain) shall be the true Hour-Lines upon such declining inclining Plain.

And this declining inclining Plain will be an Horizontal Plain in the Latitude of $17^{\circ} 26'$, and differing in Longitude from London Eastward $14^{\circ} 41'$, which will be about the Kingdom of Cassena.

VII. To project half the Sphere upon one of the Lesser Circles, (viz. of the Tropick of Capricorn, Latitude London, 51° and half North.



CONSTRUCTION.

With 60 Degrees of the Chords describe the primitive Circle representing the Equinoctial, crossing it at Right Angles with $\approx P \gamma$ for the Equinoctial, and $\approx E P \alpha$ for the Solstitial Colure; which is also the Meridian of the Place.

2. Take

2. Take the half Tangent of 11; Degrees and half (that is, the Distance of the Tropick of *Capricorn* from the North Pole of the World P;) with the same Extent on P describe the Circle ESWN, to represent the Southern Tropick or Plain of Projection.

3. Take the half Tangent of 66 Degrees and half equal to Comp. Sun's Declination (or his distance from the North Pole,) and on the Center P describe the Circle $\odot$ $\odot$ $\odot$ for the Northern Tropick of *Cancer*.

4. Divide the Space between $\odot$ and upper *Capricorn* $\odot$, into two equal parts in the Point E; so shall E be the Center, with the distance E $\odot$, to describe the Circle of the Ecliptick γ $\odot$ ω $\odot$.

5. Take the half Tangent of 51 Degrees and half (the Latitude) and set it from P to R, and through the Points γ R ω describe the Oblique Circle ω R γ B for the Horizon, intersecting the North part of the Meridian.

6. Take 28 Degrees and half the Comp. Latitude from the Tangents, and set it upon the Meridian from P Southward to H; on which Point H you may describe the Horizon to cut the Ecliptick in the Points γ and ω .

Next, to divide the Ecliptick into the Twelve Signs is thus:

7. Divide the Equinoctial $\text{Ae} \omega \gamma$ into Twelve equal parts: then laying a Ruler to E (the Center of the Ecliptick, and every of those Divisions of the Equinoctial) it will cut the Ecliptick Circle in the Points γ , δ , II , $\odot$, Q , IX , ω , M , X , $\odot$, III , XII ; and according as you divide the Equinoctial into every 10th, 5th, or every single Degree: So you may divide the Ecliptick also.

8. For the Azimuths, or Vertical Circles, to the Radius of the Equinoctial Circle; Take the half Tangent of $38^{\circ} \frac{1}{2}$, and set it from P to Z (towards the South part of the Meridian,) so shall Z be the Zenith-point, where all the Azimuth Circles must concur; and how to project them, is in this manner;

9. Take

9. Take $51^{\circ} \frac{1}{2}$ (the Latitude from the Tangents) and set it from P to X; and through X draw a Tangent-Line parallel to γP , extending it infinitely on both sides of the Meridian.

10. Make ZX, a new Radius, equal to a Tangent of 45 Degrees; and from that Tangent-Line, from X both ways, set the Tangent of 10° , 20° , 30° , 40° , &c. which Points with their respective Distances, from Z the Zenith, shall be the Centers whereon the Azimuths must be described.

11. For the Circles of Position, draw a Right Line through H, the Center of the Horizon, parallel to γP , (the Equinoctial Colure) then divide the South semi-circle of the Equinoctial, into six equal parts; a Ruler laid from P the Pole, to each of these six Divisions, will cut the former parallel Line in the Points *, *, *, &c. and those shall be the Centers whereby to describe the Circles of Position.



A Brief Synopsis of this Projection.

P The Pole of the World; as also, of the Equinoctial and the two Tropicks.

PV, equal to half Tangent of $113^{\circ} 30'$.

P $\odot$ equal to half Tangent of $66^{\circ} 30'$.

P R, equal to half Tangent of the Latitude $51^{\circ} 30'$.

H, equal to half Tangent of the Comp. Latitude $38^{\circ} 30'$.

PZ, the Center of the Horizon.

P E, equal to the Tangent $23^{\circ} 30'$, the Sun's greatest Declination.

E, the Center of the Ecliptick.

R, the Center through which the Circles of Position are to pass.

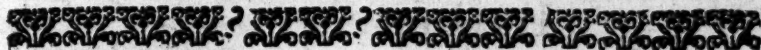
The small Circle drawn (with a prick'd Line,) with the half Tangent of 23° deg. and half; represents the Northern Polar Circle.

If

If into this Projection the Parallels of Latitude to the Ecliptick be drawn (and an Index made to move on the Center, divided as the half Tangents to the Radius of the Projection, of which I shall shew more in my *Comprehensive Treatise of Astronomy, of the Doctrine of the Planets and Eclipses*, which I am preparing for the Press. It will be an absolute Instrument to find the Rising and Setting of the Moon, and other Planets that have Latitude.

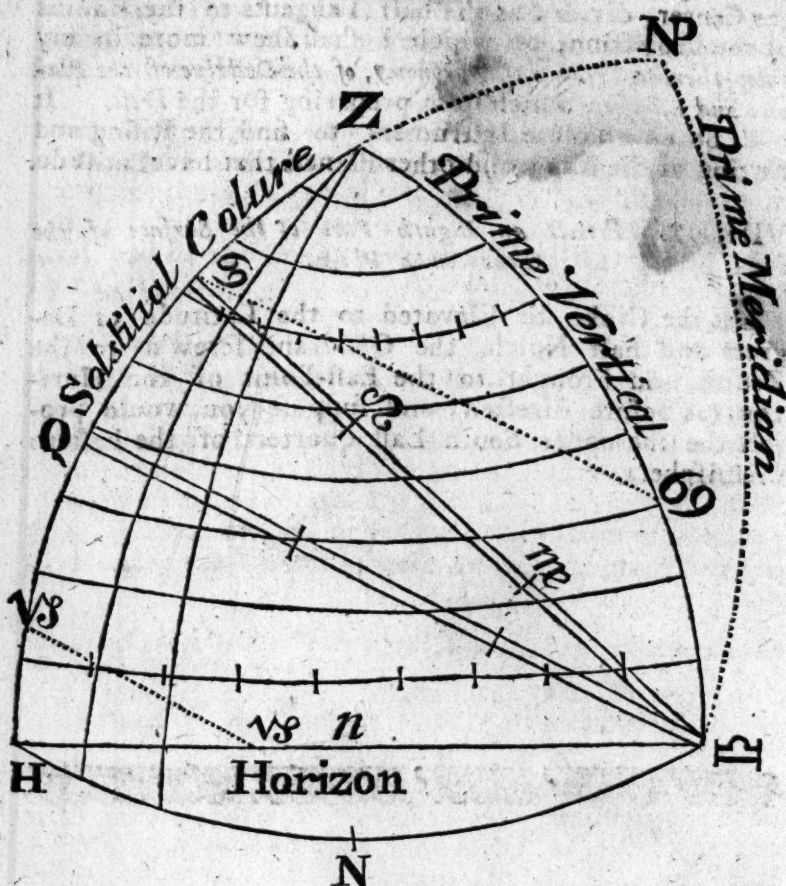
VIII. To Project an Eighth Part of the Surface of the Globe on a Plain.

Let the Globe be Elevated to the Latitude 51 Degrees and half North, the Quadrant screw'd to the Zenith, and brought to the East-Point of the Horizon, (as before directed) and suppose you would project the the upper South East quarter of the Eastern Hemisphere.



CON-

CONSTRUCTION.



In this Figure Z is the Zenith, H $\sphericalangle$, a quarter of the Horizon, all the Arches concentring at Z are Azimuths, and those Arches parallel to H $\sphericalangle$ are Parallels of Altitude.

1. Make a Spherical, Equilateral or Quadrantal Triangle, H Z $\sphericalangle$ with one Extent of the Compasses (*Prob. 97. page 234. Mathematical Recreations*) divide each side into 90 equal Parts or Degrees.

2. Let fall a Perpendicular Z n, from Z to the Middle (or 45°) of H $\sphericalangle$, which divide likewise into 90 Degrees.

3. Thro'

3. Through every point in the Perpendicular Line, to the Corresponding Points in ZH, the Solstitial Colure, and Z Σ , the prime Vertical, draw Arches, for the Almicanter or Parallels of Altitude.

4. Divide these Arches also into the like Number of Equal Parts, and through those Points draw Arches from Z to the Horizon H Σ for the Azimuth. And,

5. Count 38 Deg. and half from H to Q, and draw a Great Circle from Q to Σ for a Quadrant of the Equinox.

6. Count 62 Degrees (i. e. 28 Degrees and half + 23 Degrees and half) from H to Θ ; and from Θ to Σ , draw a Great Circle's Arch for a Quadrant of the Ecliptick.

7. Count (38 Degrees and half — 23 Degrees and half =) 15 Degrees from H to VS, and draw the prickt Line VS-VS, for part of the Southern Tropick.

8. From Θ draw the Prickt Line Θ , both parallel to the Equinoctial Q Σ , and the Projection is finish'd.

By this Method may be drawn any Eighth Part of the Earth; the Places between the 30th Parallel of Latitude, and the Pole, will be represented more true and agreeable to the Globe than by any way else in *Plano* (except the Mercatorial Chart;) for Countries are nearer their True Figure and Proportion; and their Distances to one and the same common Measure: Besides, it will, if apply'd to Nautical Uses, admit of Arches of a Great Circle very useful in Circle-Sailing.

But if you make a Terrestrial Map on this Projection, twill be convenient to make Z, the North Pole, H Σ the Equator, and then the Arches parallel to it, will be Parallels of Latitude, and the other Arches centering in Z will be Meridians.

Lastly, The Extent of the Compasses between any two Places, apply'd to either of the Outward Meridians, or the Equator, gives the Distance exact enough, especially if the Places be remote; but I shall not say any more
of

304 *Projection of the Sphere Orthographick: Or,*

of Maps in this place; because it is a Degression from my present purpose, of the Projection of the Celestial Sphere in Plane.

I pass on therefore to *Orthographick, Projection of the Sphere* by some Call'd the ANALEMMA, or Planisphere.

Definition. Orthographick Projection, supposes the Eye perpendicular to a Plain passing through the Center of the Sphere, posited in a Perpendicular to that Plain erected from the Center, but at an infinite Distance; or (which comes to the same thing) supposes that there were Perpendiculars let fall from each Point in the Circles to be projected, to the Plain, on which the Projection is to be made.

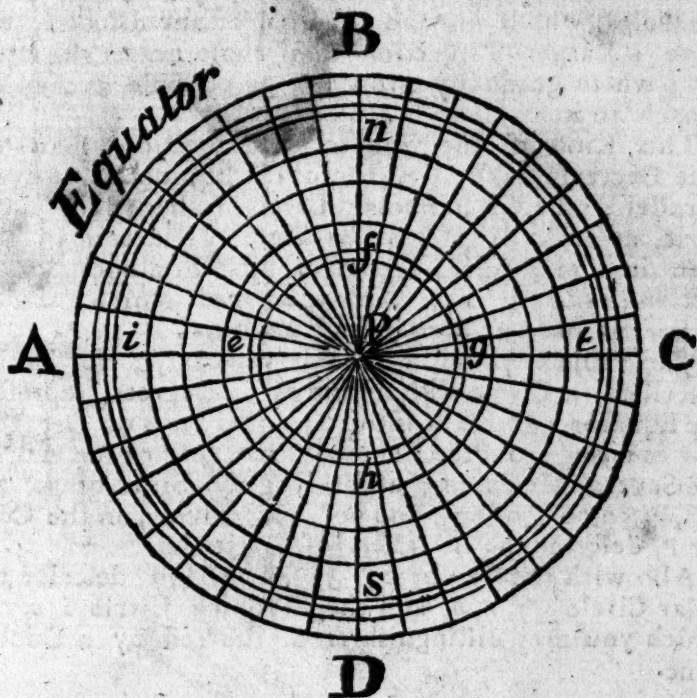
These projected Points will fall in Right-Lines on the Plain, when the Circles (great or small) to be projected, are perpendicular to the Plain of the Projection; but if Oblique, in *Ellipses*. So that we may have as great a Variety of these Kinds of Projections as on the *Stereographick*; but they being more troublesome, by reason of the Oblique Circles being projected in *Ellipses*. Mathematicians have confin'd themselves to the most Simple of these kinds, viz. either upon the Plain of the Equinoctial, or upon the Plain of the Solstitial Colure; which is admirably Useful and Easy; as shall be shewn hereafter.

I. To project the Sphere Orthographically on the Plain of the Equator.

CON-

CONSTRUCTION.

In this Terrestrial Projection, the Situation of the Eye



is supposed to be at an infinite Distance in the Axis, extended from the Center of the Earth, viz. 200 Semidiameters thereof, according to Opticks; and the Glass, or Transparent Medium, (which we suppose to be at Right Angles with this fictitious visual Ray, or Line) 100 Semidiameters from the Center, the Length thereof of one Semidiameter of the Earth. But because this is impracticable, and may be apt to confound a Young Student, therefore instead of the Earth it self, we will conceive its Representation, or the Mathematical, or Terraqueous Globe; whereof, if the Diameter be two Foot, the Eye must be posited 200 Feet distant from its Center, the Glass or Medium 100, the Breadth of which will be one Foot: And hence arises the different Figure of the Earth in this and the Equinoctial Projection foregoing: For by reason of the Eye's Position in the former, the Optick Rays or Lines, do so cut the interpos'd Medium, as that the Parallels of Latitude are so much the farther asunder, by how much nearer they are

306 *Projection of the Sphere Orthographick: Or,*

are to the Equinox: On the contrary, in this Projection, the Eye, because of the Globe's Convexity, is suppos'd to be at an Infinite Distance, before it can distinctly perceive the Equator; and consequently the Places about the Poles, which may be discern'd at any distance, will have a Larger Projection than those nearer the Equator; which gradually grow less perceptible, as they approach to it.

This Kind of the Equatorial Projection, shews the true Decrease of the Degrees, of the Equinoctial in every Parallel; the Circumpolar Lands (when laid down on it, as in a Map) may be better delineated in this than in the former, Stereographick Equinoctial projection; and so may *Tartary*, or the North Part of *Europe*, as *Sweden*, *Norway*, and *Muscovy*.

The Projection, on a Plain is very easy; for having described, a Circle with a Sine of 90 Degrees, represent the Equator; divide it into 360 equal Parts or Degrees; and drawing from the Center (P) the Pole of the World) the Several Meridians; and taking the Sines of 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90°, successively, on the Center P, describe the Parallels of Latitude.

Also with the Sine of 23° 30' and 66° 30', describe the polar Circle *e, f, g, h*, and the Tropick Circle *i, n, r, s*, which you may distinguish from the rest by a Double Line.

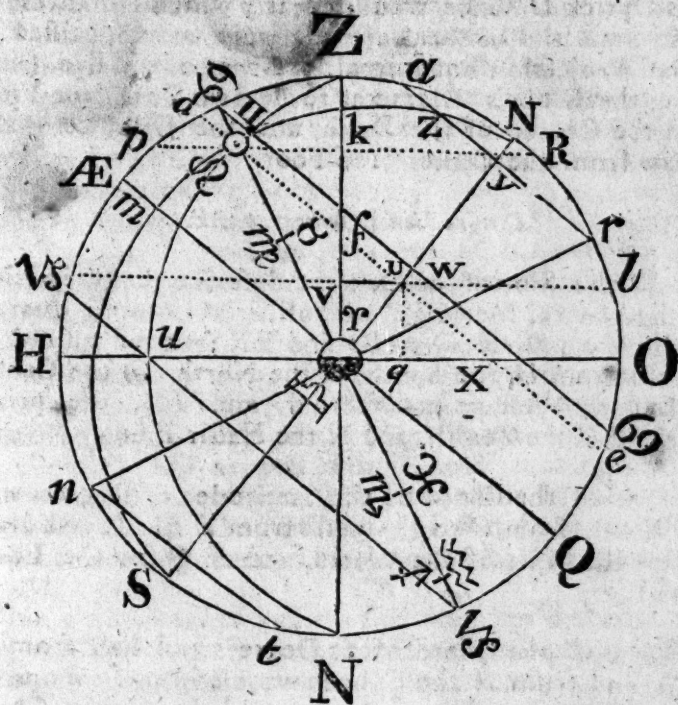
II. *To Project the Sphere Orthographick, on the Plain of the General Meridian or Solstitial Colure, Latitude 52 Degrees and half North Sun's present Declination, 21 Deg. North, his Greatest Declination, 23 Degrees and half.*

D E.

the I
frei
Para
dians
tude,
but S
the P
cular
phere.
Tha
suppo
tor, ov
and D
preced
each P
the Ser
PART

DEFINITION.

In this Projection, the Parallels, of Latitude, Declination Polars, and Tropicks are Right Lines, parallel to



the Equinoctial ; the Parallels of Altitude are also streight Lines parallel to the Horizon ; likewise the Parallels of Latitude of the Stars ; but, all the Meridians, Azimuth-Circles, and Circles of the Stars Longitude, are neither Right Lines, nor Arches of Circles, but Semi-Ellipses ; which Construction is formed, if, in the Plain of the first Meridian, you conceive perpendicular Lines to fall from all Points of each Hemisphere.

That you may the clearer apprehend this, we will suppose ONHZ the Equator, O the Place in the Equator, over which the Eye is situated in the same Manner and Distance as is shewn in the Equatoreal Projection preceding : Therefore Lines falling perpendicular from each Point in the upper Hemisphere, represented by the Semi-circle ZON, to the Corresponding Points in

the Obverse Hemisphere, represented by the Semi-circle ZHN, shall cut the Plain of the Meridian, or the Diaphanous Medium, which in this Position of the Globe, will be ZON, in the same Points as the Rays drawn from the same points in the Surface ZON, to the Eye in its Optick Distance would cut it; which Distances of the Eye and the Medium, are the same as is specified in the last Projection Equatoreal, viz. that of the Eye (supposing the Globe's Diameter to be two Foot) 200 Foot from the Center of the Earth, and the Distance of the Medium from the Center 100 Foot.

Construction Orthographick.

1. With a Sine of 90 Degrees describe the Primitive for the general Meridian, or Solstitial Colure; quarter it with Two Diameters HO and ZN; then shall HO be the Horizon, H the South, O the North, $\odot$ the Center the East, or West points thereof, and ZN, the prime Vertical, Z the Zenith, and N the Nadir Point.

2. Set off the Chord of the Latitude 51 Degrees and half North from O to H, and from Z to $\mathcal{A}$, and draw NS for the Axis of the World, and $\mathcal{A}$ Q for the Equinoctial.

3. Set off the Chord of 20 Degrees and half from $\mathcal{A}$ to $\mathcal{G}$, and from N and S. both ways to a and r n and t ; draw from the Point $\mathcal{G}$ through the Center $\odot$ the Ecliptick line $\mathcal{G}$ VS; which divide into Signs from the Line of Sines, by setting off from the Center $\odot$ both ways the Sine of 30 and 60 Degrees, and place the Characters of the Twelve Signs as you see in the Figure.

4. Draw the Right Lines a , r , n , t , $\mathcal{G}$, $\mathcal{G}$ and VS, VS each of them parallel to $\mathcal{A}$ the Equinox; so shall a , r , and n , t . represent the Polar Circles, and $\mathcal{G}$, $\mathcal{G}$, the North, VS, VS, the Southern Tropic.

5. Set off the Chord of 21 Degrees from $\mathcal{A}$ to d , and draw the Prickt Line $d e$ parallel to the Equinoctial, for the parallels of the Sun's Present Declination: After the same manner you may draw any Parallel of Latitude of any Degrees given, towards either Poles, according as your Latitude is North or South. In like manner you must draw the Parallels of Latitude of the Stars; which

which must be parallel to the Ecliptick Line $\odot$ VS ; and so of the Parallels of Altitude of the Sun or Stars; they must all be drawn parallel to the Horizon HO ; as the Parallel of Altitude pr , passing through $\odot$ the Sun.

6. To describe the Meridians, they are all Semi-Ellipses, drawn through every Degree, of the Equinoctial to co-incide in the North and South Poles; N and S , NS being the Transverse Diameter to them all.

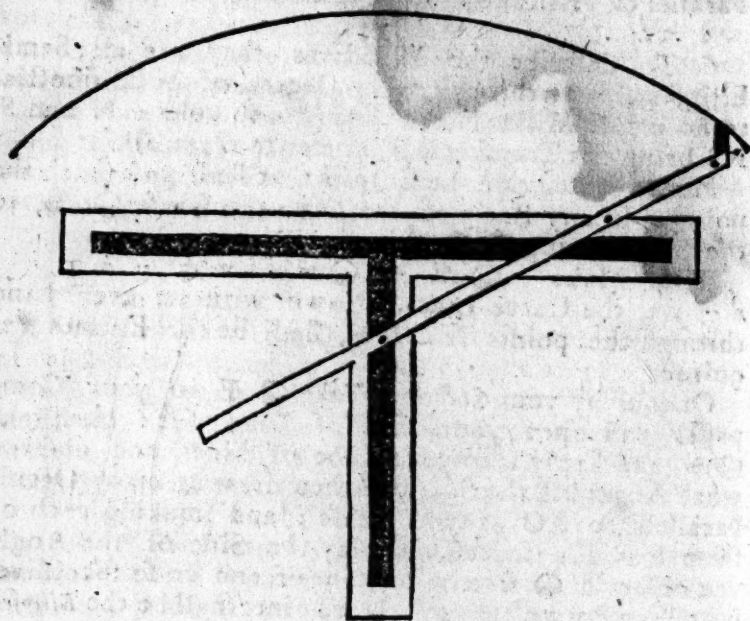
Hence to describe the Ellipses that shall pass thro' the point m , in any Ratio of the Semi-transverse $\odot$ $\mathcal{A}$, to the Semi-conjugate $\odot$ m .

Thus Make $\odot$ $\mathcal{A} :: \odot$ $m :: m$ $\odot :: m$ $\Pi :: \gamma$
 $a :: \gamma z$, the Curve-Lines, drawn with an even hand through the points $N z \Pi m$, shall be the Ellipses required.

Or thus by your Sector; Take $\odot$ $\mathcal{A}$ in your Compasses, and open your Line of Sines to it: Then take $\odot$ m , and apply it to your Line of Sines, and observe what Angle 'tis the Sine of; then draw as many Occult Parallels to AO as you please; and making each of them Radius's successively, lay the Sine of the Angle you observ'd $\odot$ m to be the Sine of, and do so successively, a Line drawn through these Points shall be the Ellipses, desir'd.

But a much easier and readier way, to describe this Ellipsis in the Projection is thus, by the Elliptical Instrument, the Figure of which is here annex'd.

Having divided the Equinoctial by a Line of Sines both ways, from $\odot$ by setting of $75^{\circ} 60'$, 45° , $30^{\circ} 15'$,



it will mark out on the Equinox, the respective Hour-Arches, Points thereon; then fixing your Instrument on the Projection, you may easily describe the Ellipses through N and S and the foresaid Points.

By the same Method, as you draw the Meridian Ellipses; so you may describe the Azimuths, and Longitudes of the Stars the Ellipses must pass through Z, the Horizon HO, and N, for the Azimuths, and through r ; the Ecliptick therefore being divided into every 10 Degrees, or every single Degree, from a Line of Sines proceed, as before directed for the Meridians.

A Brief Synopsis of this Projection.

1. Arches of the Primitive, measur'd upon the Chords $ON = ZE = HS = NQ$ is the Latitude 51° Degrees 30 Minutes.

$NZ = \text{ÆH} = SN = QO$ is the Complement of the Latitude 38° Degrees and 30 Minutes.

$\text{ÆS} = QV$, the Sun's greatest Declination, 23° Degrees 30 Minutes, equal to $Ra = Sn$, $Rr = St$, the Distance of the Polar Circles, and Poles of the Ecliptick from the Poles of the World.

Δd , the Sun's present Declination 21 Degrees North.
 $\odot l$ the Sun's Altitude at 6 a-Clock 16 Degrees.

2. Arches of Right Circles measur'd on the Sines.

$\odot v$, the Sun's present Declination 21 Degrees.

$\odot X$ the Sun's Amplitude 34 Degrees from the East.

$\odot \odot$ the Sun's Longitude from γ , 60 Degrees being in II.

$\odot f$ the Sun's Altitude when East or West 27 Degrees and half.

$\odot K$ the Sun's Altitude when in the point $\odot$ (i.e. II,) 39 Degrees and half.

$\odot V$ (being equal to $q v$) is 16 Degrees Sun's Altitude at 6.

3. Arches of lesser Circles to be reduc'd to the Primitive measur'd on the Chords.

$p \odot$ the Sun's Azimuth from the South 2 in the Point

$\odot k$ the Sun's Azimuth from the East 5 $\odot$

$F v$ the Hour from 6 that the Sun in Due East 16 Degrees and half.

$V v$ the Sun's Azimuth from the East or West at 6, 12 Degrees.

$v X$ the Ascensional Difference 26 Degrees and half.

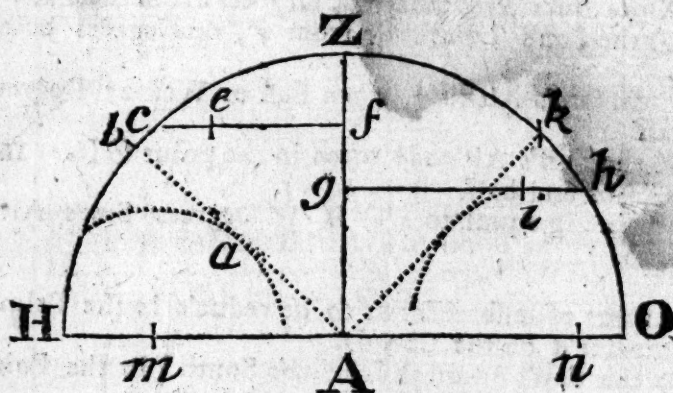
$\odot v$ the Right Ascension 50 Degrees.

P R O B. - I.

Let it be Required to reduce a lesser or parallel Circle to the Primitive, so as any part of the Parallel may be measur'd upon the Chords.

CONSTRUCTION.

1. Take half the Parallel in your Compasses, and set it off from the Center of the Primitive upon any



Right Circle, where there is most Room or Convenience; and where the other Foot of the Compasses fall, make a Mark.

2. Take in the Compasses, the Part of the Parallel to be measured; and with one Foot in the aforesaid Mark, turn the other Foot about, and make an Occult Arch.

3. Lay a Ruler from the Center of the Primitive, so as just to touch the Extremity of the said Arch; and observe where the Ruler cuts the Primitive, making a Mark.

4. Extend your Compasses from the Mark upon the Primitive, to the Point where the Right Circle, by which the Work was done, cuts the Primitive; that Extent Measur'd on the Chords, gives the Quantity of the Arch of the Parallel required.

EXAMPLE I.

Suppose I wou'd reduce the Parallel *ef*, to the Primitive, in order to measure the Arch, or Segment of it *ef*.

CON-

CONSTRUCTION.

1. Take the Extent fc and set it from A to m , upon the Right Circle OAH .

2. Take ef in your Compasses; and with one Foot in m , describe the Arch a ; a Ruler laid from A to touch the Extremity of the Arch a , will cut the Primitive in b ; then the Arch Hb measur'd on the Chords, gives the Quantity of the Arch $ef = 43^\circ$.

R O B. II.

Example. Let be required to set off 47 Degrees from g , to i , upon the Parallel, gb :

This is only the Reverse, of the last *Example*.

The Demonstration of the Axioms used in Oblique Spheric Triangles.

AND this will give Solution to the First and Second Cases.

LEMMA I.

Half the Versed Sine Multiplied by Radius, is equal to the Square of the Sine of half that Arch; and equal to half the Sine of the Arch multiplied by the Tangent of half the Arch; that is, $\text{half } V \times R = \text{Sn. } q. \text{ Arch} = \text{half Sn. Arch} \times \text{T half Arch}$.

1. $EB. BN :: AB. FB :: (\text{half } AB) BN. \text{half } FB.$

2. $V :: \text{Sn. half Arch} :: \text{Sn. half Arch} \times \text{half.}$

Therefore $\text{half } VR = \text{Sn. } q. \text{ half Arch.}$

3. $\text{half } AE. BN :: AE. AB :: BN. BA.$

Therefore $\text{half } AE \times BA = \text{Sn. } q. \text{ half Sn. Arch} \times \text{T. half Arch.}$

LEMMA II.

Putting v . for the versed Sines. Complement, (or Supplement to 180° .) Then,

$\text{Half } R \times v = \text{Co Sine } q \text{ half Arch} = \text{half Sine} \times \text{Co. Tang. half Arch.}$

PART II.

U 4

1. Half

RULE I.

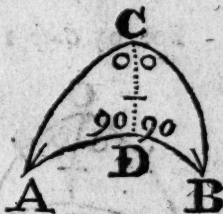
The Co-sines of the Angles at the Base, are proportional to the Sines of the Angles at the Vertex: For by Case 5. Operation 4. Right-Angled Spherick-Triangles,

$$\text{As Co.S. DC} :: R :: \text{Co.S. } \angle A ..$$

$$\text{S. } \angle C.$$

$$\text{As Co. S. DC} :: R :: \text{Co. S } \angle B ..$$

$$\text{S. } \angle C. \quad \text{the Angles at the Vertex.}$$



Therefore by Theor. 1. the two first Terms being the same, 'twill be,

$$\text{As Co. S. } \angle A :: \text{Co. S. } \angle B :: \text{ACD} :: \text{S } \angle DCB.$$

RULE II.

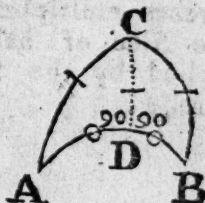
The Co-sines of the Sides are proportional to the Co-sines of the Bases — For by Case 7. of Right-Angled Spherick Triangles,

$$\text{As Co. S. DC} :: R :: \text{Co S. AC} ..$$

$$\text{Co.S. AD.}$$

$$\text{As Co.S. DC} :: R :: \text{Co.S. BC} ..$$

$$\text{Co.S. BD,} \quad \text{the two first Terms in each are the same.}$$



Therefore by Theor. 1,

$$\text{As Co.S. AC} :: \text{Co.S. BC} :: \text{Co.S. AD} :: \text{Co S. BD,}$$

the Bases.

RULE III.

The Sines of the Bases are reciprocally proportional to the Co-Tangents of the Angles at the Base.

For by Case 6. of Right-Angled Spherick-Triangles,

$$\text{As T.DC} :: R :: \text{S.AD} :: \text{T.C.DAC.}$$

$$\text{As T.DC} :: R :: \text{S.BD} :: \text{T.C.DBC.}$$

by Theor. 1, Expunging the two first Terms in each, 'twill be,

$$\text{As S. AD} :: \text{S. BD} :: \text{T.C. DAC} ..$$

$$\text{T.C. DBC.} :: \text{TA} :: \text{TB by Lemma 3.}$$

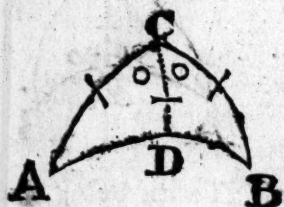


RULE IV.

RULE IV.

The Co-Tangents of the Sides, are reciprocally proportional to the Co-Sines of the Angles at the Vertex.

For by *Case 13.* of *Right-angled Spherick Triangles*,



As $R :: T. DC :: T.C. AC :: S.C. ACD.$

As $R :: T. DC :: T.C. BC :: S.C. BCD,$ by *Theor 1.* Casting away the two first Terms in each, it will be,

As $Co.T. AC :: Co.T. BC :: Co.S. ACD :: Co.S. BCD.$ Or $T. AC :: T. BC :: S.C. ACD :: S.C. BCD.$ *Lemma 3.*

LEMMA V.

The Difference of the Versed Sines of two Arches multiplied by half the Radius, is equal to the Sine of half the Sum of those Arches Multiplied by the Sine of the half Difference of the Arches.

DEMON

LEMMA VII.

As the Sum of the Co-Sines, is to the Difference of the Co Sines; So is the Co-Tang. of half the Sum of the Arches, to the Tangent of the half Difference of the Arches.

That is, A Z. Co-Sines .. X Co-Sines :: Co-Tang. half Arcs .. Tangent half X Arcs, If the Arcs are less than 90 Degrees, (for in the foregoing Figure, the two Arcs being AF, and AF,) then GC and CH are their Co-Sines; and it will be,

$$\text{As } CH + CG \dots CH - CG :: \frac{CH + CG}{2} \dots \frac{CH - CG}{2}$$

That is,

$$\text{As } Cu \dots uH :: w \odot \dots \odot F \text{ and } :: rn \dots nK, \\ (=Sy) = (yF) \quad \text{Therefore,}$$

$$\text{As } cu \dots uH :: rn \dots nK.$$

But if the Arcs are bigger than 90 Degrees,

As Z Co-Sines .. X Co-Sines :: Tangent half Z Arcs .. Co-Tang. half X Arcs.

The Tangents of two Arcs .. being reciprocally proportional to their Co-Tangents, by the third Lemma.

LEMMA VIII.

As Z. Tangents .. X, Tangents :: Sine Z Arcs .. Sine X Arcs.

Let the Arches be n A and n E as before.

$$\text{As } PM \dots KM :: EL \dots FL \text{ and } :: EG \dots FH.$$

Therefore,

$$\text{As } PM \dots KM :: EG \dots FH, \quad Q. E. D.$$

LEMMA IX.

$$S, \text{ Arc} = \frac{S. \text{ half Arc } X \text{ Co. } S. \text{ half Arc.}}{\frac{1}{2} R}$$

LEMMA X.

The Sines of Two Arcs A, E, are proportional to the Rectangles under the Sines and Co-Sines of their half Arcs; that is,

$$\text{As } S.A \dots Sn.E :: (\text{by Lem. 9.}) \frac{S. \text{ half } AX \text{ Co-Sn. } \frac{1}{2} A \dots}{\text{half } R}$$

$$\frac{Sn. \text{ half } E \times \text{Co-Sn. half } E}{\text{half } R} \text{ and}$$

So is, Sn. half A X Co Sn. half A .. Sn, half E X Co-Sn, half E.

LEMMA

LEMMA XI.

The Tangents of two Arches A, E, are proportional to the Rectangles under the Sines and Secants of those Arcs; that is,

$$\text{As } T. A \dots T. E, :: S. A \times \sec. A \dots SE, \times \sec. E.$$

For,

$$\begin{aligned} \text{As } T. A \dots T. E :: S. A \times R \dots SE \times R \text{ and } :: S. A. \times \text{Co-} \\ \text{Sn. E.} \times R. \quad \text{Co-SA} \quad \text{Co-SE} \\ \dots SE, \times \text{Co-SA} \times R :: S. A \times \text{Co-S. E} \dots SE \times \text{Co-SA} :: \\ S. A \times \sec A \dots S. E \times \sec E; \end{aligned}$$

For by the fourth *Lemma*, the Co-Sines of two Arcs are reciprocally proportional to their Secants.

RULE V.

In any Oblique Spheric-Triangle having let fall a Perp. from the Vertex to the Base, The Tangents of Bases are proportional to the Co-Tangents of the Angles at the Vertex.

For by *Case 6. of Right angled-Spherick Triangles*,

$$\text{As } TAD \dots R :: S. DC \dots T.C. ACD, \text{ and}$$

$$\text{As } T. DB \dots R :: S. DC \dots T.C. BCD,$$

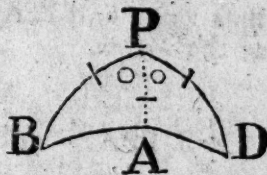
the two Middle Terms in each Proportion are the same; Therefore by *Theor. 2.* casting both away, and 'twill be,

$$\begin{aligned} \text{As } T. AD \dots T. DB :: T.C. ACD \\ \dots T.C. BCD. \end{aligned}$$



RULE VI.

As S, Z. Sides .. S. X Sides :: Co-Tang. half Z Angles at the Vertex .. Tangent half x Angles at the Vertex :: Co-Tang. half x Angles at the Vertex; .. Tang. half Z Angles at the Vertex.



DEMONSTATION.

For by *Rule 4.*

$$\text{As } T. BP \dots T. DP :: \text{Co-Sn. DPA} \dots \text{Co-Sn. BPA.}$$

There-

Therefore,

$$\text{As } T. BP + T. BP - T. DP :: \text{Co-Sn. DPA} + \text{Co-Sn. BPA} :: \text{Co-Sn. DPA} - \text{Co-Sn. BPA}.$$

That is, by *Lemma*, 8 and 7.

As S. Z Sides .. S X Sides :: Co-Tang. half Z Angles; at the *Vertex* .. T. half X Angles at the *Vertex*.

RULE VII.

.. T. half Z Sides .. T. half X Sides :: T. half Z Angles, at the *Base* .. T. half X Angles, at the *Base*.

DEMONSTRATION.

Therefore,

2. Co-Tang. half Sides .. Co-Tang. half X Sides :: T. half X Angles, at *Base* .. T. half Z Angles at the *Base*.

2. By *Axiom* 4.

$$\text{As } S. BP :: S, S, B :: S, D :: S, B.$$

Therefore,

$$\text{As } S. BP + S, DP :: S, BP - S, DP :: S. D + S. B :: S, B :: S, D - S, B,$$

Therefore by *Lemma* 6.

$$\text{As } \frac{T. BP + DP}{2} :: \frac{T. BP - DP}{2} :: \frac{T. D + B}{2} :: \frac{T. D - B}{2}.$$

RULE VIII.

As Co-Tang. half Z Angles, *Vertex* .. T. half Z Angles, *Base* :: T. half X Angles at the *Base* .. T. half X Angles, at the *Vertex*.

For by *Rule* 1.

$$\text{As } S. BPA :: S, DPA :: \text{Co-S. B} :: \text{Co-s. D.}$$

Therefore,

$$\text{As } S. BPA + DPA :: S, BPA - DPA :: \text{Co-S. B} + \text{Co-S. D Co-S. B} - \text{Co-S. D.}$$

There-

Therefore, by the 6th and 7th Lemmas,

$$\frac{\text{As } T, BPA + DPA}{2} \cdot \frac{\text{.. } T, BPA - DPA}{2} :: \frac{\text{Co-T. } B + D}{2} \cdot \frac{\text{T. } B - D}{2}$$

Therefore,

$$\frac{\text{.. } T, B - D}{2} \cdot \frac{\text{.. } T, BPA - DPA}{2} :: (\text{Co-T. } B + D \cdot \text{T. } BPA + DPA) :: \frac{\text{Co-T. } BPA + DPA}{2} \cdot \frac{\text{T. } B + D}{2} \quad \text{Q.E.D.}$$

P R O P. I.

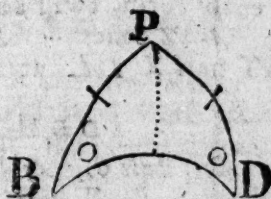
In any Oblique Spherick-Triangle BDP, given; the two Sides BP, and DP. with the contain'd Angle BPD; to find the other two Angles DBP, BDP. at the Base.

A X I O M. V.

1. As the Sine of half the Sum of the Sides, $\frac{(BP + DP)}{2}$ is to the

Sine of half their Difference, $\frac{(BP - DP)}{2}$ or $\frac{(DP - BP)}{2}$ So is the

Co-Tangent of half the Contained Angle $\frac{(\angle BPD)}{2}$ to the Tangent of half the Difference, of the other two Angles.



Again,

2. As Co-Sine of half the Sum of the Sides .. is to Co-Sine of half their Difference; so is the Co-Tangent of half the contained Angle, to the Tangent of half the Sum of the other two Angles. That is,

As S half Z Sides .. S half X Sides :: Co-Tangent half $\angle PBD$ (of the Vertex) .. T half Z $\angle$'s (at the Base.)

And Co-S half Z Sides .. Co-S half X Sides :: Co-Tangent half $\angle BPD$ (at the Vertex) .. T half Z $\angle$'s. (at the Base.)

DEMONSTRATION I.

By Rule 6. SZ Sides .. SX Sides :: Co-Tangent half $\angle P$.. T half $X \angle \angle$, at the Vertex. And,

Rule 7. T half Z Sides .. T half X Sides :: T half $Z \angle \angle$, Base .. T half $X \angle \angle$ Base.

Rule 8. T half $Z \angle \angle$ Base .. T half $X \angle \angle$ Vertex :: Co-Tangent half $\angle P$.. T half $X \angle \angle$ Base.

Therefore, Multiplying the Homologous Terms into each other, and Dividing the 1st and 3d Products; as likewise the 2d, and 4th, by what is an equal Component in each, and we have,

As SZ Sides X T half Z Side .. SX Sides X T half X Sides :: Co-T q half P .. T q half $X \angle \angle$ Base.

Therefore (by Lemma 1.)

As $S q$ half Z Sides .. $S q$ half X Sides :: Co-T q half P .. T q half $X \angle \angle$, at the Base.

Now if Squares be proportional, their Roots will be proportional; therefore extracting the Square Root of each Term 'twill be,

As SZ Sides .. SX Sides :: Co Tangent half $\angle P$.. T half $X \angle \angle$ at the Base. Q.E.D.

DEMONSTRATION II.

By Rule 6. As SZ Sides .. SX Sides :: Co-Tang. half $\angle P$.. T half $X \angle \angle$, Vertex.

By Rule 7. As Co-Tangent half Z Sides .. Co-Tang. half X Sides :: T half $X \angle \angle$ Base, .. T half $Z \angle \angle$, Base.

By Rule 8. As T. half $X \angle \angle$ at Base .. T half $X \angle \angle$. Vertex :: Co-T. half P .. T half $Z \angle \angle$ Base.

Therefore Multiplying the Homologous Terms into each other, and Dividing the 1st, and 3d, Products, as likewise the 2d, and 4th, by what is an Equal Component in each, we have,

As $S. Z$ Sides X Co-T half Z Sides .. $S X$ Sides X Co-T half X Sides :: Co-T. q half P .. T q half $Z \angle \angle$, at the Base.

Therefore (by Lemma 2.)

As Co-S q half Z Sides .. Co-S. q half X Sides :: Co-T q half $Z \angle \angle$ at the Base.

There.

PART

Therefore the Square Roots of these Quantities are Proportional,

As Co. S. half Z Sides .. Co-S. half $\times$ Sides :: Co-T. half L. P. .. T. half Z L L, Base. Q. E. D.

Then the half Z $\pm$ half X = $\left\{ \begin{array}{l} \text{greater} \\ \text{lesser} \end{array} \right\}$ Angles, at the Base Required.

Note, This Demonstration supposeth the Angle P = Z L L, at the Vertex; i. e. that the Perpendicular falls within the Triangle; but the same Proportions will arise, if it falls without.

PROP. II.

In any Oblique Spherick-Triangle given, two Angles and the interjacent Base (or Side included between those Angles;) to find the other two Sides, and the Remaining Angle.

AXIOM VI.

1. As S half Z L L .. S half $\times$ L L :: T, Base (Side) .. T, half $\times$ Sides.

2. As Co-S half Z L L .. Co-S half $\times$ L L :: T, half Base (Side) .. T half Z Sides.

This *Axiom* may be Demonstrated, by the same Rules and *Lemmas* that the preceding was, but will be needless if you consider the 20th Definition of the Spherick-Triangles foregoing, in changing the Triangle into its Supplemental; and esteeming the Angle's Sides, and the Side given, an Angle.

PROP. III.

In any Oblique Spherick-Triangle given the three Sides, to find an Angle.

AXIOM VII.

In a Spherical Triangle, if half the Difference of the Sides containing an Angle, be Added to half the Side opposite to that Angle, and likewise subtracted from the same, and the Sum and Remainder Noted; Then,

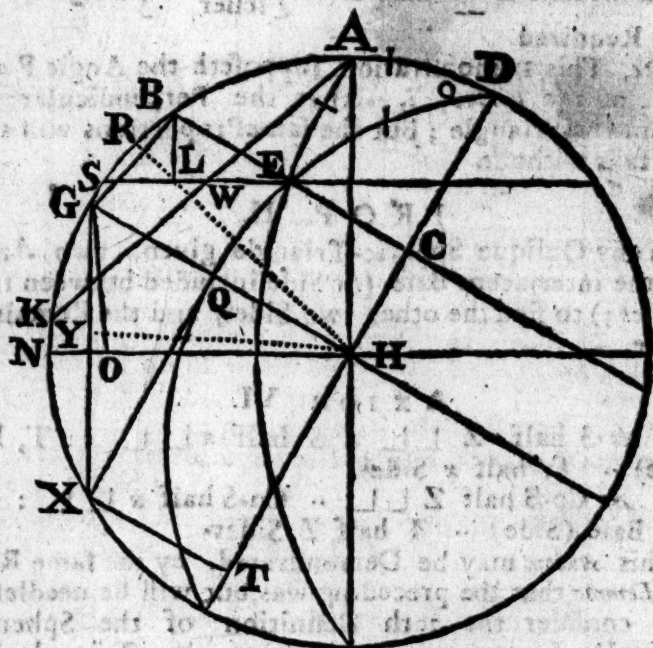
As the Rectangle of the Sines of the Containing sides, Is to the Square of Radius;

So is the Rectangle of the Sines of the foresaid Sum and Remainder,

To the Square of the Sine of half the Contained Angle.

CONSTRUCTION.

Let the Triangle be ADE, and D be the Contained Angle, and let AB be the Difference of the Sides



AD and ED the containing Angle (for DB is equal to ED,) and let $AE = AS$, be equal to the Side opposite to the Angle at D; then making $SK = AB$, draw the Subtendents AK and BS; and dividing the Arch AK and BS equally in R, draw from the Center the Obscure Line HR; then drawing QX parallel to HP, and BL and GO parallel to AH, &c.

DEMONSTRATION.

GQ is the Versed Sine of the Angle ADE; as also of the Arch GX. Therefore the Arch GX, is the Measure of the Angle ADE. But QX is the Right Sine of the Arch GX; therefore QX is also the Right Sine of the Angle ADE.

And seeing AS is equal to the opposite side AE, and SK to AB, the difference of the containing Sides; therefore the whole Arch $AK = AE$ and AB: Therefore the half thereof AR is = to the Sum of the Halves of AE and AB; that is, of half the opposite side, and of half

half the Difference of the containing Sides, the Sine whereof is AW: And if the Difference AB be taken from the side AE, that is, from AS, the remainder is BS, the half whereof is BR; so that if the half of AB be subtracted from the half of AE or AS, the remainder is BR; and seeing GN is equal to AD, GO, the Sine of GN is also the Sine of AD and BC is the Sine of DE or DB. So that BC and GO are the Sines of the containing sides AD and ED, and AW and BR are the Sines of the aforesaid Sum and Remainder, and GY the Sine of half the Angle at D. I say then, that as the Rectangle of the Sines of the containing Sides AD and ED .. is to the Square of Radius :: So the Rectangle of the Sines of the Sum and Remainder AR and BR, to the Square of the Sine of half the Angle ADE, namely, to the Square of the Sine of half the Arch GX.

That is,

As the Rectangle of GO and BC, .. is, to the Square of GH ::

So is the Rectangle of AW and BR to the Square of GY.

For it's largely demonstrated by *Pitiscus, Lib. 3*, that as GH, the Semidiameter of a great Circle, .. is in Proportion to BC the Semidiameter of a lesser Circle ::

So is QH the Sine of a Certain Arch in the greater ..,

To EC, the Sine of the like Arch in the lesser ..,

And so is GQ the versed Sine in the one, .. to BE the Versed Sine in the other.

Therefore,

As GH .. BC :: GQ .. BE. And GH .. GO :: B .. BL,

For the Triangles GOH and BLE are equiangled, and similar.

Therefore,

As GH q .. BCXGO :: GQXBE .. BLXBE,

And dividing the two last Rectangles by BE; then as GH q .. BCXGO :: GQ .. BL. Or the Converse, namely,

As BC X GO .. GH q :: BL .. GQ.

Again, seeing that AK is parallel to BS, and BL parallel to AH,

Therefore the Angle SBL, is equal to the Angle HAW.

Therefore the Right-angled Triangles SBL and HAW are equiangled and similar. Likewise seeing the Right-angled Triangles YGH, and QGX, have the Angle YGH common to them both, therefore they are also equiangled and similar.

Therefore,

As AW .. AH Radius :: BL .. BS. And;

As GY .. GH Radius :: GQ .. GX.

Therefore the Rectangle of AW in BS is equal to that of BL in Radius.

Also the Rectangle of GY in OX, is equal to that of GQ in Radius.

Therefore,

As AWXBS .. GYXGX :: BLX Radius .. GQX Radius. Then,

As AWXBS .. GYXGX :: BL .. GQ. But,

As BL .. GQ :: (as before is proved) BCXHO .. GH; that is, to the Square of Radius.

Therefore,

As BCXGO .. Radius Squar'd :: AWXBS .. GYXGX. But,

As AWXBS .. GYXGX :: AWX half BS (that is, BR) .. GYX half GX (that is, GXY.)

Therefore,

As the Rectangle of BC in GO, (for BC, GO, are the Sines of the containing sides DE, AD,)

Is to the Square of GH, (which is the Radius, or Semidiameter, Squared,)

So is the Rectangle of AW in BR, the Sines of the foresaid sum, and Remainder,

To the square of GY; that is, the square of the sine of half the Angle ADE, Q.E.D.

To work this Axiom, by Logarithm-sines it hath been before proved in Logarithmetick-Arithmetick, that the Rectangle or Product of Two Numbers, is the Sum of the correspondent Logarithms, and a square Number = to its Logarithm doubled;

There-

Therefore,

If unto the Sines of the foresaid Sum and Remainder, be Added twice Radius; and from that Total be subtracted the Sines of the containing sides; half The Remainder is the Sine of half the contained Angled required.

Or more Briefly thus:

If instead of subtracting the Sines of the containing sides, we Add their several Complements Arithmetical, the Total is more than the Remainder (last cited) would have been by twice Radius.

Therefore,

Leaving out twice Radius: If to the several Complements Arithmetical of the Sines of the containing sides, be added the Sines of the foresaid Sum and Remainder, half total is the Sine of half the contained Angle required.

PROBLEMS in Geography and Great Circle-Sailing.

IT is agreed upon by all Artists in Navigation; that Sailing by the Arch of a Great Circle is the exactest of all kinds of Sailing, that have ever been us'd or can be propos'd, both for the Trueness of the Course and (Exactness, and) Nearness of the Distance: For since of all Figures, that made by the Surface of the Earth approacheth nearest to the Convex Surface of a Sphere; the nearest Distance, *Ergo* betwixt two Points is the Arch of a Great Circle; is as on a Plain Surface, the nearest Distance betwixt two Points in a Right Line.

It plainly follows, that such who steer their Course from one part of the World to another in the Arch of a Great Circle, go the nearest way possible to their desired Port; but 'tis the most difficult, and hardly possible for a Ship exactly to sail by, Men being drawn from it by Conveniences of Winds and Currents; or else forced from it by Cross Winds or Interposition of some Head-Lands, Shoals, or Islands, &c.

But yet it may be of good Advantage to keep conveniently near it, especially in a Parallel, or East or West Course; as in sailing from the Lands End of *England* for *Newfound-Land*, there's great Advantage in the distance to raise and depress the Pole 10 or 12 Degrees, besides a Benefit it yields in correcting or rectifying your Account by Dead Reckoning.

P R O B. I.

If the two Places over which the Great Circle is to pass, be both under the Equinoctial, the Equinoctial is the Great Circle, and the Difference of Longitude between the two places is the Distance, and the Course between is directly East or West.

P R O B. II.

PROB. II.

If the two places lie both under the same Meridian, that is, directly North and South, and one of them situate under the Equinoctial, then are the Degrees of Latitude of the other Distance.

PROB. III.

If both Places be under the same Meridian and both Latitudes of the same Denomination, that is, both North, or both South, then the difference of Latitude is their Distance.

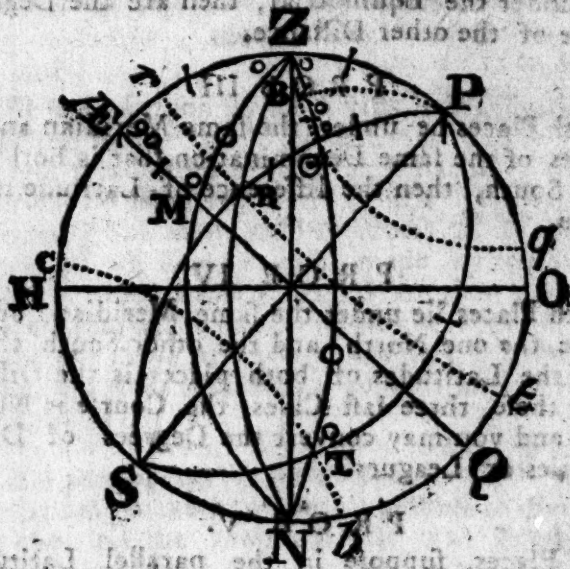
PROB. IV.

If both Places lie under the same Meridian, but have Latitude, the one North, and the other South, then the Sum of the Latitudes of both places is the Distance; and in these three last Cases the Course is North or South; and you may convert the Degrees of Distance into Miles or Leagues.

PROB. V.

Two Places, suppose in the parallel Latitude of 50 Degrees North differing in Longitude 47 Degrees 26 Minutes West. What are the Angles of Position, or Bearings, their Distance in a great Circle, and the greatest Latitude the Arch shall pass through?

Let P represent the Lizard one Place, and © the other,
 $2P = P \odot$ the Comp. of their Latitudes = 40 Degrees



In the Triangle ZPB there are given ZP=40 Degrees and $\angle ZPB = 23$ Degrees 53 Minutes (and half a Long.) to find the $\angle PZ\odot = P\odot Z$, the Angles of Position; Z $\odot$ the Distance and PB the Arch of the greatest Latitude the great Circle shall pass through.

1. *As* TC L ZPA .. R :: SC ZP .. TC L PZR.

2. $AsR_2 \rightarrow ZP :: sL \rightarrow ZPB \rightarrow ZB.$

As $S 90^{\circ} E$ — $S 40^{\circ} E$ — $S 23^{\circ} 13' E$ — $35^{\circ} 05'$
Doubled, is 30 Degrees 10 Minutes; Or 1810 Miles, the
Distance; which in parallel Sailing will be found to be
1842 Miles.

3^d. At TC ZP .. R :: S.C. $\angle$ ZPB .. TPB.

At TC 40 Degrees — S 90 — SC — 23 Degrees 53 Minutes .. $\angle$ 37 Degrees 30 Minutes.

Whole Complement to 90 Degrees is the Arch's greatest Latitude, viz. 52 Degrees 30 Minutes.

So that the Ship must sail North $71^{\circ} 16'$ West, or 905 Miles, till she comes into the Latitude of 52 Degrees 30 Minutes North, then South 71 Degrees 16 Minutes 905 Miles, to arrive at her intended Port; which in this Example is a Voyage from the Lizard in England, to Penguin-Island in Newfound-Land.

PROB. VI.

Suppose that one Place is in the Latitude 50 degr. North another under the Equinoctial, or on the Equator; Latitude $00^{\circ} 00'$, their Difference of Longitude, $42^{\circ} 40'$ West. I demand the Angles of Position and the Great Circle's Distance between them?

In the Right-angled Spherick-Triangle ZÆM there are given the Latitude. (or Difference of Latitude) ZÆ = 50 degr and Difference of Longitude ÆM equal $42^{\circ} 40'$ West, to find the Angles of Position ÆZM and ÆMZ; also the distance ZM.

1. At T. ÆM 42 deg. 40 min. .. R-S 90 deg. :: SÆZ 50 deg. .. TC $\angle$ Position, ÆZM equal 39 deg. 44 min. whole Complement to 90 deg. is 50 deg. 16 min. the Angle of Position from the Lizard to the Amazons, or near South West half West.

2. At TZE 50 deg. .. R S. 90 deg. :: S. MÆ 42 deg. 40 min. .. TC $\angle$ ÆMZ 29 deg. 38 min. whose Complement to 90 deg. is 60 deg. 22 min. or near N.E. 6. E. half E. the Angle of Position from the River of Amazons to the Lizard.

3. At R S. 90 deg. .. SC ÆM 42 deg. 40 min. :: SCÆZ 50 .. SCZM 34 deg. 17 min. whose Complement to 90 deg. is 55 deg. 43 min. or 3143 Miles, the distance from the Lizard to Amazons in the Arch of a Great Circle. Whereas by Mercator's Sailing the Course is South 36 deg. 23 min. West. distance 43.8 Miles; which is 975 Miles more than the former.

PROB. VII.

P R O B VII:

Admit the *Lizard* in Latitude 50 deg. North and *Barbadoes* in Latitude 13 deg 10 min. North, their Difference of Longitude 52° West, were given to find the Angles of Position, the great Circle's distance, between them, and the greatest Latitude the Arch shall pass through.

In the Oblique Spherical Triangle ZRP there are given the side ZP 40 deg. Comp. Latitude *Lizard*, side PR 76 deg. 50 min. Comp. Latitude *Barbadoes*, and the Contained Angle ZPR 52°, their Difference in Longitude, to find the Angles of Position PRZ and PZR, distance ZR, and Perpendicular PB, the greatest Latitude of the Arch.

O P E R A T I O N.

$$\begin{array}{l} 1. \text{ Side PR } 76^{\circ} 50' + \text{ Side PZ } 50^{\circ} 00' \} Z 116^{\circ} 50' \\ \text{Angle } \} \text{ ZPR } 52^{\circ} \text{ its half } = 26^{\circ} \text{ Comp. to } 90^{\circ} \text{ is } 64^{\circ} \\ \text{half } = \left\{ \begin{array}{l} 58^{\circ} 25' \\ 18 \quad 25 \end{array} \right. \text{ i. e. } \left\{ \begin{array}{l} \text{of } 116^{\circ} 50' \\ 36 \quad 50 \end{array} \right. \end{array}$$

Then,

$$2. \text{ As S. half Z Sides } 58^{\circ} 25' \text{ .. S. half } \times \text{ sides } 18^{\circ} 25' \\ \therefore \text{ T.C. } \frac{1}{2} \angle \text{ ZPR } 64^{\circ} \text{ .. T. } \frac{1}{2} \times \angle \text{ s } 37^{\circ} 15'$$

$$2. \text{ As S half sides } 58 \text{ deg } 25 \text{ min. .. S half } \times \text{ sides } 18 \text{ deg. } 25 \text{ min.} \\ \therefore \text{ TC half } \angle \text{ RPR } 64 \text{ deg. .. T half } \times \angle \text{ s } 37 \text{ deg. } 15 \text{ min.}$$

$$\begin{array}{rcl} 3. \text{ As SC half Z sides } 31 \text{ deg. } 35 \text{ min. .. SC half } \times \text{ sides } 71 \text{ deg. } 35 \text{ min.} & \therefore & \text{ TC half } \angle \text{ ZPR } 64 \text{ deg.} \\ \text{half Z } \angle \text{ s } & 74^{\circ} 55' & 74^{\circ} 55' \\ \text{Add half } \times & 37 \quad 15 & 37 \quad 15 \text{ ZRP} \end{array}$$

$$\begin{array}{rcl} \text{Sum is } \angle \text{ Posit. PZR } 112 \quad 20 & & 37 \quad 40 \times \text{ s is the Posit.} \\ & \text{at the } \textit{Lizard}. & \text{at } \textit{Barbadoes}. \end{array}$$

$$4. \text{ As S } \angle \text{ PRZ } 37 \text{ deg } 40 \text{ min. .. side ZP } 40 \text{ deg.} \\ \therefore \text{ S } \angle \text{ ZPR } 52 \text{ deg. .. side ZR } 56 \text{ deg. or } 336 \text{ Miles the distance requir'd.}$$

$$5. \text{ As S. PZ } 40 \text{ deg.} \\ \therefore \text{ S } \angle \text{ PZB } 67 \text{ deg. } 50 \text{ min.} \\ \therefore \text{ SPB } 36 \text{ deg } 32 \text{ min. Comp. is the Arch's greatest Latitude.}$$

P R O B VIII.

P R O B VIII.

Suppose one Place in the Latitude of 42 deg. 10 min. North, the other in Latitude 34 deg. 25 min. South, their Difference of Longitude 87 deg. 20 min. Then in the Triangle ZPT are given the side ZP 47 deg. 50 min. Comp. Latitude side PT equal 90 deg. + 34 deg. 25 min. viz. side PT 124 deg. 25 min. and the $\angle$ ZPT 87 deg. 20 min. to find as in the last Problem the greater Angle of Position 118 deg. 20 min. the Lesser 52 deg. 14 min. Great Circle's distance is 6633 Miles, and Arch's Greatest Latitude 49 deg. 17 min.

1. If this Sailing was made practicable, there would excellent Advantages accrue to the Publick therefrom.

As, for instance:

Suppose two Places in the Parallel of Latitude 60 Degrees, their Difference of Longitude 112 Degrees 30 minutes, or 6750 Miles: By parallel Sailing, the distance between them is, East and West 3375 Miles: Whereas their distance on the Arch of a Great Circle is only 2948 Miles, being less than the former distance by 427 Miles, or 142 Leagues. All which distance is saved by sailing by the Arch of a Great Circle; which is very considerable, being almost the sixth part of the whole distance.

And the nearer you approach the Poles, and the greater your difference of Longitude is, the Advantage of this Sailing still increaseth, and is more sensible.

2. As for the Course from one Place to the other, it is plain, that two Places lying in the same parallel of Latitude, according to *Mercator's* Sailing, they must bear exactly East and West from each other: Whereas by Great Circle-Sailing, you must steer between the North, and the Points of East and West, for one half of the Way; and between the South, and Points of the East and West for the other half of the Way, till you arrive at your desir'd Port.

I have in a Manuscript by me, made this Sailing as easy as Plain Sailing; with a plain Method of working a Traverse, and keeping a Sea-Journal thereby: But want of Room will not permit me here to insert it.

3. By

3. By this ingenious Way of Sailing, the cautious and prudent Merchant may easily escape the Snares laid by Enemies or Pirates; who being inform'd of his intended Voyage, may way-lay him, as in the Example above.

If a Ship bound from one of our supposed Ports to another, should be way-laid by Pirates cruising about half way, upon the Rhumb; and the Merchant-Ship keeping in the Arch of a Great Circle, the Pirate so steering his Course, will miss of the Merchant-Ship by 733 Miles: For the Pirate being about half way on the Rhumb, would still be in the Latitude of 60 degrees; and the Merchant-Ship keeping in the Arch of a Great Circle, at half way would be in the Latitude of 72 degrees 13 minutes North; which is 12 degrees 13 minutes, or 733 Miles to the Northward of the Pirate.

And if at the same time the Pirate, missing of his Prize, should endeavour to rectify his Mistake, and sail towards the Ship's intended Port, upon the Rhumb, he would still lose his Labour: For the Merchant-man being as far advanced in the Arch of a Great Circle as the half distance in the Rhumb, viz. 1687 Miles and half (the whole distance in the Arch of a Great Circle being only 2948 Miles,) it is plain the Merchant-Ship has only 1260 Miles and half to make on the Arch of a Great Circle, to arrive at her desired Port; and the Pirate has still 1687 Miles and half to make on the Rhumb; which is more than the distance of the Merchant-Ship by four hundred twenty seven Miles; and so much the Merchant-man is a-head of the Pirate; and therefore may justly hope to arrive at her desired Port, before the Pirate comes within an Hundred Leagues of him, tho' the Pirate sail better and faster than the Merchant-man.

4. Lastly, If Great Circle Sailing had no other good Quality to recommend it, but that of shortening a Voyage considerably, I humbly conceive, it ought to be preferr'd to all others, especially in Long Voyages, between places that have much Difference of Longitude; as, from the Ports of Europe, to the Western parts of America; and in the East India Voyages.

But

But above all, when you have the Benefit of Trade-Winds : For then you may steer your Ship in the Arch of a Great Circle with all the Exactness that is requisite. For which reason I thought it proper not wholly to omit this Spherical or Circular Sailing in this Treatise.

The Author has prepar'd for the Press, A Comprehensive Treatise of Astronomy : In which are shewn, the Various Systems of the World, Exhibited by the Ancient Philosophers ; and the True System accounted for, and demonstrated : Together with the Theories, Calculations, and Geometrical Projections of all the various Motions of the Planets at large ; perform'd Trigonometrically and Logarithmically : Also the Mathematical and Judicial Parts of Astrology Explain'd. With the whole Calculations, Projections and Types of Solar and Lunar Eclipses ; and the General Appearance of the Sun's Eclipse in several Parts of the World remote from London. And the Transit of *Venus* and *Mercury* over the Sun's Disk, &c. Likewise the Method of finding the Calendrical Notes both of the *Julian* and *Gregorian* Accounts ; and an Essay offer'd for the Emendation of the *Julian* Calendar. With a Perpetual Almanack of the said Notes ; and Tables of the Days of the Months and Weeks for ever. The Whole made Easy and Plain to the meanest Capacity, in a Method never before attempted.



THE Use of the GLOBES

I N Problems of Geography, Navigation, and Astronomy.

DEFINITION.

OF GLOBES there are Two sorts, *viz.* Terrestrial and Celestial. The Terrestrial Globe is a lively Representation of the Terrestrial Mass of the Earth and Water, under the real Form of Roundness; with all the Countries, Rivers, Lakes, Cities, Towns, Hills and Lines necessary thereunto; as, the Meridians, Parallels of Latitude, Poles, Equinoctial, Ecliptick, Tropicks, Polar Circles.

Of the Poles of the GLOBES.

A Body cannot be made round without Circular Motion; and there is no Circular Motion, without some Rest; and that Rest cannot be simply within the *Globe*. Therefore in the outward Surface of the *Globe* (imagine) there are Two Points, on which it resteth whilst it is moved; which Points are called the *Poles of the Globe*, opposite one to another, and equidistant from the Center of the *Globe*, the one North, and the other South.

Of the AXIS.

From the Center of the Globe, imagine a Line to proceed through both of the *Poles*; which Line is the *Axis*, and is represented by the two Wires in the *Poles* of the Globe. Upon these two Wires the Globe is turned round, even as the Heavens are imagined to move upon the Axis of the World.

Other Appendants belonging to either Globe, required to make it fit for Operation, are the *Brazen Meridian*, by which the Globe is hung by the Axis at both the Poles, and is divided into 360 Degrees, or four times 90 Degrees; because the Elevation of the Pole, or Latitude of any place cannot exceed 90 Degrees.

The Uses of this are, to find the Latitude of Places, the Right Ascension and Declination of the Sun and Stars, their Meridian, Altitudes, Depression, &c. to Elevate the Globe to any Latitude.

The Hour-Circle and Index;

Whose Center is the pole of the Globe, and is divided into the Hours of the Day and Night; which, with its Index, is to shew the time of the Sun or Stars Rising and Setting, and the Time of the Day in any part of the World.

The Quadrant of Altitude;

Which is a Thin Brass Plate, divided into 90 Degrees, and fitted with a Nut and Screw, to move to any Degree upon the Brass Meridian. Its Use is to measure Distances, Altitudes, Azimuths, &c.

The Wooden Horizon.

The upper Plane of this is a Broad Circle encompassing the Globe, having two Notches, the one in the North, the other in the South part, being made fit to contain the Brazen Meridian, that the Globe may move into any position.

In the upper Plan whereof, are delineated,
The First or Inner Circle, which is divided into twelve Equal Parts, called Signs of the Zodiack, or Ecciptick; every Sign having its Name, Nature, and Planet it governs annex'd to it: Whose Names and Characters are, *Aries*, the Ram γ ; *Taurus*, the Bull σ ; *Gemini*, the Twins II ; *Cancer*, the Crab S ; *Leo*, the Lyon L ; *Virgo*, the Virgin M ; *Libra*, the Ballance M ; *Scorpio*, the Scorpion M ; *Sagittarius*, the Archer A ; *Capricornus*, the Goat VS ; *Aquarius* the Water-Pourer, W ; *Pisces*, the Fishes X ;

Each of these Signs is again divided into thirty equal Parts or Degrees, and numbered with 10, 20, 30.

Next to this Circle of Signs, is the *Julian* Kalender, or Old Stile still used in *England*, divided into the twelve Months of the Year viz. *January*, *February*, &c. to *December*, every Month being subdivided into its Number of Days; to which are also annexed, the Names of the fixed Feasts.

To this, is the *Gregorian* or foreign Account, instituted by Pope *Gregory* 13th; in which since 1700 the Months begin 11 Days before us. The last is the Points of the Compass, which shews to us the Quarters of the Heavens and the Coast and Bearings of Countries, or Places from each other, also to distinguish the Day from the Night, the Rising, Setting, Azimuth, Amplitude of the Sun, or Stars, the Twilight, &c.

The Semi-circle of Position,

Is a half Circle of Brass fixed in the Intersection of the Meridian and Horizon, and is to move up and down between the Meridian and Horizon; and is divided from either end with 10, 20, 30, &c. to 90. And is chiefly used in *Astrology* and *Dialing*, and therefore is not usually sold with the Globes, unless desired.

PROB I.

To find the Latitude of any Place.

PRACTICE.

Bring the Place to the Brazen Meridian, the Degrees of the Meridian over the Place proposed, shews the Latitude Northerly towards the North Pole, and Southerly towards the South Pole.

Ex-

Red
Places
titude
Place
the Gl
of Alci
the De
Minute
PART

in Geography, Navigation, &c.

EXAMPLE.

So the Latitude of *London* will be 51 Degrees and half North.

PROB. II.

To find the Longitude of any Place.

PRACTICE.

Bring the Place under the Brass Meridian; the Degrees of the Equinoctial, or Equator, cut by the said Meridian, is the Longitude of the Place required; which is accounted, in our latest and best Globes, from the Meridian of *London*.

EXAMPLE.

The Longitude of *London* is $0^{\circ} 0'$ and of the *Lizard* $5^{\circ} \frac{1}{4}$ West.

In like manner, the Longitude of *Barbadoes* is 57 Degrees 54 Minutes West.

PROB. III.

To find the Angle of Position between two Places, that is, to find the Angle, the Arch of a Great Circle (passing through the two given Places) makes with the Meridian of one of the Places.

EXAMPLE.

I demand the Angle of Position between *Barbadoes* and the *Lizard*?

PRACTICE.

Rectifie the Globe to the Latitude of one of the Places (suppose the *Lizard*) fixing the Quadrant of Altitude to that Latitude (50 Degrees;) bring the said Place to the Brass Meridian; where staying the Body of the Globe, lay the Graduated Edge of the Quadrant of Altitude over the other Place (in this Case *Barbadoes*) the Degrees of the Horizon, (South 71 Degrees 30 Minutes West, or nearest W. S. W. half W.) counted

PART II.

Y

from

The Use of Globes

from the South, (or North) towards the West (or East) is the Angle of Position, or Angle, the Arch of a Great Circle passing over the the said Places, makes with the Meridian (of the *Lizard*) the first Place.

Note, This is not the Rumb, or Point of the Compass from the *Lizard* to *Barbadoes*, which may thus be proved : If you rectifie the Globe to the Latitude of *Barbadoes* and proceed as directed above, you will find the Angle of Position between *Barbadoes* and the *Lizard* to be North 37 Degrees 30 Minutes East, which is 34 Degrees less than the Position between *Barbadoes* and the *Lizard* ; whereas the Rumb, or Course is South 49 Degrees 32 Minutes West from the *Lizard* to *Barbadoes*, and North 49 Degrees 32 Minutes East from *Barbadoes* to the *Lizard* ; so that the Angle of Position between either is not the true Point of the Compass leading from one to another.

For the better understanding hereof we shall shew these few Propositions, *viz.*

1. The Mariners Compass pointeth out the common Intersection of the Horizon and Meridian, the one respecting the North, and the other the South.

2. If we sail upon any Point of the Compass, except North or South, we often change our Horizon and Meridian.

3. The same Rumb cutteth all Meridians of all Places at equal Angles, and respecteth the same Quarters of the World in every Horizon.

4. The Portions of the same Rumb intercepted between any two Parallels, whose Difference of Latitude is the same, are also equal to each other.

Hence an Equal Segment of the same Rumb equally changeth the Difference of Latitude in all places ; so that in an equal space passed in one and the same Rumb, one of the Poles is equally elevated, and the other depressed.

5. A great Circle drawn through the *Vertex* of any Place that is distant from the Equator, cannot cut diverse Meridians at equal Angles.

6. A Circle drawn thro' the Vertical Point of any Place, and inclining to the Meridian, maketh greater Angles wth all other Meridians, than with that from whence it was drawn.

7. From all which we may observe, that if a Ship sail on the North or South Rumb, her Course will be always under the same Meridian; if East or West, she will either describe the Equator, or a Circle parallel thereto: For if your Vertical Point be under the Equinoctial, your Ship will describe a Segment or Arch of the Equator; but if your Vertical Point or Zenith be distant from the Equator either North or South, your Course will then describe a Parallel as far distant from the Equator, as is the Latitude, of the place you sailed from: But if a Ship sails on a Rumb, which inclineth to the Meridian, her Course will then be neither a greater nor lesser Circle; but she will describe a Crooked or Spiral Line, which if never so far continued, does not pass thro' the Poles, but wind about the Poles until they lose themselves.

PROB IV.

Any two Places given, to find the Rumb or Point of the Compass between them (if the Rumb-Lines are drawn on the Body of your Globe.)

PRACTICE.

Find the said two places on the Globe, and see what Rumb-Line passeth thro' both of 'em; and that is the Rumb-point, Course or Bearing of those two places one from the other: For

EXAMPLES.

1. You'll find the *Lizard* and *Cape-Cod* in *New England* to bear from each other W.b.S. and E.b.N. also *Cape del Gadé* on the Coast of *Zanquebar* and *Comorin*, will be found to bear off each other W.S.W. and E.N.E.

2. But If you cannot find a Rumb-Line that passeth through both the Places, then observe what Rumb-line on the Globe runneth nearest to a parallel to both Places,

PART II.

Y 2

and

and that is the Rumb-Point or Course from one of the Places to the other.

EXAMPLE.

1. So you'll find the nearest Parallel-Rumb for the *Lizard* and *Barbadoes* is South West half West; And consequently the *Lizard* bears from *Barbadoes* North East half East. Also after the same manner *Sierra Leona* in *Africa*, and the Island *St. Helena*, bear North North West and South South East.

PROB. V.

To find the Distance between any two Places.

PRACTICE.

Lay the Quadrant of Altitude over the two Places (i.e. the beginning of the Degrees being on one of Places) the Degrees intercepted, is the Distance; which you may reduce to Miles, or Leagues by multiplying by 60, or 20.

EXAMPLES.

1. So you'll find the Distance between *London*, and *Charlton-Island* is 46 Degrees 20 Minutes, or 2780 Miles, and the Distance of the *Lizard* and *Barbadoes* is 56 Degrees or 3360 Miles, by the Arch of a Great Circle, which is less than in the Rumb by 61 Miles.

PROB. VI.

The Latitude and Longitude (of your Ship or) of any Place being given; to find the true Situation of it, tho' not expressed upon the Globe.

PRACTICE.

Find the Degrees, &c. of the Longitude of the Given place on the Equator, on the Globe, and bring the said point to the Brazen Meridian; then reckon the Latitude of the said place upon the Degrees of the said Brazen Meridian, from the Equator North or South, according

according to its Latitude; and right under the Degree and Minute upon the Meridian is the Point or Place, (of the Ship) required.

PROB. VII.

The Time of the Day given, at one Place, to find the Time of the Day in any other Part of the Earth.

EXAMPLE.

Let the Place be London, and it's required when it's 12 a-Clock at London, what a-Clock it is at Charlton Island?

PRACTICE.

Bring London to the Brazen Meridian, and set the Hour-Index to the upper 12 of the Hour-Circle; then turn the Body of the Globe Eastward (because London is Eastward of Charlton-Island, till you bring Charlton-Island to the Brazen Meridian; then shall the Index point on the Hour-Circle to 6h. 45m. A.M. before Noon; and in like manner when it's Noon at London, 'tis at Surat in the East-Indies 5h. 45m. P.M. or Afternoon.

PROB. VIII.

To find out the different Habitations of the Antaci, Periaci, and Antipodes, of any Place (suppose to London, Latitude 51 Degrees and half North.)

PRACTICE.

Bring London to the Brazen Meridian; it shews 51 Minutes and half North Latitude. Then Number 51 Degrees and half of South Latitude below the Equator, and you meet with the Antaci (if any be.) Remove London 180 Degrees from the Brazen Meridian, either Eastward or Westward, and you'll have the Periaci, under the Meridian where London was before, and the Antipodes is now in the Place where their Antaci stood before.

PROB IX.

The Sun's Place given, to find what places of the Earth the Sun is Vertical to (or in their Zenith) at any Time assign'd.

PRACTICE.

Find the Sun's Place given, in the Ecliptick on the Globe, which bring to the Brazen Meridian, and observe what Degree of the Meridian it cuts; then moving the Globe round about, you'll see what Places of the Earth are in that Parallel of Declination, (for they will all come successively to that Degree of the Brazen Meridian) and those are the Places or parts of the Earth or Sea to which the Sun will be Vertical, or in their Zenith.

But *Note*, This must be understood of such only as inhabit in the Torrid Zone; or between the Tropicks of 23° and 23° : For the Sun is never in the Zenith of any (Place or) people that inhabit either in the Temperate or Frigid Zones, viz. between the Latitude of 23° and half to 90° either North or South.

PROB. X.

The Day of the Month and Hour of the Day, of either a Solar or Lunar Eclipse being given, to find by the Terrestrial Globe, all those places of the Earth, in which the same will be Visible.

PRACTICE.

Observe the Sun's place in the Ecliptick (on the Globe) for the Day given, and also the Opposite point thereto, which is the Moon's place at that Time.

Then find that place of the Globe to which the Sun is Vertical at the Time or Hour given, and bring the same to the Vertical point, (or Pole) of the wooden Horizon; and staying the Globe in that Situation, observe what places are in the upper Hemisphere; for in most of them, will the Sun be visible during the Eclipse. Thus for an Eclipse of the Sun. But,

2. For the Moon's Eclipse, do thus ;
Find the *Antipodes* of that place, which hath the Sun Vertical at the Hour given ; and bringing the same to the Pole of the Wooden Horizons, mark what places are in the upper Hemisphere of the Globe ; for in them will the Moon be visible during her Eclipse, except those that are precisely in the Horizon, or very near it.

PROB XL.

1. The Day of the Month and Hour of the Day, being given, to find those places on the Globe, in which the Sun riseth.

2. Those in which he then Setteth.

3. Those to whom it's Mid-day.

And *Lastly*, Those Places that are actually enlightned, and those that are in Darkness.

PRACTICE.

Mark that place on the Globe, to which the Sun is Vertical at the given time, and bring the same to the Brazen Meridian ; elevate the Pole according to the Latitude of the said place ; and there staying the Globe, observe what Places touch the Verge of the Western Semi-circle of the Wooden Horizon ; for in them the Sun riseth at that Time.

2. Those in the Eastern Semi-circle, for in them the Sun setteth.

3. Those that are exactly under the Brazen Meridian ; for in them 'tis Noon or Mid-day. And,

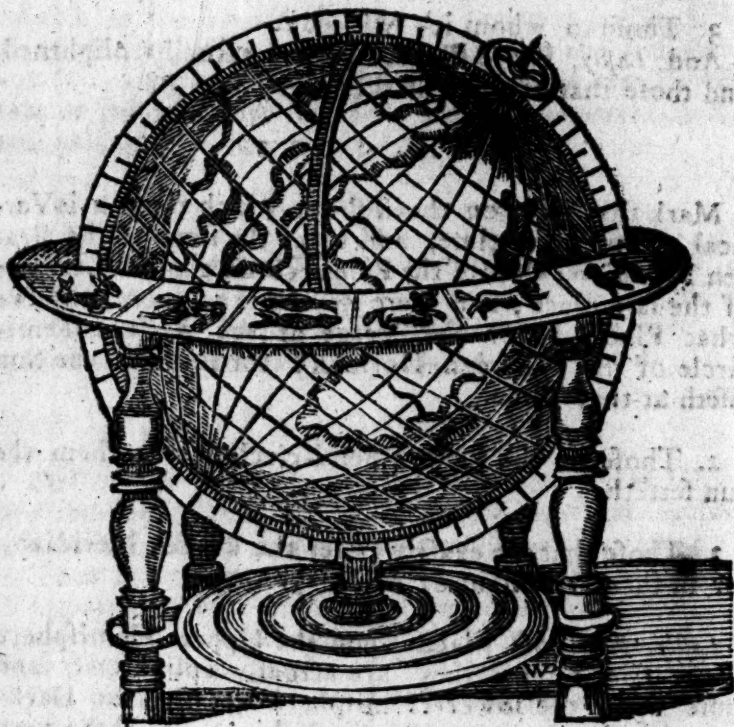
Lastly, All those places upon the Upper Hemisphere of the Globe ; for they are actually enlightned, and those upon the lower Hemisphere are then in Darkness, or Deprived of the Sun's Light at that very Time.

*The Use of the Celestial GLOBE in Astronomical
PROBLEMS of the Sun, and Fixed Stars.*

I. Of the SUN.

PROB. I.

THe Month and Day, of the month, either according to the *Julian* or *Gregorian* Account, be



ing given; to find the Sun's Place in the Ecliptick.

EXAM-

EXAMPLE.

What's the Sun's Place at Noon, on the 3d of September in the Julian Account; or September 14th, (by the Gregorian?)

PRACTICE.

Look for the Day of the Month September 3d. (Julian) or September 14 (Gregorian) which is all one Day on the Wooden Horizon, or Calender, and right against the said Month, stands the Sun's place (in this Case) 21° of XXI of Virgo. W. W. R.

NB. All the Problems relating to the Sun may as well be wrought on the Terrestrial, as Celestial, the same Circles being described on both: But 'tis more natural to work the Problems of Geography and Navigation by the Terrestrial, and the following Problems of the Sun and Stars by the Celestial Globe; on which diverse Problems may be resolv'd, the Globe, being in any position; yet 'twill be more convenient to have the Globe accommodated with all its necessary Appendants, before you use it, and also rectifie the Celestial Globe for the Latitude of London 51 Degrees and half North, and September 3d, 1732, by Prob. 1. and 4. of the Terrestrial Globe: Set the Brazen Meridian to the Latitude 51 Degrees and half North, and screw the Quadrant of Altitude fast to the Zenith or 90 Degrees and half on the said Brass-Meridian: Then having found the Sun's place for the given time September 3d to be 21 Degrees of XXI on the Ecliptick of the Globe; which bring to the Brass-Meridian, and there staying the Globe, move the Hour-Index or Pointer to the upper XII on the Hour-Circle, so that it may go round with the Globe; and your Globe is then Rectified for the Given Place and Time, and fit to resolve the Problems following.

[Vide the Print of the Celestial Globe, Page 356.]

PROB. II.

The Latitude (51 Degrees and half North) and the Sun's place (16 Degrees γ) Given, to find the Sun's Declination?

PRA-

PRACTICE.

Bring 16 Degrees of γ (on the Ecliptick of the Globe) to the Brass-Meridian, the Degrees of the said Meridian cut by the Sun's Place, being counted from the Equinoctial, you'll find to be 6 Degrees North, which is the Sun's Declination ; because he is in a Northern Sign.

PROB III.

The Sun's Place (16 Degrees of Ω .) Given ; to find his Right Ascension.

PRACTICE.

Bring 16 Degrees of Ω to the Brass-Meridian, the Degrees of the Equinoctial cut thereby is the Sun's Right Ascension (138 Degrees 26 Minutes) to be counted from the Beginning of γ .

PROB IV.

The Sun's Place (16 Degrees of Ω) and Latitude (51 Degrees and half North) given as before ; to find the Oblique Ascension.

PRACTICE.

Bring the Sun's Place (16 Degrees of Ω) to the East side of the Wooden Horizon ; the Degrees of the Equinoctial, cut by the said Horizon, is the Oblique Ascension required, in this case 117 Degrees 10 Minutes.

PROB V.

The same things given as before ; to find the Oblique Descension.

P R A C T I C E .

PRACTICE.

Bring the Sun's Place (16 Degrees of $\mathcal{Q}$) to the West Side of the Horizon, (representing Sun-Setting; the Degrees of the Equinoctial, cut by the Horizon is the Sun's Oblique Descension.

PROB VI.

The Sun's Oblique Ascension, and Descension, being given, to find his Ascensional Difference.

PRACTICE.

Subtract the lesser Number from the greater, and the Remainder is the Ascensional Difference required.

PROB VII.

The Latitude (51 Degrees and half North) and Sun's Place (16 Degrees of $\mathcal{Q}$) given, to find the Time of Sun-rising and Setting.

PRACTICE.

Bring the Sun's place (16 Degrees of $\mathcal{Q}$) to the Brass-Meridian, and the Hour-Index to the upper 12 in the Hour-Circle; then bring the Sun's place (16 Degrees of $\mathcal{Q}$) to the East side of the Horizon, and the Hour-Index will point to (4h. 35m.) the Sun's Rising in the Morning.

2. Again, Turn the Body of the Globe Westward till the Sun's place (16 Degrees of $\mathcal{Q}$) touch the West side of the Horizon, and the Hour-Index will point to (7h. 25m.) the Sun's Setting in the Evening.

PROB. VIII.

The same things given, as in the last Problem, to find the length of the Day and Night.

P R A-

PRACTICE.

Bring the Sun's place (16 Degrees of $\mathcal{N}$) to the East side of the Horizon, and the Hour-Index to the undermost XII; then turn the Body of the Globe Westward, till the Sun's place (16 Degrees of $\mathcal{N}$) touch the West side of the Horizon, and the Index will point to the Time less, or more than 12 Hours, in this Case 2h. 50m. more than 12 Hours; that is, the Day is 14h. 50m. long. Again, If you count the Hours between the lower 12, and the Hour-Index, 'twill give (9h. 30m.) the Length of the Night required.

P R O B. IX.

To find the Beginning, Ending, and Continuance of the *Crepusculum*, or Twilight. *Examp.* Let the Sun's Place be 29 Degrees of $\mathcal{O}$ *Taurus*, the Latitude 51 Degrees and half North (and the Twilight begins and ends when the Sun is 18 Degrees below the Horizon.

PRACTICE.

Rectify the Globe, as before taught, by putting the Brass-Meridian to the Latitude 51 Degrees and half North, and Quadrant of Altitude to the Zenith, or 51 Degrees and half; and there screw it fast.

And seeing the Quadrant of Altitude will not reach 18 Degrees below the Horizon, find therefore the Opposite Point to $\mathcal{O}$ 29 Degrees, and that is $\mathcal{M}$ 29 Degrees; which last Point, as also the Quadrant of Altitude, bring to the West side of the Brazen Meridian; and moving the Globe, and Quadrant of Altitude, then observe how many Hours the Hour-Index is moved from XII (in this Case 1h. 8m.) for that shews that Twilight begins at 8 Minutes after 1 in the Morning; this subtracted from the Time of the Sun's Rising (4h. 11m.) found by *Prob. 7.* remains 3h. 3m. before Sun-rising, which is the Length of the *Crepusculum*, or Continuance of the Twilight:

Again, the Beginning of the Twilight (1h. 8m.) being doubled, gives (2h. 16m.) the Length of dark Night.

2. Also, if you would find at what Time the Twilight endeth after Sun-setting, bring the said opposite Point to the Sun in 29 Degrees, to 18 Degrees of the Quadrant of Altitude on the East side of the Brazen Meridian; and then will the Hour-Index point to, or shew 10h 52m. the Time that Twilight ends in the Evening.

Or thus; subtract the beginning of Twilight, (1h. 8m.) from 12h, the Remainder shews (10h. 52m.) the Continuance of Twilight, as before.

N. B. From May 12th, to July 12th, (i. e. while the Sun is perambulating from 12 Degrees of II Gemini , to 30 Degrees of S Cancer) you'll find by the Globe, that there will be no Twilight at all, but continual Day: For all that Space of Time the Sun never descendeth so much as 18 Degrees under the Horizon in the Latitude of London, which is 51 Degrees and half North.

P R O B. X.

The Sun's Place, (16 Degrees of Q) in Latitude 51 Degrees and half North; What's the Sun's Amplitude?

P R A C T I C E.

The Globe Rectified, bring the Sun's place (16 Degrees of Q) to the East or West side of the Horizon, and the Degrees intercepted between the Sun's place and the East or West Points of the Horizon (accounted from the East or West towards the North or South) is the Sun's Amplitude, (in this Case East 26 Degrees North) that is, he rises East 26 Degrees North, and sets West 26 Degrees North or E.N.E. $\frac{1}{4}$ N. and W.N.W. $\frac{1}{4}$ N. because the Sun is in a Northern Sign.

P R O B. XI.

The Sun's Place (16 Degrees of Q) and Latitude (51 Degrees and half North) given; to find his Meridian Altitude, and Depression at Midnight.

PRACTICE.

The Globe being rectified, bring the Sun's Place (19 Degrees of $\mathcal{A}$) to the Brass-Meridian, and count the Degrees from the Point over the Sun, in the Brazen Meridian, to the South Part of the Horizon, (in North Latitude) but down to the North part of the Horizon in South Latitude;) the Degrees intercepted (in this Case 54 Degrees 36 Minutes) are the Sun's Meridian Altitude required.

2. Bring the Sun's Place ($\mathcal{A}$ 16 Degrees) to the opposite Point of the Meridian below or under the Horizon; the Degrees intercepted, between that Point of the Meridian and the upper Edge of the Horizon (in this Case 22 Degrees 24 Minutes) is the Sun's Depression at Midnight.

PROB. XII.

The Sun's Place (suppose 29 Degrees of $\mathcal{S}$) Latitude (51 Degrees and half North) and Hour of the Day (9. A.M. or 3 P.M.) given; to find the Sun's Altitude, at the said Place and Time.

PRACTICE.

Rectifie the Globe, and Quadrant of Altitude, screw it to the Zenith, and bring 29 Degrees $\mathcal{S}$ to the Brass-Meridian, the Hour-Index to the upper XII; then turn about the Globe, till the Hour-Index points to the Given Hour (9. A.M. or 3. P.M.) there staying the Globe, lay the Quadrant of Altitude over the Sun's Place ($\mathcal{S}$ 29 Degrees;) the Degrees cut by the Quadrant of Altitude thereby, is the Sun's Altitude (in this Case 43°).

PROB. XIII.

The same things given as in the last Prob. to find the Sun's Azimuth.

PRACTICE.

Perform the last Problem and observe then the Degrees of the Horizon, cut by the Quadrant of Altitude, is

is the Sun's Azimuth, to be accounted from the North or South part of the Horizon towards the East or West.

PROB. XIV.

The Sun's Place (29 Degrees of $\odot$) Latitude (51 Degrees and half North) and Sun's Altitude (12 Degrees *A.M.*) given, to find the Hour of the Day.

PRACTICE.

Rectifie the Globe to the Latitude, the Sun's place being brought to the Brass-Meridian, the Hour-Index to 12, and the Quadrant of Altitude fix'd on the Zenith. Then bring the Sun's Place ($\odot$ 29 Degrees) to the Given Altitude, (on the Quadrant of Altitude) before (or after) Noon; then shall the Hour-Index point at (5h. 36m. *A.M.*) (or if *P.M.* at 6h. 24m.) the Hour of the Day required.

PROB. XV.

The same things given, as in the Last Problems, to find the Sun's Azimuth.

PRACTICE.

perform the last Problem, the Degrees of the Horizon cut by the Quadrant of Altitude, is the Sun's Azimuth from the North or South, (in this Case 107 Degrees 8 Minutes) from the South, or 72 Degrees 52 Minutes from the North Eastward; which reckon'd by the Points of the Compass on the Horizon, will be very near E.N.E. half E. if in the Morning, or W. N.W. half W. if the Observation had been in the Evening.

PROB. XVI.

The Sun's Place ($\odot$ 26 Degrees) Latitude 51 Degrees and half North given; to find the Sun's Altitude at the Hour of 6, Morning or Evening.

PRA-

PRACTICE.

Rectifie the Globe (as in *Prob. 12*) by bringing the Sun's Place $\mathcal{Q}$ 16 Degrees to the Meridian and Hour-Index to 12; then turn about the Globe Eastward or Westward, till the Index points at 6 o'Clock in the Hour-Circle; there holding the Globe, lay the Quadrant of Altitude, just over the Sun's place ($\mathcal{Q}$ 16 Degrees) and it will cut the Quadrant of Altitude in 12 Degrees 32 Minutes (in this Case) the Sun's Altitude at 6. A.M. or P.M. W.W.R.

PROB. XVII.

The same things given, as in the last *Problems*, to find the Sun's Azimuth.

Perform the last *Problem*, the Degrees of the Horizon, cut by the Quadrant of Altitude accounted from the North is (N. 79 Degrees 49 Degrees E. in the Case above, or E. b. N. 1 Degree 04 Minutes Easterly) the Azimuth at the Hour of 6, required.

PROB. XVIII.

The same things given, as in *Prob. 16*. to find the Hour of the Day, when the Sun shall be due East or West.

PRACTICE.

The Globe rectified, (as before taught) bring the beginning of the Degrees of the Quadrant of Altitude to the East Point of the Horizon, and turn the Globe about, till the Sun's place ($\mathcal{Q}$ 16 Degrees) do touch the Degrees of the Quadrant of Altitude, and then will the Index point at (6h. 57m. A.M.) the Time the Sun is due East. Also, if you carry the Quadrant of Altitude to the West point of the Horizon, turn about the Globe till 16 Degrees of $\mathcal{Q}$ touch the Edge of the Degrees, the Hour-Index will point at (5h. 03m. P.M.) the Time the Sun will be due West.

PROB. XIX.

P R O B. XIX.

The same things given, as in *Prob. 16.* to find the Sun's Altitude, when he is in the East or West Azimuths.

P R A C T I C E.

Rectifie the Globe (as before) and bring the Quadrant of Altitude to the East or West-points of the Horizon; then turning about the Globe till the Sun's place (♌ 16 Degrees) cuts the graduated Edge of the Quadrant of Altitude (in this Case 20 Degrees 19 Minutes) which reckon'd upon the Quadrant of Altitude, from the Horizon to the Sun's Place, is the Sun's Altitude when he is upon the East or West Azimuths. *W.W.R.*

✍ *Note.* - These Four last Problems are only of use when the Sun is in the six Northern Signs: For the Sun is never above the Horizon, at the Hour of 6, or on the East or West Azimuths when he is in the Six Southern Signs, of the Zodiack or Ecliptick.

S E C T. II.

*The Use of the Celestial Globe in Problems of the
Fixed STARS.*

PROB. I.

TO find a Star's (suppose * *Aldebaran*, or the Bull's South Eye) Right Ascension. (Latitude 51 Degrees and half North.



PRACTICE.

The 1st, 2d, 3d, 4th, 5th, and 6th Problems following, viz. to find a Star's Right Ascension, Declination, Oblique Ascension, Oblique Descension, Amplitude and Meridian Altitude, are performed in the same manner as the

the 3d, 2d, 4th, 5th, 10th, and 11th Problems of the Sun, by bringing the Center of the Star to the Brazen, Meridian; or Wooden Horizon, as the Problem requires; so that the Bull's South Eye being brought to the Meridian, the said Meridian cuts the Equinoctial in 64 Degrees 17 Minntes from γ ; or in Time, 4h. 16' for its Right Ascension required.

PROB. II.

To find a Star's (admit *Aldebaran's*) Declination, bring it to the Brass-Meridian, as before; the Degrees cut thereby, shew its Declination 15 Degrees 50 Minntes North.

PROB. III.

To find a Star's (suppose the South Eye of the Bull's) Oblique Ascension.

Bring the Star to the East side of the Horizon; the Degrees of the Equinoctial cut by the said Horizon are his Oblique Ascension required.

PROB. IV.

To find a Star's (as admit the Bull's South Eye) Oblique Descension.

Bring the said Star to the West side of the Horizon; the Degrees of the Equinoctial cut by the Horizon, give his Oblique Descension.

PROB. V.

To find a Star's (suppose *Aldebaran's*) Amplitude.

Bring the said Star to the Horizon, on either side; 'twill cut it in 25 Degrees 56 Minutes from the East or West North ward; or in Points of the Compass on the same near N. E. $\frac{1}{2}$ E. or N. W. $\frac{1}{2}$ W. the said Star doth rise, and set from the North Eastward or Westward.

PROB. VI.

To find a Star's Meridian Altitude, (as suppose *Aldebaran's*.)

PART II.

γ 2

Bring

Bring the Star to the Meridian; the Degrees intercepted between that point on the same and the South part of the Horizon is the Star's Meridian Altitude.

P R O B. VII, VIII, and IX.

The Latitude (51 Degrees and half North) and Time of the Year, (or Sun's Place, *viz.* January 1. the Sun being in 22 Degrees of VS) given; to find the Time of the Day or Night (*Aldebaran* or) any Star rises, comes to the Meridian, and sets.

P R A C T I C E.

Elevate the Globe to the Latitude (51 Degrees and half North) Bring VS 22 Degrees) the Sun's place to the Brass-meridian, and the Hour-Index to XII; then bring the said Star (*Aldebaran*) to the East and West side of the Horizon; also the Brass-meridian, and the Index will accordingly point to the said Star's Rising, (1h. 15m. P.M.) Setting 4h. 9m. A.M.) and his Culminating or coming to the Meridian (8h. 42m.)

P R O B. X.

To find (*Aldebaran's* or) any Star's Ascensional Difference.

P R A C T I C E.

The Globe Elevated to the Latitude (51 Degrees and half North) bring the Star (*Aldebaran*) to the Brass-Meridian, and the Hour-Index to XII. Then the Star-being brought to the East or West side the Horizon, the Hours and Minutes that are contained between the Hour-Index and 6 a-Clock is the Stars Ascensional Difference, in this Case, 1h. 27m.

P R O B. XI.

To find (*Aldebaran's*) or any Star's Semidiurnal Arch, or (more properly) half its Time or Continuance above the Horizon.

PRACTICE.

The Globe set to the Latitude (51 Degrees and half North) as before, bring the Star (*Aldebaran*) to the Meridian, and the Hour-Index to 12. Then turn the Globe till the said Star touches the Horizon, and the Index shall point at (7h. 27m.) the Time the Star (*Aldebaran*) is above the Horizon, before he comes on the Meridian, and continues the same time he passes the Meridian till he sets in the West. Therefore 7h. 27m. is the Semidiurnal Arch; which doubled is (14h. 54m.) the Time of the Star's continuance above the Horizon.

PROB. XII.

The Latitude (51 Degrees and half North) and Day of the Month *November* 16th,) or Sun's Place 5 Degrees 24 Minutes of $\mathcal{M}$) given; to find what Stars shall be upon the Meridian, at 12 a-Clock that Night.

PRACTICE.

The Globe rectified, by bringing the Sun's place (5 Degrees 24 Minutes of $\mathcal{M}$) to the Meridian, and the Hour-Index to the upper 12: Turn the Body of the Globe till the Hour-Index points to the lower 12; there staying the Globe, observe what Stars are then exactly under the Semi-circle of the Brass-Meridian; for those are Stars that will be on the Meridian at Midnight, at that Time, of which *Aldebaran* is the Chief.

PROB. XIII.

To find what Stars will come to the Meridian at any Hour of the Night,

Example. *January* 5th the Sun in 25 Degrees 33 Minutes of $\mathcal{V}$ S, what Stars will Culminate at 8h. 26m. that Night.

PRACTICE.

Bring 25 Degrees 33 Minutes of $\mathcal{S}$ (the opposite Point to $\mathcal{V}$ S) to the Meridian, and the Hour-Index to XII. Then turn the Globe about Westward, till the Index

point at 8h. 26m ; and there staying the Globe, all the Stars which lie under the Brass-Meridian, are then upon the Meridian, of which *Aldebaran* or the Bull's Eye is the Chief.

P R O B. XIV.

The Sun's Place (20 Degrees of μ) and the Altitude of a known Star (suppose *Aldebaran*, 30 Degrees high to the Eastward, or wanting the Meridian,) given, to find the Hour of the Night.

P R A C T I C E.]

Rectify the Globe, and bring (20 Degrees of μ) to the Meridian, and the Hour-Index to 12, and the Star (*Aldebaran* to the given Altitude (30 Degrees) (either wanting, or past the Meridian;) there staying the Globe; observe at what Hour the Index points, (as here at 9h 2m.) for that is the Hour of the Night required.

P R O B. XV.

The Latitude (51 Degrees and half North, the Altitude of *Aldebaran* (42 Degrees short of the Meridian, or any other Star given, to find the said Star's Azimuth.

P R A C T I C E.

Bring the Star to the given Altitude (42 Degrees) on the Quadrant of Altitude, (it being screw'd to the Zenith) either past, or wanting the Meridian, the Degrees of the Horizon cut by the Quadrant shew the Star's Azimuth, from the North or South, in this Case S. 57 Degrees E. or near S. E. *b.* E. in Points of the Compass.

P R O B. XVI.

The Latitude (51 Degrees and half North) And *Aldebaran's* or any other Star's Azimuth (S. 57 Degrees E.) given, to find its Altitude?

PRACTICE.

Bring the Quadrant of Altitude to cut (S. 57 Degrees E.) on the Horizon the Azimuth ; then move the Globe till the Star cuts the graduated Edge of the Quadrant of Altitude, and it will shew on the same (42 Degrees) the Stars's Altitude required.

P R O B. XVII.

To find the Distance between any two Stars.

PRACTICE.

Set the Graduated Edge of the Quadrant of Altitude (it being loose) to the Centers of both the Stars, and the Degrees intercepted, on the Quadrant between the two Stars are their Distance.

Note, If the Quadrant of Altitude be too short, take their Distance with a pair of *Calope-Compasses*, and measure their Distance upon the Equinoctial of the Globe.

Thus you will find that *Orion's* Right Shoulder and the Right Shoulder of *Auriga* are distant asunder $37^{\circ} 38'$, and *Marchad* in the Wing of *Pegasus*, and *Lyræ* the Harp are distant 63 Degrees.

P R O B. XVIII.

To find the Longitude and Latitude of a Star.

EXAMPLE.

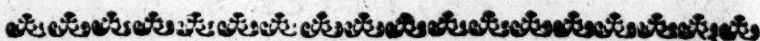
What is the Longitude and Latitude of the bright Star *Capella*?

PRACTICE.

Skrew the Quadrant of Altitude over that Pole of the Ecliptick, which is nearest to the Star, whose Longitude you would find ; that is, it must be fixed over the polar Circle, or 23° and half distant from the Pole of the Globe.

Then laying the Quadrant of Altitude exactly over the Center of the Star (*Capella*), observe what Degrees of the Ecliptick (reckoning them from the Beginning of γ) and those Degrees, (*i. e.* $77^{\circ} 16'$) are the Star's Longitude from γ .

2. The Quadrant being fitted, as before, and laid over the Center of the Star (*Capella*) the said Star shall cut the Quadrant of Altitude in 22 deg. 50 min. the Star's Latitude from the Ecliptick North; because in this Case the Star is to the Northward of the Ecliptick.



S E C T. III.

Of the Poetical Rising and Setting of the Fixed Stars, viz.

The { Cosmical
Achronical } Rising and Setting of the Stars.
Heliacal

P R O B. XIX.

1. **T**O find the Cosmical Rising and Setting.

E X A M P L E.

In Latitude 51 deg. and half North, what Stars do rise and set Cosmically on *May 27*, the Sun being in 17° of Π ?

P R A C T I C E.

Rectify the Globe to the Latitude, and bring the Sun's place $\Pi 17$ deg. to the East part of the Horizon. Then observe what Stars are in the Verge of the Eastern Semi-circle of the Horizon; for there those Stars, (*i. e.* *Aldebaran*, and divers other smaller Stars) do rise Cosmically that Day. Also those Stars which touch the Western Semi-circle of the Horizon do set Cosmically at that time.

Which

Which you'll find to be *Serpentarius's* right Leg; and divers other small Stars are setting Cosmically.

P R O B. XX.

2. To find the Achronical Rising and Setting of the Stars.

E X A M P L E.

In the Latitude of 51 Degrees and half North, *October* 18, the Sun is in 5° of *m*. What Stars do then rise and set Achronically?

The Globe rectified, bring the Sun's place, 5 degr. *m* to the West part of the Horizon; then all those Stars which touch the Eastern Side of the Horizon are rising Achronically, (*i. e.* the Whale's Tail, and other small Stars; also those that are in the Verge of the Western parts of the Horizon (as the Lion's Tail, South Ballance, and other small Stars) are then setting Achronically.

P R O B. XXI.

3. To find the Heliacal Rising and Setting of the Stars.

1. The Globe being rectified to the Latitude, and the Quadrant of Altitude fixed to the Zenith; then bring the Star given (suppose *Aldebaran*, which is a Star of the first Magnitude, and may be seen, by the Rule of the Ancients, when the Sun is but 12 Degrees below the Horizon) to the East Side of the Horizon, and the Quadrant of Altitude to the West Side thereof, and observe what Degree of the Ecliptick doth cut the Quadrant of Altitude in 12 Degrees, which you shall find to be 15 Degrees of *Capricorn*, the Opposite Point, Sign and Degree to which is 25 Degrees of *Cancer*; to which when the Sun cometh (which will be about the 26th of *June*) then will *Aldebaran* rise Heliacally.

2. Bring the Star to the West Side of the Horizon, and turn the Quadrant of Altitude to the East Side, and see what Degree of the Ecliptick is Elevated upon the Quadrant (which is 13 Degrees 11 Minutes of *Scorpio*.) The opposite Degree to this is

13 Degrees 82 Minutes of *Taurus* ; to which when the Sun comes, which is about *April 22*, the said Star (*Aldebaran*) shall then set Heliacally.

P R O B. XXII.

The Latitude, Month, Day, and Hour of the Night given, to find what Stars are then rising or setting, what are on the Meridian, and how they are above the Horizon, and on what Azimuth, or Point of the Compass they are ; by means of which the Real Stars in the Heaven may be known and distinguished rightly by their proper Names, &c.

P R A C T I C E.

Rectifie the Globe to the Latitude, and screw the Quadrant of Altitude to the Zenith.

Then by help of a Compass, set the Globe due North and South ; and the Sun's Place being brought to the Meridian, turn about the Globe, till the Hour-Index points at the given time of the Night.

Then shall the Globe in this Position represent the Face or Appearance of the Heavens at that very time.

By which you may easily see what Stars are in the Eastern Horizon, rising, and what in the West, setting ; what are on the Meridian, which are to the North, and which are to the South, &c.

And if you lay the Quadrant of Altitude over any particular Star, 'twill shew its Altitude and Azimuth, or on what Point of the Compass it is ; by which Method of proceeding any Star may be easily known ; especially, if you have a Quadrant, or fit Instrument, to take the Altitude of any fixed Star express'd on the Globe, to see whether it agrees with that Star which is its Representative on the Globe, or not.

Lastly, If every Star on the Globe had a Hole thro' its Center, and your Eye were placed in the Center of the Globe, and looking thro' the Hole of any Star on the Globe, you would behold the same Star in the Heavens which that Star on the Globe represents : For from the Center of the Globe there doth proceed a continued Ray or right Line thro' the Star on the Globe, to the same Star in the Heavens.

P R O B.

P R O B. XXIII.

To find the Hour of the Day by the Sun's shining on the Globe.

P R A C T I C E.

Rectifie the Globe to the Lat. and set it due North and South, (by the last *Prob.*)

2. And by help of a Level or Plummert, set the Wooden Horizon parallel to the Natural Horizon; then shall the Poles and Circles of the Globe correspond with those in the Heavens.

3. When the Sun shines, observe the shadow of the Axis or Pole, upon the Hour-Circle, and it gives the Hour of the Day.

Thus may the Globe serve as an Universal Sun-Dial in any Latitude to find the Time of the Day. But,

Note, This Rule is only for the Summer half Year, viz. from *March* to *September* (in North Latitude) For if you would find the Hour for the Winter half Year, depress the Pole of the Globe, as much below the South part of the Horizon, as before it was Elevated above the North; so shall the shade of the Pole, or Axis give the Hour of the Day as before.

But in Setting the Brass-Meridian North and South by the Compass, you must allow the Variation of the Needle in that Place.

P R O B. XXIV.

Diverse Ways to find the Latitude of a Place by the Globe.

1. To find the Latitude by the Globe when the Sun shines.

P R A C T I C E.

Upon some Plan where the Sun shines, set the Globe level, (as directed in the last *Prob.*)

Then

Then turning the North Pole towards the Sun, (at Noon) until you find the Pole to cast no shade, that is, 'till the Sun's Rays fall into the very Axis of the Globe; then shall the Degrees on the Brass-Meridian, counted from the Horizon, give the Sun's Meridian Altitude: To which adding or subtracting the Sun's Declination for that Day, gives the Latitude of the Place.

PROB. XXV.

2. To find the Latitude by the Sun's or Star's Declination and Amplitude.

EXAMPLE.

Let the Sun or Star's Amplitude be 33 Degrees 20 Minutes North, and Declination 20 Degrees 5 Minutes North; what's the Latitude?

PRACTICE.

Elevate the Pole of the Globe to the Comp. of the Amplitude (56 Degrees 41 Minutes) and count the Amplitude (33 Degrees 20 Minutes) and screw the Quadrant of Altitude thereto; which is in the Zenith, (on the Brass-Meridian) and bring the Quadrant to the East or West Points of the Horizon, the Equinoctial Colure or first Point of γ being also brought to the Meridian; number on the Quadrant of Altitude (20 Degrees 5 Minutes) the Sun's Declination this (20 Degrees 05 Minutes) bring to the Equinoctial; and 'twill cut there 51 Degrees and half for the Height of the Pole or Latitude.

PROB. XXVI.

3. The Sun's Amplitude (33 Degrees 20 Minutes) and Ascensional Difference (27 Degrees 7 Minutes) given, to find the Latitude.

PRACTICE.

Elevate the Pole of the Globe to the Ascensional Difference, (27 Degrees 7 Minutes) and screw the Quadrant of Altitude to the same (27 Degrees 7 Minutes) which is in the Zenith.

Then

Then bring the first Point of Υ to the Meridian, and number on the Quadrant of Altitude upwards (56 Degrees 40 Minutes) the Comp. of the Amplitude, and bring that Degrees to the Equinoctial; then the Degrees of the Horizon (51 Degrees and half) cut by the Quadrant of Altitude counted from the East, or West is the Latitude required.

P R O B. XXVII.

4. The Sun or Star's Declination (20 Degrees 5 Minutes) and the Time when he is upon the East or West Points of the Compass (7h. 7m.) being given; to find the Latitude.

P R A C T I C E.

Elevate the Globe to the Sun's Declination (20 Degrees 5 Minutes) and screw the Quadrant of Altitude to the Zenith-point (*i. e.* 69 Degrees 55 Minutes;) then number the Hour past 6 (*viz.* 1h. 7m. or 16 Degrees 50 Minutes) upon the Horizon South-Eastward, or Westward; and to this Point bring the Quadrant of Altitude, and observe what Degrees of the said Quadrant are cut by the Equinoctial (as here 38 Degrees and half) counted from the Zenith; for that is the Complement of the Latitude, whose Complement to 90 Degrees is the Latitude (51 Degrees and half North) required.

P R O B. XXVIII.

5. The Sun's Declination (20 Degrees 5 Minutes) and his Altitude East or West (25 Degrees 55 Minutes) given; to find the Latitude.

E X A M P L E S.

Elevate the Globe to the Complement of the Sun's Altitude (64 Degrees 5 Minutes) and screw the Quadrant of Altitude to the Zenith, (64 Degrees 5 Minutes).

Then bring the Equinoctial point Υ to the Meridian, and count on the Quadrant of Altitude the Sun's Declination (20 Degrees 5 Minutes) upwards, and move it about the Equinoctial till those 20 Degrees 35 Minutes on the Quadrant touch the Equinoctial, which will be in
(38 De-

(38 Degrees and half) the Complement of the Latitude required.

P R O B. XXIX.

6. The Sun's Declination (20 Degrees 5 Minutes) and his Azimuth at the hour of 6 (12 Degrees 45 Minutes) given, to find the Latitude.

P R A C T I C E.

Elevate the Globe to the Sun's Azimuth at 6 (12 Degrees 45 Minutes) and screw the Quadrant of Altitude to the Zenith.

Then bring the First Point of γ to the Meridian, and number on the Quadrant of Altitude upwards (69 Degrees 55 Minutes) the Complement of the Sun's Declination; which bring to the Equinoctial; so shall the Quadrant of Altitude cut the Horizon in (38 Degrees and half) the Complement of the Latitude required.

P R O B. XXX.

7. A Star's Meridian Altitude (suppose *Aldebaran's*, to be 51 Degrees) given; to find the Latitude.

P R A C T I C E.

Bring *Aldebaran*, to the Meridian, (and from the said Point of the Star's Center, count his Meridian Altitude (51 Degrees) downwards; and observe that Point on the Meridian; then bring that Point to the South part of the Horizon, and you shall find the North Pole of the Globe Elevated to (54 Degrees 48 Minutes North) the Latitude required.

P R O B. XXXI.

8. Admit you observe two Stars (suppose *Erys* the Harp Rising East, and *Regular* on the Meridian) and you will find the Latitude.

P R A

P R A C T I C E.

Bring the Star (*Regulus*) to the Meridian; and there staying the Globe, move the Brazen Meridian up and down in the Horizon, till you bring the other Star (*Lyra*) to cut the East part of the Horizon, and then shall the Globe be Elevated to (37 Degrees 50 Minutes) the Latitude required.

P R O B. XXXII.

9. To find the Latitude, by Observing the Altitude of two known Stars in one Azimuth or Point of the Compass.

E X A M P L E.

Admit you observe *Scheder's* Altitude to be 66 Degrees and *Capella*, 20 Degrees when they both bear upon the North-East Point of the Compass, and you require the Latitude of the Place.

P R A C T I C E.

The Difference of the Star's Altitude (46 Degrees) is equal to their Distance.

Lay the Quadrant of Altitude over both the Stars, at their proper Degrees of Altitude you observ'd them.

Then turning the Globe in the Horizon, until the Quadrant does just touch the Horizon in the given Azimuth of the Stars (North-East;) so shall the Globe be Elevated to (40 Degrees) the Latitude required.

P R O B. XXXIII.

10. The Declination, Altitude and Azimuth of the Sun, or a known fixed Star.

E X A M P L E.

The Right Knee of *Hercules's* Declination is 47 Degrees 9 Minutes, his Azimuth Observed 122 Degrees from the North, his Altitude 60 Degrees. What is the Latitude of the Place of Observation?

P R A

PRACTICE.

The Globe Elevated to the Star's Altitude (60 Degrees) and the Quadrant of Altitude serew'd to the Zenith (60 Degrees.)

Then count (122 Degrees) the Azimuth from the North part of the Meridian, and to those Degrees bring the Quadrant of Altitude; and there keeping it, turn the Globe about, till the Star's Declination (47 Degrees 9 Minutes) counted upon the Equinoctial Colure from the Equinoctial) do cut the Quadrant of Altitude, in (71 Degrees 13 Minutes) the Latitude required.

P R O B. XXXIV.

11. The Day of the Month, and Hour of the Night, that you Observe a Star rising or setting given, to find the Latitude of the Place.

PRACTICE.

Rectifie the Globe and Hour-Index to the Time of the Night; there fix the Globe, and turn the Brass-Meridian thro' the Notches of the Horizon, till the Star observ'd comes to the East side of the Horizon, if Rising, or to the West, if the Star be Setting, so shall the Degrees of the Pole of the Globe's Elevation, cut by the Horizon, under the Elevated Pole, be the Pole's Height or Latitude required.

Thus have I exhibited diverse Ways of finding the Latitude by the Globe, which may be of good use, especially to the Mariner.

I shall therefore conclude this Section, with two General Problems, of the Sun, and a fixed Star; and leave the Operation the Industrious Learner.

P R O B. XXXV.

Of the S U N.

1. *January* 18th. Latitude 51 Degrees and half North
What's is 1 The Sun's Place?

2. The

2. The Sun's Declination ?
3. The Sun's Right Ascension ?
4. The Sun's Oblique Ascension ?
5. The Sun's Oblique Descension ?
6. The Sun's Amplitude ?
7. The Sun's Meridian Altitude ?
8. The Sun's Depression at Midnight
9. The Sun's Rising ?
10. The Sun's Setting ?
11. The Length of the Day and Night ?
12. The Sun's Altitude, suppose 10° A. M. given also ; what is his Azimuth from the South, and Hour of the Day ?

P R O B. XXXIV.

Of a Fixed Star.

August 4th, Lat. $51^{\circ} \frac{1}{2}$ North given, suppose the last Star in the Wing of Pegasus.

1. What is the Star's Right Ascension ?
2. The Star's Declination ?
3. The Star's Oblique Ascension ?
4. The Star's Oblique Descension ?
5. The Star's Amplitude ?
6. The Star's Meridian Altitude ?
7. The Star's Rising ?
8. The Star's Setting ?
9. The Star's Duration above the Horizon ?
10. The Suppose Star's Altitude 30° wanting the Meridian : What is his Azimuth from the South ?
11. The Hour of the Night ?
12. The Hour of the Night 9h. and half given ; what is this Star's Altitude ?
13. His Azimuth from the South ?



S E C T. III.

Of the Constellations, or Images, (containing the Fixed Stars) drawn upon the Celestial Globe.

THE Causes why the Ancient Astronomers assumed these kinds of Images, are probably these four, viz.

1. The Likeness those Stars have in Disposition with those Images.
2. The Efficacy which the same Astronomers superstitiously ascribed to some Constellation.
3. Immortality, to which some great Heroes were consecrated by them, and would have their Names as it were inscribed in Heaven.
4. For Instruction-sake : Things cannot be taught without Name : To give a Name to every Star, had been troublesome to the Master to devise, and for the Scholar to remember. Therefore they have reduced many Stars into one Constellation, that thereby they may tell the better where to seek them ; and being sought, how to express them.

The Number of these Constellations were by the Ancients accounted in Number Forty eight, twelve whereof we call the Twelve Signs of the Zodiack. But we have now drawn on the Globe Sixty six ; which are as follows, viz.

Zodiacal

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
14.
15.
16.
17.
18.
19.
20.
21.
22.
23.

PA

Zodiacal Constellations.

	Stars,
1. <i>Aries</i> , the Ram,	16
2. <i>Taurus</i> , the Bull,	32
3. <i>Gemini</i> , the Twins,	24
4. <i>Cancer</i> , the Crab,	16
5. <i>Leo</i> , the Lion,	33
6. <i>Virgo</i> , the Virgin,	39
7. <i>Libra</i> , the Ballance,	10
8. <i>Scorpio</i> , the Scorpion,	26
9. <i>Sagittarius</i> , the Archer,	29
10. <i>Capricornus</i> , the Goat,	21
11. <i>Aquarius</i> , the Waterman,	34
12. <i>Pisces</i> , the Fishes,	34

In all 314

Northern Constellations 23, viz.

1. <i>Ursa minor</i> , the Little Bear,	9
2. <i>Ursa major</i> , the Great Bear,	33
3. <i>Draco</i> , the Dragon,	36
4. <i>Cepheus</i> ,	11
5. <i>Bootes</i> ,	22
6. <i>Corona Borealis</i> , the Northern Crown,	8
7. <i>Hercules</i> ,	30
8. <i>Lyra</i> , the Harp,	10
9. <i>Cygnus</i> , the Swan.	17
10. <i>Cassiopeia</i> .	33
11. <i>Perseus</i> , with <i>Medusa's</i> Head,	27
12. <i>Auriga</i> ,	24
13. <i>Serpentarius</i> ,	24
14. <i>Serpens</i> ,	14
15. <i>Sagitta</i> , the Dart.	5
16. <i>Aquila</i> , the Eagle,	9
17. <i>Antionus</i> ,	7
18. <i>Delphinus</i> , the Dolphin,	10
19. <i>Equinulus</i> ,	5
20. <i>Pegasus</i> , the Flying Horse,	23
21. <i>Andromeda</i> ,	23
22. <i>Triangulum</i> , the Triangle,	3
23. <i>Coma Berenicens</i> , the Hair of <i>Berenices</i> ,	11

In all, 383

PART II,

A a 2

Besides

Besides a Constellation of Stars in a Crowned Heart added by Sir Charles Scarborough, in memory of King Charles the Martyr.

Southern Constellations.

	Stars.
1. Cetus, the Whale,	26
2. Orion, the Warrior,	39
3. Flumen Eridanus, the River,	43
4. Lepus, the Hare,	13
5. Canis Major, the Great Dog,	13
6. Canicula, the Little Dog,	2
7. Argo Navis, the Ship,	41
8. Robur Carolinum, the Oak,	11
9. Hydra, the Snake,	27
10. Crater, the Cup,	8
11. Corvus, the Crow,	7
12. Centaurus, the Bow-man,	42
13. Lupus, the Wolf,	22
14. The Crossiers, or the Cross,	5
15. Ara, the Altar,	8
16. Corona Austrina, the Southern Crown,	10
17. Columba, the Dove,	10
18. Piscis Austrinus, the Southern Fish,	11
19. Grus, the Crane,	12
20. Phoenix,	12
21. Indus, the Indian,	12
22. Pavo, the Peacock,	16
23. Aries Indica Toucan,	7
24. Apis Musca, the Fly,	3
25. Chameleon,	10
26. Triangulum Auster, the South Triangle,	5
27. Piscis volans, the Flying Fish,	7
28. Dorado,	7
29. Anser Americanus, the Indian Fowl,	10
30. Hydrus, Serpens Austrica, the Southern Serpent,	10

In all 439

So that in these Sixty six Constellations are contained 1136 Stars, besides many more that are unformed, or out of these Constellations, making in all 1378 Stars.

Note, Frederico Hautmanns, inhabiting on the Island *Sumatra*, being accommodated with the Noble *Tycho's* Instruments, observed the Latitudes and Longitudes of these Constellations, before named, *viz.*

1. The Crane; 2. The Phenix; 3. The Indian; 4. The Peacock; 5. The Bird of Paradise; 6. The Fly; 7. The Camelion; 8. The South Triangle; 9. The Flying Fish; 10. The Dorado; 11. The Indian Fowl; and 12. The Southern Serpent.

These Southern Stars have been observed and rectified by the ingenious *Dr. Edmund Halley*, Astronomer-Royal.

The Magnitudes of these Stars are express'd in a little Table on the Globe in several Shapes, *viz.* into six Degrees of Magnitude, or Bigness; the biggest and brightest are termed Stars of the First Magnitude; those next inferiour in Bigness, are Stars of the Second Magnitude, &c. unto Stars of the Sixth Magnitude.

And as touching the Reasons that induced the Ancients to bring the Stars into these divers Shapes and Figures, they are generally allowed to be these, *viz.*

1. To express some Property of the Stars; as that of the Bear, to infer Cruelty; the Dragon, Scorpion and Serpent signifies Poison; *Andromeda* chained signifieth Imprisonment, the Head of *Medusa* cut off betokens the Loss of that Member; the Ram, to be hot and dry; *Orion*, with his terrible and threatening Gesture, importeth Tempests and terrible Weather, &c.

2. Some of the Stars do in some sort (though not precisely) represent such Figures as are assigned to them. For instance; the North and South Crowns the Scorpion the Triangle, the Great and Little Bear, and the Dragon do partly represent the Figures they bear the Names of.

3. Some Figures were placed in Heaven (as is said before) for the Continuance of the Memory of some notable Men, who in regard of their worthy Deeds, had deserved of Mankind; as, *Perseus*, *Hercules*, &c.

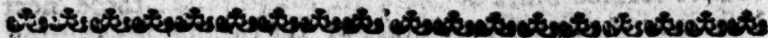
'Tis uncertain who was the First Author of these Constellations: We had them from *Ptolemy*, and he received them from the *Platonicks*; therefore they are of great Antiquity.

Thyſtus, the Brother of *Atreus*, is reported to have invented the Constellation called the *Ram*.

In *Job* 38th mention is made of the *Pleiades* (i. e. the Seven Stars, so called) *Orion*, *Arcturus* and *Mazzaroth*, which most do interpret the Twelve Signs.

Job is suppos'd to have lived about the time of *Abraham* as *Sideracutus* maketh mention in his Book *de commentariis Locorum distantis*.

The Poets have invented strange conceited Stories of these Constellations (as you may read at the Close of *Morden's Treatise of the Globes*) that by such Fictions Men might be the more in love with Astronomy.



Of the Via Lactea, or Milky Way.

VIA Lactea, or *Circulus Lacteus*, so called by the *Latines*, and by the *Greeks* *Galaxia*, and by the *English*, the *Milky Way*.

This is a broad white Circle, which is Visible in the Heavens in the North Hemisphere. It begins at *Cancer*, on each side the Head thereof, and passeth by *Auriga*, *Perseus* and *Cassiopeia*, the Swan, and the Head of *Capricorn*, the Tail of *Scorpio*, and the Feet of *Centaur*, *Argo*, the Ship, and so to the Head of *Cancer*.

As touching the Place, Matter, and efficient Cause thereof, there are sundry Opinions among the Ancient Philosophers.

But to wave those, our Moderns, by Telescope-Observations, do find this white Circle in the Heavens (which is described between two Tracks of small Pricks on the Globe) is caused by a great number of small Stars constipated in that part of the Heavens, so very small, and close together that the bare Eye can perceive nothing but a confused Light.

Also

Also about the South Pole are seen two white Spots, seeming to be two white Clouds (called by the Mariners *Nubeculae Magellanae*, or the *Magellan Clouds*) is a Number of pale very small Stars, as in the Milky Way.

Stella nebulosa is nothing but a Cluster of small Stars, as those of the Manger, called *Præsepe* in the Crab, consisting of more than Forty Stars: And the *Nebulosa* of *Orion's* Head, of Twenty one small Stars, &c.

The Fixed Stars do not at all alter their Latitude; 'because the Motion of the Equinoctial is made upon the Poles of the Ecliptick, from West to East.

But the Stars do not keep the same Longitude, they being fixed in their own Sphere, like Knots in Wood; therefore move not, but are by the *Præcession* of the Equinox left behind the Equinoctial Colure, and so are caused to alter their Longitude; as, by comparing the Observations of Ancient and Modern Astronomers together, 'twill appear: For about 346 Years before Christ, the First Star in the Ram's Head was by the *Egyptian* and *Gæcian* Astronomers observed to be in the Equinoctial Colure: And 114 Years ago, when the Noble *Tycho* observed, it was found to be in $27^{\circ} 37'$ of *Aries*. So that in about 2000 Years it is mov'd forward 28 Degrees, and will, according to *Tycho's* Opinion, finish its Revolution in 25412 Years. According to which Motion was the following Table calculated: By which you may find the Degrees and Minutes of the Equinoctial Motion, answerable to any Number of Years within the said Revolution.

Rut our Modern Astronomers do allow the Increment of the Stars Longitude $50''$ per Annum.

The TABLE.

<i>Years.</i>	<i>Deg. Min.</i>	
1	0	0 $\frac{3}{4}$
2	0	1 $\frac{1}{2}$
3	0	2 $\frac{1}{2}$
4	0	3 $\frac{1}{4}$
5	0	4 $\frac{3}{4}$
6	0	5
7	0	6 $\frac{1}{4}$
8	0	7 $\frac{1}{4}$
9	0	8 $\frac{1}{4}$
10	0	9 $\frac{1}{2}$
20	0	17
40	0	34
60	0	51
80	1	08
100	1	25
200	2	50
300	4	15
400	5	40
500	7	05
1000	14	10
2000	28	20
3000	42	30
4000	56	40
5000	70	50
10000	141	40
20000	283	20
25000	354	10
25412	360	00

The

The use of which Table is for finding the Equinoctial Position of any Star for any Year either past, present, or to come : For if you would know the Motion of the Equinox for any Number of Years, seek the said Number of Years in the first Column, and against it you have the Degrees and Minutes of the Equinoctial Motion.

But our Modern Astronomers do find the Stars increasing in Longitude about 50 Seconds *per Annum*, and make a Revolution in about 7800 Years, caused by the Præcession of the Equinox, as aforesaid ; from which they used to account their Longitude, and not by any proper Motion of their own.

Yet several are of opinion, that many, if not most of the smaller Stars do move somewhat, change their Situation and Magnitude : nay, some have been found actually to have their Revolutions of appearing and disappearing in exact spaces of time.

As for instance ; one in the Swan's Breast hath a Revolution of about Twenty Years ; another in the Neck of the Whale, which sometimes appears of the third Magnitude, the Period of whose Revolution from its greatest Appearance to the least, is finished in about Three hundred thirty three Days ; but from the exact time of its appearing first, of the sixth Magnitude, to the Day it comes again to the same Magnitude, is about an Hundred and twenty Days ; and the time of its Duration at its greatest Magnitude, is about Fifteen Days.

Another Star in *Andromeda's* Girdle, as also one among the Unformed, a little beneath the Swan's Head, whose Longitude is $1^{\circ} 52' 26''$, and Latitude $47^{\circ} 25' 22''$ North, do finish their Revolution, Visible and Invisible, increasing and decreasing, as that in the Whale's Neck ; only that makes its Revolution in Eleven Months ; but this in Ten.

In this 'tis observable, that it riseth to its greatest Splendor (*viz.* to that of the Third Magnitude, and falls from it, and rises again to it in about Thirty Days time ; which is looked upon to be a very strange Phenomenon.

And since the Year 1668, there are missing two Stars in the Poop of *Argos* the Ship, near the Great Dog.

Lastly,

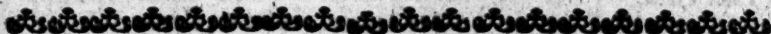
Lastly, There are more than an Hundred Stars that are found to be thus movable, by accurate Observations which have been made of them since the Year 1664. This makes some of Opinion, that Stars newly appearing are the weaker Intention of Nature ; and that when they disappear, they suffer a Dissolution. But Time hereafter must make this manifest. *Et videbitis Cælum apertum.*

I shall conclude this Section about the Fixed Stars, with the Constellations, as expressed in Verse by Captain S. Sturmy, in his *Magazine*, First or Oldest Edition, viz.

Job 26. 7. 13. He stretcheth out the North over the empty Place, and hangeth the Earth upon nothing. By his Spirit he hath garnished the Heavens, &c.

So Psal. 19. 1. The Heavens declare the Glory of God, and the Firmament sheweth his handy Work.

The Army of the Starry Sky
Declares the Glory of God most High ;
Seen and perceiv'd of every Nation
In eight and forty Constellations.



Northern Constellations.

FIRST, Near unto the Northern Pole
The Dragon and two Bears do rowl,
Whose hinder Parts and Tail contain
The Greater and the Lesser Wain,
The Hare, the Bear-ward, and the Crown,
And thence comes *Heracles* kneeling down.
Then next below, a Place doth take
Great *Serpentarius*, with the Snake.
Under the Harp of *Orpheus*
The Eagle and *Antionus*
The Silver Swan her Wings doth spread
Above the Dart and Dolphin's Head.
Then *Pegasus* comes on a-main ;
Andromeda follows in her Chain,
The Triangle below her stands,
And at her Feet in *Perseus* Hands.

The

The Georgian Head : above are seen
Her Parents, *Cephus* with his Queen
Cassiopeia, not far below
Hemicubus his Goat doth shew :
On his Left Shoulder, in his Hand
He doth the stormy Kids command.

Zodiacal Constellations.

Here in the Zodiack begins
The Ram (γ) the Bull (τ) and (II) loving Twins,
The Crab (♋) the Lion (♌) and (♍) Virgin tender,
The Ballance (♎) *Scorpio* (♏) *Sagitta* (♐) and Bow tender.
The Goat (♑) Waterman (♒) then (♓) Fishes twain,
Shall bring you round to the Ram (γ) again.

Southern Constellations.

Fifteen Images there do appear
In the Southern Hemisphere ;
The monstrous Whale above the rest,
Eridanus too wets his Breast
Over the Hare *Orion* bright
Sparkles in a cold Winter's Night.
Then comes the Great Dog, at whose Tail
The Famous *Argo* spreads her Sail.
Above, the Little Dog doth flame,
For whom the *Latines* had no Name.
Long *Hydra* on her Tail allow
Carries the Pitcher and the Crow.
The Centaur holds the Wolf by th' Heel,
The *Alter* and old *Ixion's* Wheel
Are never seen of us ; but here.
The Southern Fish brings up the Rear.

The Planets *Saturn*, *Jupiter*, *Mars*, *Venus*, and *Mercury*
may easily be known from the Fixed Stars by their not
twinkling. 2^{dly}, By their great and visible Light ; *Sa-*
turn, for that he is of a dull Leaden Colour ; *Jupiter*,
because of Splendidness and Azure Colour, being a very
large and bright Star ; *Mars*, by his red and fiery Ap-
pearance ; *Venus*, by reason of her bright Body ap-
pearing brighter than any Star in the Firmament. She
is

is seen always a little before the Sun rises, or else presently after Sun-setting; from whence she is called the Morning or Evening-Star.

Some Astronomical Paradoxes propos'd and answer'd.

1. **T**HERE is a certain Place on the the Continent of Europe, where if several of the ablest Astronomers (the World now affords) should nicely observe all the Celestial Bodies, and that at the same Instant of time; yet the Planetary Phases, and their various Aspects would be really different to each of them.

Answer. Neither at the Center, nor any Part of the Earth, can any one observe all the Celestial Bodies at one and the same Instant of time.

2. There is a Place in Great Britain, where an Astronomer may behold the Stars in the Day-time, the Sun shining, if Clouds interpose not.

Answer. In any deep Well, or Coal-pit, from the Bottom, if the Shaft be streight, and there happen to be Stars of any considerable Magnitude in, or near the Zenith, you may in a Minute or two, by Redfastly looking up, discover them. Some Astronomers have Wells for that purpose.

3. There are divers remarkable Places upon the Terrestrial Globe, whose sensible Horizon is commonly Fair and Serene; and yet 'tis impossible to distinguish properly in it any one of the intermediate Points of the Compass; nay, not so much as Two of the Four Cardinal Points themselves,

Answer. Under either of the Poles.

4. There is a certain noted Place in the vast Atlantick Ocean, where a brisk Levant is absolutely the best Wind for a Ship that is to shape a due East Course, and yet she shall still go before the Wind.

Answer. If the Place have a violent Tide or Current, as I have seen my self in the *Straits*, being in the Grand Fleet of *England*, the stronger the *Lavant*, or Easterly Wind blows, the stronger the Currents drive, you in against it.

5. It may be clearly demonstrated by the *Terraqueous Globe*, that it is not above twenty four Hours Sail from the River of *Thames* in *England*, to the City of *Messina* in *Sicily*, at a certain time of the Year, provided there be a brisk North Wind, a light Frigate, and an *Azimuth-Compass*.

Answer. This is meant by the *Artificial Globe* and the *Hand-Index*.

6. There is a certain Place on the *Terrestrial Globb* where the Sun is several times upon the same Point of the *Compass* in the *Forenoon* and in the *Afternoon*.

Answer. If a Ship or Place be on the *Equator*, then observe about the 9th of *March*, or about the 13th of *September*, when the Sun is in the *Equinoctial*, the Sun will be East all the Hours in the *Forenoon*, and West all the Hours in the *Afternoon*.

Many more might be propos'd and answer'd, would Room permit. But my Treatise drawing to a Conclusion, I shall present the Reader with one *Æigma* for his Amusement, viz.

SAY who I am, bright Sailors : You,
Or none, can prove such Paradoxes true ;
As in the subsequent Discourse you'll find,
No Mortal is more constant to his Friend
Than I : And yet on t'other hand, 'tis strange,
There's none more wavering, or more apt to range.
All known Parts of the World I travel o'er,
Tho' a Recluse, who ne'er stir out of Door :
By Sea or Land to ev'ry Coast I come,
Tho' like a Quack, I travel much at home.

To stand on Picket, which the Soldiers dread,
 Enlivens me, who otherwise am dead.
 Hanging's the last which does to some befall;
 But I, unhung, can shape no Course at all:
 Yet soon as hung, I scamper to and fro,
 Looking out sharp quite round me as I go.
 Altho' I have no Eyes, nor can I rest,
 Till I the Object find I fancy best,
 Whom I respect still with my noblest Part,
 Nay, tho' he be too of a stony Heart;
 I am remarkable for Constancy;
 Yet Fickle Mortals learn to rove from me:
 Without Doors, I in Houses am confin'd,
 And tho' I am my self opaque and blind,
 I so enlighten others, that they know
 By me, tho' senseless, where they ought to go.
 I stand divided too tho' whole and sound,
 In Quarters, which tho' old, in new are found.
 Thus I, by flat Absurdities made clear,
 Shall, tho' conceal'd, to Mariners appear.

The Use of the Celestial HEMISPHERES.

THose who have not Conveniency for, nor Ability to purchase Globes, and yet would know the Stars, their Longitudes, Latitudes, Right Ascensions, &c. may attain it with a Pair of Hemispheres; wherein are all the Constellations, and each Star according to its Longitude, Latitude, &c. placed in them, generally made near 10 Inches Diameter, to fold as a Book, or Sea-Chart, in three or four Leaves, and projected on the Plain of the Ecliptick; so that in one Hemisphere (which is as one Chart, or Opening in a Book) you have all the Constellations on the North side of the Ecliptick; and in another, all the Southern.

There are drawn in these Hemispheres, the Equinoctial, Tropicks, Polar Circles, and Poles of the World, &c. The Pole of the Ecliptick is the Center of each Hemisphere; and the Margent going round them is the Ecliptick, being divided into the 12 Signs, and each Sign into 30 Degrees, each Degree being subdivided into Halves and Quarters; and Lines are drawn from the Center (or Pole of the Ecliptick) to the Beginning of each Sign; and on one of those Lines are placed the Degrees of Latitude, and number'd from the Ecliptick with 10, 20, &c. to 90, at the Pole or Center of it.

Example of its Use, to find the Longitude and Latitude of a Star.

1. Stretch out the Silk String (fastned in the Center for that purpose) over the Center of the given Star; and it shews, or cuts the Star's Longitude in the Ecliptick.

2. With a Pair of Compasses take the Distance from the Center of the Star, to the Center of the Hemisphere: Lay that Distance on the Scale of Latitude from the Center of the Hemisphere, and it will shew the Latitude required.

Thus

Thus you'll find the Foot of the Crociera, a Star of the second Magnitude, in the Southern Hemisphere, between *Robur Carolinum* and *Centaurus*, to be in 8 Degrees of *Scorpio*, and Latitude $53^{\circ} 20'$ South.

I shall not enlarge here; but refer you to the Hemispheres themselves, where are Directions for their Use. Only I have given you in Page 288. a small Print of them, projected on the Plain of the Equinoctial, their Centers being the North and South Poles; by which, having the Right Ascension and Declination of a Star, you may, (by the Rule above) find that Star's Longitude, Latitude, &c.

Exitus acta probat, Finis non pugna coronat.



P I N I S

the
sen
of

mi-
se.
of
neir
ch,
tar,
ngi-

ni
ed
ono
Nash
edso
T
on T
ent
sag
kari
of
the
the
the

on of

at

.T.

for

ad-h

at

Ed

Ye

ma

dupe